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THE VOLUME-BASAL AREA LINE

A Study in Forest Mensuration

By

F. C. HUMMEL, M.A., D.Phil.
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LONDON: HER MAJESTY'S STATIONERY OFFICE

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FOREWORD

THIS bulletin is a study in forest mensuration which has arisen out of investigations into the timber content of British woodlands.

It deals primarily with the relationship that exists between the volume of a tree and its sectional area at breast height ; a relationship here called, for convenience, the volume-basal area line. Studies of this relationship have made possible the production of general volume tables and general tariff tables, which facilitate the rapid and accurate estimation of the timber content of certain types of plantations, and are therefore of considerable practical importance to foresters working in the field.

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PREFACE

THE term *volume-basal area line*, in this paper, is used to denote the relationship that exists between the volumes of trees and their sectional areas at breast height.

Part I, which deals with this relationship when the trees are taken from different stands scattered throughout a country, describes the use of the volume-basal area line in the preparation of general volume tables. The recently published general volume tables for coniferous species in Great Britain (Hummel, Irvine and Jeffers 1950 (1), (2), (3); 1951 (1), (2), (3)) are used as examples to illustrate the procedure. At the time when these tables were being compiled Professor Spurr in America also started to work on similar lines and arrived at similar conclusions. In his book (Spurr 1952) he states: 'When many (500 or more) trees have been measured, the general method should be that used in the recent British volume tables', and in another place, 'This method' (i.e. the method used in the British volume tables) 'is much the same as that worked out contemporaneously by the present writer and described in the following chapter'.

Part II of this paper deals with the volume-basal area line within a stand. After a discussion of the errors to which the line is subject, its variation with species, age, site and thinning treatment are examined. The main outcome of this part of the investigation are the *general tariff tables* discussed in chapter VI and reproduced in appendix III. 'General tariff tables' are a series of related volume tables based on breast height girth alone, the particular table applicable to a stand being determined from sample trees or the mean girth or height of the stand. There are several general tariff tables in current use on the Continent of Europe, which are described and discussed in a recent paper by Loetsch (1952). The idea of having general tariff tables is thus not new, but the methods described in the present paper, of preparing and applying general tariff tables, differ from the methods described elsewhere.

Throughout this investigation considerable use has been made of statistical methods, but the details of computations and of statistical analyses have usually been omitted except where these details were considered essential to the argument: they were considered unnecessary in a bulletin on a forestry subject particularly as the statistical methods employed are not advanced and are described in the standard textbooks on statistics.

ABSTRACT

In 1948, it was decided that general volume tables based on breast height girth and total height should be prepared for the more important coniferous species planted as forest crops in Great Britain. Various methods of preparing such tables were tested, and eventually a volume table for Scots pine (*Pinus sylvestris* L.) was prepared from the formula

$$\frac{Y}{H} = a + b X \qquad (5)^*$$

where Y = volume of tree
H = total height of tree
X = basal area of tree (i.e. sectional area at breast heights).
a and *b* are constants.

This formula was, however, found to be unsatisfactory when it was applied to European larch (*Larix decidua* Mill., syn. *L. europea* D.C.). It was then decided to investigate the possibility of using the volume-basal area line as a means of preparing general volume tables. This method depends on the regression of volume on basal area being linear for trees of a given total height. This linearity was tested and confirmed in seven species :

Scots pine (*Pinus sylvestris* L.)
European larch (*Larix decidua* Mill.)
Norway spruce (*Picea abies* (L.) Karst.)
Corsican pine (*Pinus nigra* var. *calabrica* (Loud.) Schneid.
Sitka spruce (*Picea sitchensis* (Bong.) Carr.)
Douglas fir (*Pseudotsuga taxifolia* (Poir.) Rehder.)
Japanese larch (*Larix leptolepis* (Sieb. and Zucc.) Murr.)

This linear regression of volume on basal area within each height class may be expressed by the equation :

$$Y = a + b X \qquad (6)$$

It was also found that the regression constants *a* and regression coefficients *b* in the above equation vary with height ; these further relationships may be expressed by the polynomial equations :

$$a = a_1 + a_2 H + a_3 H^2 + a_4 H^3 + \ldots (7)$$
$$b = b_1 + b_2 H + b_3 H^2 + b_4 H^3 + \ldots (8)$$

By substituting (7) and (8) in (6) a general volume equation is obtained :

$$Y = a_1 + a_2 H + a_3 H^2 + a_4 H^3 + \ldots$$
$$+ b_1 x + b_2 x H + b_3 x H^2 + b_4 x H^3 + \ldots (9)$$

This volume equation (9) is simplified if either equation (7) or (8) or both are linear, and a further simplification may occur if a_1 or b_1, or both these constants, are zero.

A volume table may be calculated direct from equation (9). This is the most objective and mathematically correct method, but if the volume equation turns out to have many terms the computational work may be considerable.

An alternative method of using the volume-basal area line, is to prepare the volume table in four stages as follows :—

1. Within each height class determine the linear regression of *Y* on *X*.

2. Using the values of *b* found in these regressions of *Y* on *X* determine the regression of *b* on *H*.

3. Similarly determine the regression of *a* on *H*.

4. The adjusted values of *b* and *a* obtained in stages (2) and (3) can now be used to construct the volume table, either by direct calculation or by reading the appropriate volumes from the replotted regression lines.

A slight refinement which has proved useful may be interposed between stages (2) and (3) : it is to recalculate the values of *a* in the equations : $Y = a + b X$ by inserting in these equations the adjusted values of *b* obtained in stage (2). These recalculated values of *a* are then used in determining the regression of *a* on *H*. The advantage of this refinement is that, with the recalculated values of *a*, the regression of *a* on *H* follows a more clearly defined trend than if the original *a* values are used.

General volume tables for six species were compiled by this method. In two species all stages were calculated, while in the other four species one or more of the stages were dealt with graphically.

In the six species examined it was found that the regression of *b* on *H* is either linear, or if it is not linear, that the assumption of linearity will not appreciably detract from the precision of the resulting volume table.

The regression of *a* on *H* was more variable. In Norway spruce, Douglas fir and Japanese larch it was a concave curve ; in European larch and Sitka spruce the lower portion of the curve was also concave but there was a change in the direction

* This and succeeding bracketed numbers are formula reference numbers, as used in main text.

of the curvature, the upper part of the curve being convex, and in Corsican pine all the *a* values were around zero. The recalculations of *a*, by inserting in the equations : Y=a+bX, the values of *b* obtained from the linear regression of *b* on *H*, removed the point of inflection in the case of European larch. A similar recalculation of *a* in Corsican pine gave a regression of *a* on *H* resembling the original curve of *a* on *H* in European larch, i.e. a curve with a point of inflection. The general rule thus seems to be that the regression of *a* on *H* takes a concave form in the lower height classes, and that near the upper limit of the height range there may, in some species, be a point of inflection.

Thus a study of the material examined suggests that the form which the general volume equation (9) will normally take is likely to be :

$$Y = a_1 + a_2H + a_3H^2 + b_1X + b_2XH \qquad (9b)$$

but there may be additional terms with a_4H^3 and a_5H^4.

The question arises, whether it was preferable to prepare the volume tables by the method described, or whether it would have been better to prepare the tables direct from equation (9b), with a probable minimum of 5 terms and a probable maximum of 7 terms. Direct calculation would have been quite impracticable on account of the coding required, unless the data had been summarised into very broad girth groups with a resulting loss in precision. Had this been done, it is difficult to say whether or not there would have been a saving in time. The advantage would have been complete objectivity, the main disadvantage, that the insight into the data given by the various graphs would have been lost. It is also impossible to say whether anything was gained by not depending entirely on graphical solutions, once the regressions Y = a + b X in each height class had been calculated. It would appear, however, that when the trends of the regressions of *a* and *b* on *H* are as clearly defined as they were in European larch, Norway spruce, Douglas fir and Japanese larch, little is to be gained by calculating these regressions or by recalculating the values of *a* from the adjusted values of *b*. If, however, there is some doubt about the trends, it appears best to assume the regression of *b* on *H* to be linear and to recalculate the values of *a* accordingly.

Part II deals with the volume-basal area line within a stand, and more particularly, within even-aged coniferous stands of a single species. It has been known for some time that this relationship is usually linear or nearly linear. The important difference between the volume-basal area line relating to heterogeneous data, as described in Part I, and the line derived from data relating to one stand, is that, with heterogeneous data, the regression of volume on basal area is linear for a given height, while if all the trees are from a single stand, the regression is usually linear irrespective of height.

The object of the present investigation was to find out how the volume-basal area line within a stand changes with species, site, age and thinning treatment.

The material for studying these changes was provided by the permanent sample plot records of the Forestry Commission. These plots are normally thinned and remeasured at intervals of three to six years. At each remeasurement, the volumes of about eight sample trees are determined. These sample trees are distributed over the range of girth in the plot, but the method of selection is subjective*, the aim being to choose trees which, in stem form, height and taper appear to be 'representative' of the crop at the time of measurement. No attempt is made to select the identical sample trees at successive remeasurements. The regression of volume on basal area is estimated from these sample trees. Up to a few years ago this was done graphically ; more recently the regression has been calculated from the equation :

$$Y = a + b X$$

where Y = volume of tree
X = basal area of tree
a and *b* are constants.

Errors in the estimates of the regression constant *a* or the regression coefficient *b* or of both these factors, may be caused by :

(i) errors in the actual measurements—these are known to be too small to be of consequence.
(ii) errors due to drawing the regression line by eye instead of calculating it. This source of error was examined on eighty volume-basal area lines where the regression had been calculated as well as having been drawn by eye. With one or two exceptions the differences between the lines drawn by eye and the calculated lines were very small.
(iii) errors arising from the fact that the regression is estimated from a sample of trees instead of being determined from all trees. This source of error was found to be the most important.

In addition to the normal sampling errors, there is also the possibility of bias because of the subjective selection of the sample trees, but there was no evidence of any such bias in four plots where a complete measurement of all trees enabled this point to be examined.

* A more objective method of selecting sample trees has since been introduced.

It was found that with about eight sample trees, the total errors of *a* and *b* from all three sources may lead to maximum errors of between five and ten per cent in the estimate of total volume in a plot, and up to twice that amount in the volume estimates for the extreme girth classes.

In studying the changes of the volume-basal area line : $Y = a + bX$, the regression coefficient *b* was considered first. For each of the seven species dealt with in this investigation, the value of *b* in each plot at each measurement was plotted over top height. The successive values of *b* in each plot were connected by straight lines, the thinning treatments being differentiated by the type of line and symbols used. A study of these graphs led to the following observations :

(i) *b* is not affected by thinning treatment. This suggests that *b* is probably not very closely correlated with girth, because girth is known to be greater in plots that have been thinned heavily over a period of years than in plots that have been thinned lightly.

(ii) The regression of *b* on height appears to be linear to a top height of eighty feet, above which the scatter of the points is too great to indicate the trend very clearly.

(iii) The scatter of individual values of *b* from the mean value for a species at a particular height appears to be due mainly to the errors in determining *b* rather than to any variation in *b* with site. Nevertheless, there are a few plots with consistently high or low values of *b* at consecutive remeasurements, which suggests that, in these particular stands, *b* does differ genuinely from what it is elsewhere. In some species, the scatter of the points is greater than in others. It is low in Sitka spruce and Corsican pine, and great in Douglas fir.

(iv) At any given height, the average value of *b* is similar in all the species examined, and it is not possible to say whether such differences between species as are evident on the graphs are genuine or whether they are due solely to random variation between sites within species and between trees within sites.

Instead of studying the changes of *a* with top height it was found preferable to examine the changes of z', z' being the point on the volume-basal area line at which : $Y = o$. The relationship between *a* and z' is given by the equation :

$$z' = \frac{-a}{b}.$$

The reason for preferring to work with z' instead of *a* was because a preliminary inspection of the data suggested that, while *a* normally decreases with top height, z' remains constant. Therefore z' was plotted over top height in the same way as has been explained for *b*. From a study of these graphs of z' over top height it was apparent that, up to a top height of about 70 feet, the average value of z' remains constant at about 0·03 square feet in all species, on all sites, and under all thinning treatments covered by the data ; and that there are very few stands where z' has deviated appreciably from this mean value of 0·03 square feet. Above a top height of 70 feet, z' becomes more variable, and although it has remained more or less constant in the majority of plots, there are some plots, mainly in Douglas fir and Sitka spruce, where z' differs from 0·03 square feet by a greater amount than could safely be ascribed to error.

These findings relating to the regressions of *b* and z' on top height have been made use of in the preparation of the so-called *general tariff tables* which are given in Appendix III, and which are designed to provide a new and simple method of estimating the volume of standing timber.

These general tariff tables represent the tabulated values of a series of volume-basal area lines ; these have in common the point : $z = 0{\cdot}03$ square feet and the values of *b* are chosen so that the volume interval between successive lines is one hoppus foot at a basal area of one square foot (i.e. 12 inches breast height quarter girth).* Thus, table 18 shows a volume of 18 hoppus feet and table 19 a volume of 19 hoppus feet at a B.H.Q.G. of 12 inches. The table appropriate to a particular stand is then determined from the volumes of sample trees.

The main advantages and limitations of the general tariff tables as a means of estimating the volumes of standing trees are as follows :

Advantages

(i) An estimate is obtained not only of the total volume but also of its distribution by breast height girth classes.

(ii) There is no need to choose the sample trees from any particular girth classes although it is advisable to adopt a sampling procedure which is objective (e.g. every tree in every *n*th row) and will ensure that the sample trees are distributed over the whole range of girth in the stand or plot. If this is done, then any departure of *z* from 0·03 square feet will cause no great error in the total volume estimate, because an underestimate in volume at one end of the girth range will be countered, although not necessarily exactly balanced, by an over-estimate at the opposite end.

(iii) There is no need to calculate the mean basal area of the stand or of the sample trees.

* For definition of terms, see Appendix I, page 58.

(iv) The precision of the volume estimate may be estimated without much difficulty provided that a suitable method of sampling is adopted.

(v) The method may help in overcoming some of the difficulties encountered in applying the Mêthode du Contrôle to even-aged high forest.

Limitations

(i) The method is not quite as easy to understand as some of the 'mean sample tree' methods. This is an important consideration when the volume is to be determined by junior personnel in the field.

(ii) Whether or not the general tariff tables are applicable under conditions other than those covered by this investigation is not yet known. It seems probable, however, that the tables may safely be applied to all coniferous species in Great Britain, provided that the top height of the stand is not more than 80 feet ; and preliminary tests suggest that these tables may also be used in young hardwood stands with mean breast height quarter girths up to about 6 inches, but not to old hardwood stands where, in the few stands examined, z' departed very markedly from 0·03 square feet. It may be possible to prepare a different set of general tariff tables for old hardwood stands ; this is a problem which requires further study.

PART I. THE VOLUME-BASAL AREA LINE FOR HETEROGENEOUS TREE POPULATIONS

Chapter 1

TRIALS OF VARIOUS METHODS OF PREPARING GENERAL VOLUME TABLES

WHEN it had been decided to prepare general volume tables for the more important coniferous species planted in Great Britain the question arose as to which was the most suitable method to adopt.

The choice of method was limited in the first place by the fact that the tables were to be based on breast height girth and total height alone. This eliminated procedures applicable when form or taper are also taken into account. The main reason for using two characters instead of three was the desire to keep the tables as simple as possible. They are intended mainly to assist in preparing estimates of growing stock for purposes of forest management. A high degree of accuracy is usually not essential, nor would it be obtainable in practice, because the majority of foresters and landowners who have to make such estimates have neither the time nor the training in mensurational theory to use elaborate methods.

The choice of method was further restricted by the nature of the available data. These came from three main sources :

1. Temporary sample plots established mainly between 1917 and 1919 for the purpose of preparing the yield tables published in Forestry Commission Bulletin No. 3 (1920).

2. Measurements taken in the course of fellings during the first and second world wars. These measurements were carried out by the timber supply Departments, the Forestry Commission, and the University research parties who kindly permitted the use of their data.

3. The permanent sample plot records of the Forestry Commission.

The particulars available for each tree were as follows :

(i) Breast-height quarter girth (abbreviated as B.H.Q.G.) measured at 4 ft. 3 inches above ground level, on the upper side of the tree on slopes.

(ii) Total height, measured to the nearest foot and in some instances to the nearest half foot.

(iii) Hoppus volume over bark calculated from the mid quarter girth and the stem length from ground level to the point where the over bark diameter of the stem is 3 inches. The whole length was thus treated as one section and no allowance was made for the stump.

Branch wood of all dimensions was ignored.

The above conventions of measurement apply to all the figures quoted in Part I of this paper unless a specific statement to the contrary is made.

In addition to the above measurements, there were measurements of girth at 10 foot intervals up the stem for those trees which had been used as sample trees in the permanent sample plots ; but, as most of these permanent sample plots are still young, there are few records of taper for large trees. Owing to this scarcity of information on taper, the general volume tables could not have been prepared by von Wülfing's method (1949) or by any other method requiring detailed data on taper.

The graphical method described by Chapman and Demeritt (1936) was rejected as being too subjective and also because it has other disadvantages which are mentioned by Bruce and Schumacher (1950). It was then decided to try a method recommended by the latter authors, which is based on the equation :

$$Y = G^a H^b c \qquad (1)$$

where Y = Volume of tree,
G = breast height girth,
H = total height of tree,
a, b, c are constants.

In order to calculate a volume table from this equation it is necessary to transform it into its logarithmic form :

$$\log Y = k + a (\log G) + b (\text{Log } H) \; ; \qquad \textbf{(1a)}$$

where $k = \log c$.

TABLE 1

SCOTS PINE (SCOTLAND) : COMPARISON OF ACTUAL DATA WITH THOSE CALCULATED FROM EQUATION (1b)

True girth at breast height inches (1)	Total height in feet (2)	Calculated volume hoppus feet (3)	Actual volume hoppus feet (4)	Number of trees (5)	Number of trees × difference + (6)	− (7)
11·5	23·5	·52	·45	36	2·52	
12	28·5	·69	·60	39	3·51	
12·5	37	·98	·88	18	1·80	
15	24·5	·98	1·07	7		·63
16	28·5	1·32	1·37	72		3·60
16	39	1·79	1·87	60		4·80
16·5	47	2·29	2·34	13		·65
19	30·5	2·06	2·22	28		4·48
19·5	42·5	3·02	3·14	69		8·28
20·5	48	3·80	3·99	44		8·36
20	56·5	4·21	4·88	2		1·34
23	42·5	4·36	4·46	39		3·90
24	49·5	5·56	5·64	71		5·68
24	57·5	6·43	6·71	11		3·08
27	42·5	6·24	6·07	7	1·19	
27·5	49·5	7·53	7·53	60	—	—
28	57·5	9·07	8·97	25	2·50	
26·5	66	9·16	9·64	2		·96
31·5	52	10·70	9·80	25	22·50	
32	58	12·31	11·54	30	23·10	
31·5	66·5	13·58	13·58	3	—	—
34·5	52	13·10	11·65	3	4·35	
35	59·5	15·43	14·50	23	21·39	
35·5	68	18·11	15·42	4	10·76	
39	59·5	19·63	16·86	7	19·39	
40	67	23·32	19·62	3	11·10	
44	63	27·17	23·31	2	7·72	

A volume table was prepared by this method from the 703 Scots pine trees (*Pinus sylvestris* L.) which had been measured in the course of the past thirty years as sample trees in the permanent Scots pine sample plots in Scotland. Twenty-eight sample plots are represented in this material, ranging from quality class I to IV. The breast-height girths had been recorded to the nearest half-inch true girth, and total heights from ground level (no allowance for stump) to the tip of the tree to the nearest half-foot. The data were grouped into 4-inch true girth classes (i.e. 1-inch quarter girth classes) and 10-foot height classes. The mean girth and mean height of the trees in each such group were actually calculated, because these means may differ from the theoretical means of the class, particularly when the number of trees is small. For example, the 16-inch girth and 50-foot height group had an actual mean girth of 16·5 inches and a mean height of 47 feet. These means, together with the mean volume of each group, are shown in the first three columns of Table 1.

Using these data the volume table was calculated from equation (1a).

The regression with its standard errors was found to be :

$$(\log Y - 0{\cdot}5158 \pm 0{\cdot}0013) = (2{\cdot}232 \pm 0{\cdot}019) \times (\log G - 1{\cdot}312) + (0{\cdot}971 \pm 0{\cdot}022) \times (\log H - 1{\cdot}616) \qquad (1b)$$

or converted and without standard errors

$$Y = \frac{G^{2{\cdot}232}\, H^{0{\cdot}971}}{3{\cdot}981}$$

Columns 4 to 7 of Table 1 show that the calculated volumes gave overestimates in the smallest and largest girth classes, and underestimates in the intermediate girths. Taking the data as a whole, the total calculated volume is 2·5 per cent greater than the actual total. The same method was then tried on 1,356 European larch trees (*Larix decidua* Mill.). This material represents all the sample trees in the permanent sample plot records for the whole of Great Britain. The measurements are summarised in the first three columns of Table 2.

The regression calculated from equation (1a) was :

$(\log Y - 0{\cdot}6441 \pm 0{\cdot}0008) = (\dot{2}{\cdot}059 \pm 0{\cdot}013) \times (\log G - 1{\cdot}325) + (1{\cdot}131 \pm 0{\cdot}016) \times (\log H - 1{\cdot}697)$ (1d)

or converted and without standard errors

$$Y = \frac{G^{2.059}\ H^{1.131}}{4{\cdot}004} \qquad (1e)$$

Columns 4 to 7 in Table 2 indicate that in European larch there were discrepancies between calculated volumes and actual volumes similar to those in Scots pine : overestimates in the smallest and largest girths, underestimates in the intermediate girths ; and taking the data as a whole, a considerable positive bias.

There appear to be two main reasons for these unsatisfactory results. First, the regression of (log Y) on the two independent variables (log G) and (log H) is assumed to be linear, but in both the species examined there is a pronounced departure

TABLE 2

EUROPEAN LARCH : COMPARISON OF ACTUAL DATA WITH THOSE CALCULATED FROM EQUATION (1e)

True girth at breast height inches (1)	Total height in feet (2)	Calculated volume hoppus feet (3)	Actual volume hoppus feet (4)	Number of trees (5)	Number of trees × difference + (6)	Number of trees × difference − (7)
11	22·5	·47	·44	9	.27	
11·5	30·5	·72	·60	72	8·64	
12·5	38·5	1·12	1·05	58	4·06	
13	48·5	1·57	1·53	4	·16	
14·5	24	·89	·93	2		·08
15	31	1·27	1·32	37		1·85
16	39·5	1·91	1·93	151		3·02
16·5	49	2·60	2·59	70	·70	
17	56·5	3·24	3·35	7		·77
18	31	1·85	2·14	4		1·16
19·5	41	2·99	3·06	97		6·79
19·5	49·5	3·71	3·96	151		37·75
20	57·5	4·62	4·97	50		17·50
20	66	5·41	5·68	1		·27
23	42·5	4·39	4·38	27	·27	
23·5	50	5·51	5·69	125		22·50
24	58·5	6·87	7·00	100		13·00
24·5	67·5	8·41	8·50	10		·90
27·5	52	7·96	7·77	41	7·79	
27·5	59·5	9·27	9·41	99		13·86
27·5	68·5	10·86	11·09	21		4·83
29	77·5	13·93	13·54	7	2·73	
31	54·5	10·74	10·66	2	·16	
31·5	61	12·59	12·11	52	24·96	
31·5	67·5	14·13	13·74	24	9·36	
32	78·5	17·30	16·29	29	29·29	
30·5	87	17·62	16·73	1	·89	
35	62	15·92	15·55	7	2·59	
35·5	68	18·24	16·78	12	17·52	
35·5	80·5	22·03	20·71	28	36·96	
35	85	22·75	20·22	3	7·59	
38	64·5	19·77	18·72	1	1·05	
38·5	72	23·01	20·48	2	5·06	
39·5	81·5	27·86	25·40	24	59·04	
40·5	86	31·19	30·26	4	3.72	
43·5	82	34·28	31·68	10	26·00	
44	87	37·50	32·75	8	38·00	
47·5	82	41·02	33·87	1	7·15	
47·5	86·5	43·55	38·09	3	16·38	
51·5	88	52·48	47·18	2	10·60	

from linearity. By adding a quadratic term to the equation, it is probable that a volume table could be produced which follows the trend of the data more closely, but this addition would greatly increase the computational work, which is very considerable even on the assumption of linearity.

Secondly, to quote Sampford (personal communication): 'The overall positive bias results from the fact that positive deviations from the line, for the most part, corresponded to large volumes and negative deviations with small volumes, combined with the fact that in the process of taking antilogarithms an error of given size in the logarithm is increased approximately in proportion to the size of the antilogarithm.

Thus, though the deviations from the line added to zero on the logarithmic scale, the positive deviations are increased proportionately more than the negative deviations on conversion to the scale of true volumes'.

It might perhaps be argued that as volume is three dimensional, the sum of constants (a + b) in the equation $Y = G^aH^bc$ should equal 3, while according to equation (1b) for Scots pine (a + b) = 3·203, and in European larch (a + b) = 3·190. It would be possible to make the equation satisfy the condition (a + b) = 3 by making b = 3 — a, but Spurr (verbal communication) has found in other species that, even if this condition is satisfied, the volume tables calculated from the equation $Y = G^aH^bc$ are not necessarily satisfactory.

The laborious computations, which would have been necessary in order to eliminate the bias and to improve the fit of the equations to the data, were not carried out, because it was thought preferable to search for a simpler method of preparing general volume tables. Hammersley, of the Lectureship in the Design and Analysis of Scientific Experiment at Oxford University, in a personal communication, suggested testing the following formulae:

$$\frac{Y}{G^3} = a + b\left(\frac{H}{G}\right) + c\left(\frac{H}{G}\right)^2 + \ldots \quad (2)$$

$$\frac{Y}{H^3} = a + b\left(\frac{G}{H}\right) + c\left(\frac{G}{H}\right)^2 + \ldots \quad (3)$$

Owing to the amount of computational work involved, both in converting the original data into terms such as $\frac{Y}{H^3}$ and $\frac{G}{H}$, as well as in the calculation of the regressions, only forms of these equations with two terms were tried. It was considered that, if additional terms had to be added, these formulae would be of little practical value. The equations that were actually tested are:

$$\frac{Y}{G^3} = a + b\left(\frac{H}{G}\right) \quad (2a)$$

$$\frac{Y}{H^3} = a + c\left(\frac{G}{H}\right)^2 \quad (3a)$$

Two other formulae, which suggested themselves to the writer at the time and had the advantage of being simpler to handle because they involve less coding, were also tested. They were:

$$\frac{Y}{X} = a + bH \quad (4)$$

$$\frac{Y}{H} = a + bX \quad (5)$$

Where X is the basal area. (In hoppus measure, $X=G^2$)

5,677 Scots pine trees were used to test the usefulness of these formulae for preparing general volume tables. This material includes the 703 trees referred to in Table 1 as well as 4,974 trees from other sources. These other sources consist of data from temporary sample plots and war-time fellings in the whole of Great Britain, and the permanent sample plot records for England and Wales. The number of trees in each girth and height class is given in the published Scots pine volume table (Hummel, Irvine and Jeffers 1950 (1)).

The regressions calculated for the four formulae were as follows:

(2a) $$\frac{Y}{G^3} = a + b\left(\frac{H}{G}\right) \quad \text{or:}$$

$$Y = aG^3 + bG^2H$$

$$Y = \frac{G^2(176{\cdot}861H + 13{\cdot}68212G)}{10^6}$$

(3a) $$\frac{Y}{H^3} = a + c\left(\frac{G}{H}\right) \quad \text{or:}$$

$$Y = aH^3 + cG^2H$$

$$Y = \frac{H(0{\cdot}08320H^2 + 185{\cdot}145G^2)}{10^6}$$

(4) $$\frac{Y}{X} = a + bH \quad \text{or:}$$

$$Y = aX + bXH$$

$$Y = \frac{X(4{\cdot}17464H + 4{\cdot}85044)}{10}$$

(5) $$\frac{Y}{H} = a + bX \quad \text{or:}$$

$$Y = aH + bHX$$

$$Y = \frac{H(425{\cdot}68393X - 0{\cdot}10535)}{10^3}$$

Implied in these formulae are the following assumptions concerning the form factor (F) :

$Y = FXH$; therefore :

(2a) $F = a\left(\dfrac{G}{H}\right) + b$

(3a) $F = a\left(\dfrac{H}{G}\right)^2 + c$

(4) $F = \dfrac{a}{H} + b$

(5) $F = \dfrac{a}{X} + b$

It was expected that the two formulae in which the form factor is made to vary with the ratios of $\left(\dfrac{G}{H}\right)$ or $\left(\dfrac{H}{G}\right)^2$ would be preferable to the other two in which the form factor changes either with height alone or with girth alone ; but this was not confirmed by a test in which a random sample of 100 trees, stratified by girth and height, was taken from the 5,670 trees, and the actual volumes in the sample compared with the volumes calculated by each of the four formulae. The results are as shown in Table 3. Column 1 in this table gives the formulae, column 2 the mean differences (calculated minus actual volumes) expressed as percentages of the actual volumes, and column 3 the standard deviations of the differences between the individual tree volumes and the calculated volumes. The figures in both columns 2 and 3 were obtained by expressing the difference between calculated and actual volume for each tree as a percentage of the actual volume and using these individual percentage figures in the subsequent calculations.

TABLE 3

BIAS AND PRECISION OF SCOTS PINE VOLUME TABLES CALCULATED FROM FORMULAE 2 TO 5

Formula	Difference %	Standard deviation %
(1)	(2)	(3)
(2a) $\dfrac{Y}{G^3} = a + b\left(\dfrac{H}{G}\right)$	−4·30	20·18
(3a) $\dfrac{Y}{H^3} = a + c\left(\dfrac{G}{H}\right)^2$	−3·91	19·87
(4) $\dfrac{Y}{X} = a + b\,H$	−4·25	20·61
(5) $\dfrac{Y}{H} = a + b\,X$	−3·21	19·56

Table 3 shows that, for each formula, the standard deviation of the differences between the individual tree volumes and the calculated values was approximately 19 to 20 per cent of these calculated volumes. Table 3 also indicates that in each case the calculated volumes were at an average 3 to 4 per cent *less* than the actual volumes, a difference which was found to approach significance at the 5 per cent probability level. Inspection of the measurements of the 100 trees constituting the sample revealed that nearly half of this negative bias was due to two trees in the 50 and 60 foot height classes which had quite abnormally low volumes in relation to their height and breast height girth.

The figures in Table 3 were taken to suggest that the precision and bias of volume tables prepared by any of the above formulae might be expected to be similar and that, in choosing between the formulae, it was mainly a matter of weighing the probable theoretical advantages of formulae (2a) and (3a), in which the form factor varies with both height and girth, against the practical advantages of formulae (4) and (5) which involve less coding (avoidance of terms such as $\dfrac{Y}{H^3}$) and therefore less computational effort. Ultimately, it was decided to prepare the volume table from equation (5) which was given preference over (4) because of the slightly smaller bias and standard deviation which it gave in the test.

The volume table thus calculated from the equation $\dfrac{Y}{H} = a + bX$ has been published (Hummel, Irvine and Jeffers 1950 (1)). When tested on 1,900 of the trees on which it was based, the following results were obtained :

Calculated volume	46929·19 hoppus feet	
Actual volume . .	46332·47 ,, ,,	
Difference . .	596·72 hoppus feet	1·29%
Mean difference per tree . .	0·31 ,, ,,	

This volume table was also tested on the sixty-five permanent Scots pine sample plots of the Forestry Commission. In each of these, the volume corresponding to the mean girth and mean height of the plot was obtained from the volume table and multiplied by the number of trees in the plot. The aggregate volume thus calculated for the sixty-five plots was 1·08 per cent less than the volume determined by direct measurement according to the Forestry Commission sample plot procedure. Of the sixty-five plots, thirty-seven had a stocking of over 1,000 hoppus feet per acre. In thirty of these thirty-seven the volumes estimated from the volume tables were within 10 per cent, and in the remaining seven plots between 10 and 14 per cent, of the measured volumes. In the twenty-eight plots with a stocking of less than 1,000 hoppus feet, the estimates were less precise. In fifteen plots they were within 10 per cent, in nine they were between 10 and 20 per cent, and, in the remaining four plots, between 20 and 27 per cent of the measured volumes.

The formula used for the Scots pine volume table, i.e. $\frac{Y}{H} = a + bX$, was then also used for preparing a volume table from 4,903 trees of European larch, but the results were less satisfactory than in the case of Scots pine. The standard deviation of individual tree volumes from the calculated values was 24·6 per cent, compared with 19·6 per cent in Scots pine; in contrast to that species, there was a pronounced bias in the smaller height classes. Similarly disappointing results were obtained from the formula $\frac{Y}{X} = a + bH$.

It was after the failure of these two formulae in the case of European larch that the writer decided to investigate the possibility of using the volume-basal area line as a means of preparing general volume tables.

Chapter 2

THEORETICAL BACKGROUND TO THE PREPARATION OF GENERAL VOLUME TABLES FROM THE VOLUME-BASAL AREA LINE

THE method of preparing volume tables, which is described in this chapter, depends on the regression of volume on basal area being linear for trees of a given total height. This linearity was tested on the following data:

Species	*Number of trees*
Scots pine	5,677
European larch	4,903
Norway spruce	2,617
Corsican pine	1,108
Sitka spruce	946
Douglas fir	1,472
Japanese larch	1,389

The data were summarised by 10-foot height classes and 1-inch breast-height girth classes (i.e. ¼ inch quarter girth). Within each 10-foot height class, the mean volume of each 1-inch girth class was plotted over its mean basal area. The points for the 30-foot, 50-foot and 70-foot height classes for each of the seven species examined are reproduced in Figures 1 to 7, which also show the calculated linear regressions through these points. The points for the 40-foot and 60-foot height classes have been omitted for the sake of clarity, and the points for the height classes between 70 and 110 feet, which was the maximum height covered by the data, were omitted owing to difficulties of scale. The scale was too large to accommodate these classes, and a smaller scale would have been unsuitable for the smaller height classes.

Figures 1 to 7 illustrate the fact which was observed also in the other height classes not shown in these figures, that *the regression of volume on basal area can be adequately approximated by a straight line within each height class.* The comparatively large deviations of some points from the line near the upper limit of the girth range in each height class is explained by the fact that these points usually represent very few trees. In the middle of the girth range, where each point represents the mean value of a large number of trees, the deviations are smaller. If the volume-basal area line within each height class is linear, then:

$$Y = a + bX \qquad (6)$$

where Y = volume of tree,
X = basal area, and
a = regression constant
b = regression coefficient

It is also evident from Figures 1 to 7 that in each species the regression constants *a* and the regression coefficients *b* in equation (6) vary with height. These relationships may be expressed by the polynomial equations:

$$a = a_1 + a_2 H + a_3H^2 + a_4H^3 + \dots \qquad (7)$$
$$b = b^1 + b_2H + b_3H^2 + b_4H^3 + \dots \qquad (8)$$

By substituting (7) and (8) in (6) a general volume equation is obtained:

$$Y = a_1 + a_2H + a_3H^2 + a_4H^3 + \dots + b_1X + b_2XH + b_3XH^2 + b_4XH^3 + \dots \qquad (9)$$

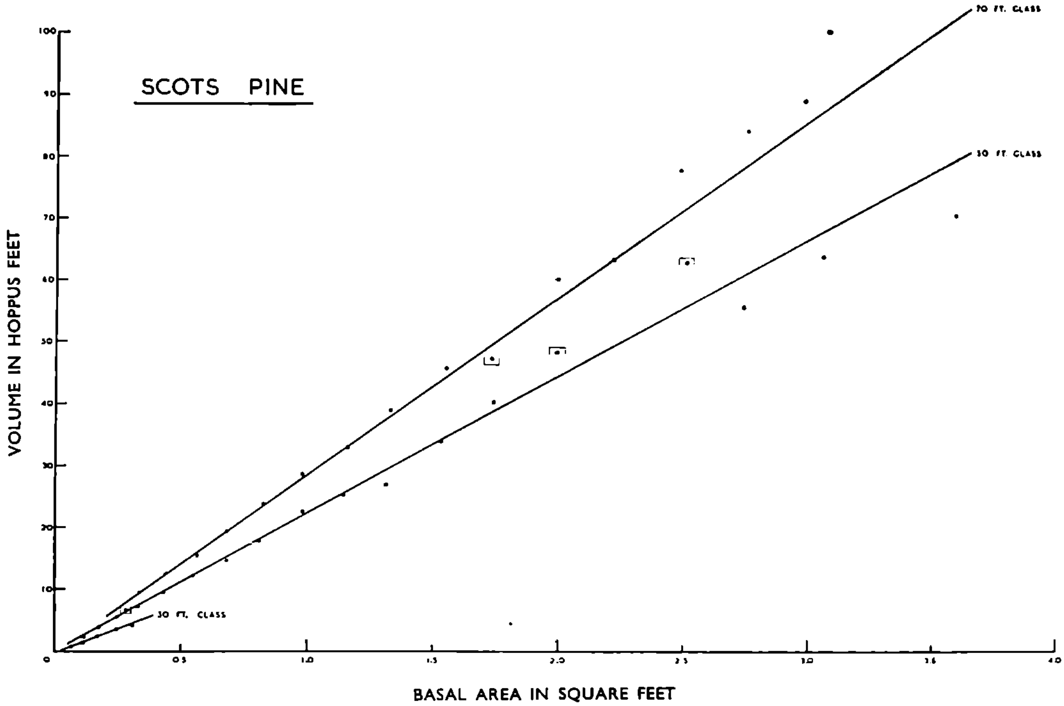

FIGURE 1. Scots pine. Relationship between volume and basal area for the 30, 50 and 70 foot height classes.

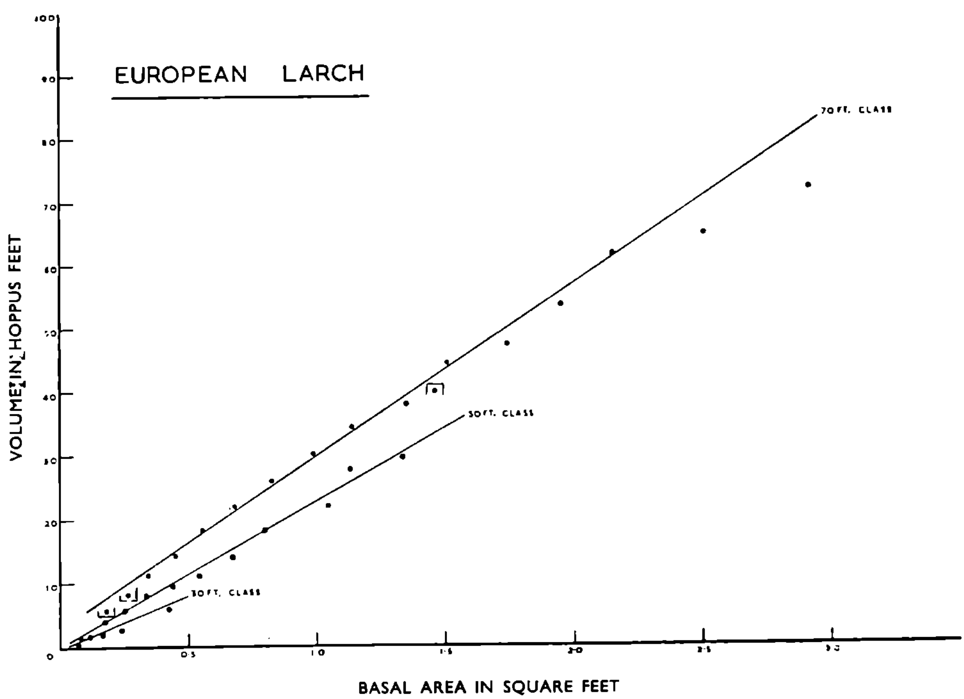

FIGURE 2. European larch. Relationship between volume and basal area for the 30, 50 and 70 foot height classes.

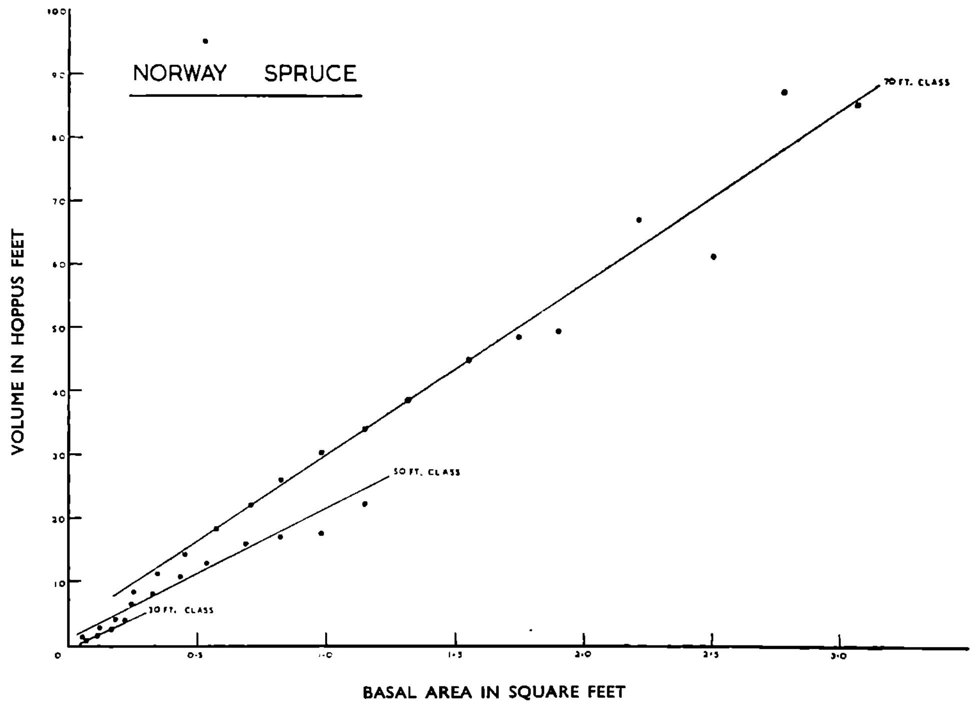

FIGURE 3. Norway spruce. Relationship between volume and basal area for the 30, 50 and 70 foot height classes.

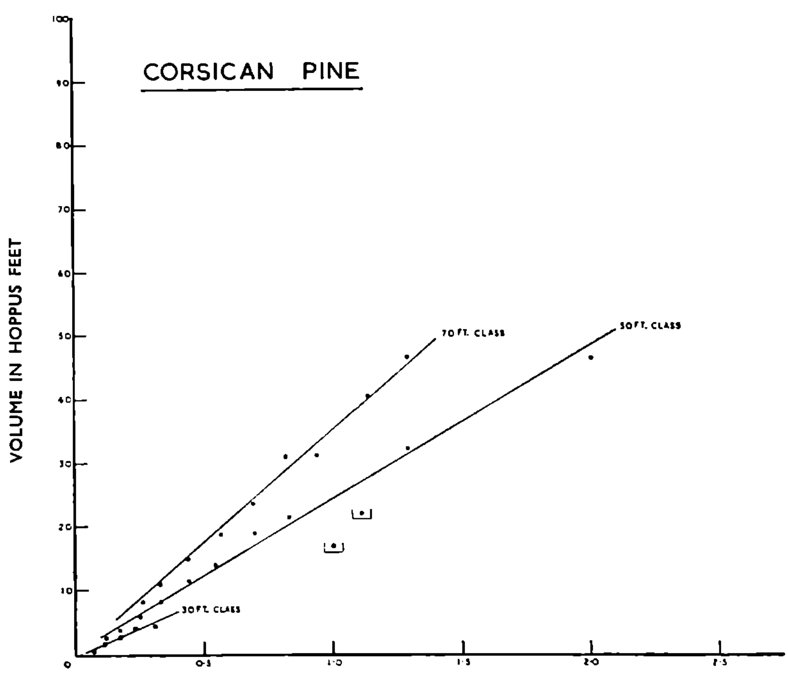

FIGURE 4. Corsican pine. Relationship between volume and basal area for the 30, 50 and 70 foot height classes.

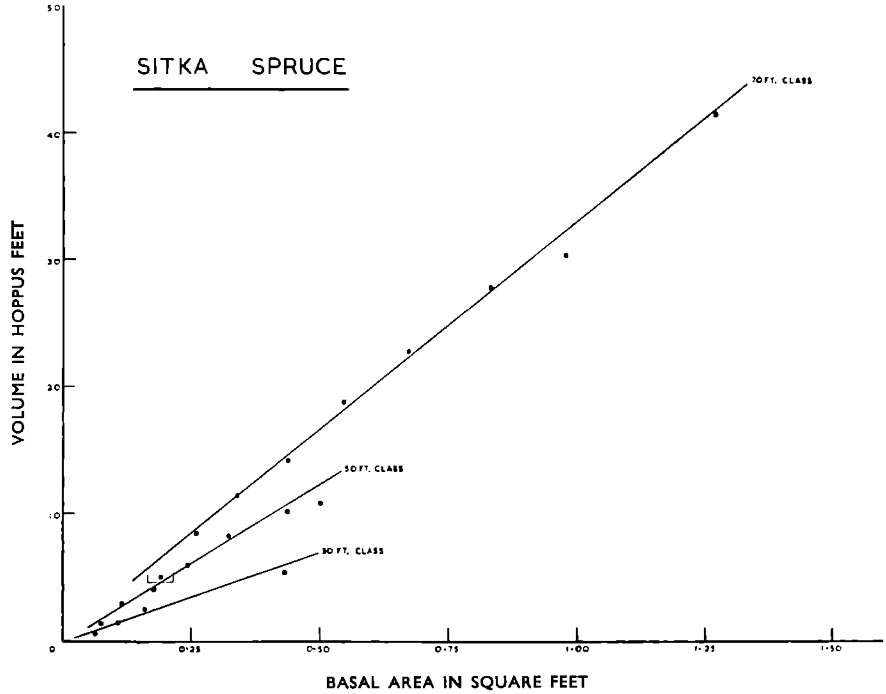

FIGURE 5. Sitka spruce. Relationship between volume and basal area for the 30, 50 and 70 foot height classes.

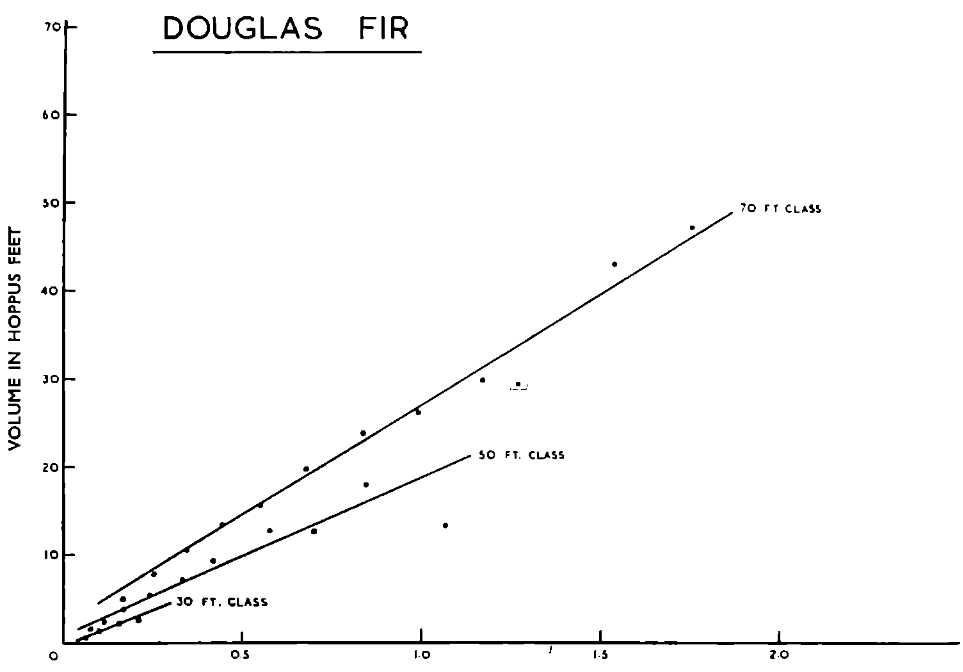

FIGURE 6. Douglas fir. Relationship between volume and basal area for the 30, 50 and 70 foot height classes.

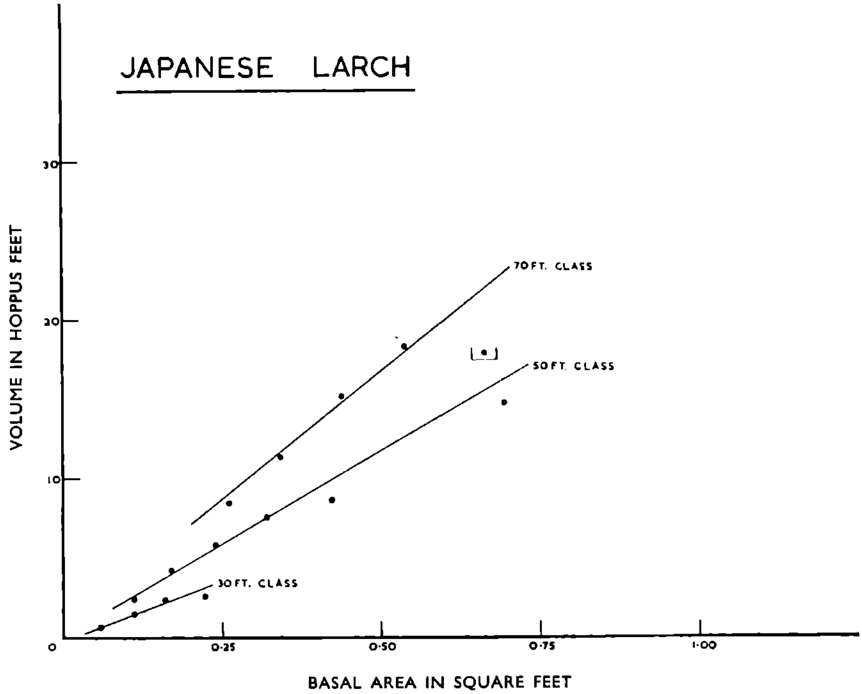

FIGURE 7. Japanese larch. Relationship between volume and basal area for the 30, 50 and 70 foot height classes.

This volume equation (9) is simplified if either equation (7) or (8), or both, are linear, in which case the volume equation becomes :

$$Y = a_1 + a_2H + b_1X + b_2XH + b_3XH^2 + \dots \quad (9a)$$

$$Y = a_1 + a_2H + a_3H^2 + a_4H^3 + \dots + b_1X + b_2XH \quad (9b)$$

$$Y = a_1 + a_2H + b_1X + b_2XH \quad (9c)$$

A further simplification may occur if a_1 or b_1 or both these constants are zero :

$$Y = a_2H + b_1X + b_2XH \quad (9d)$$

$$Y = a_1 + a_2H + b_2XH \quad (9e)$$

$$Y = a_2H + b_2XH \quad (9f)$$

It will be observed that equation (9f) is the one from which the Scots pine volume table was prepared, which is mentioned in Chapter 1 (Hummel, Irvine and Jeffers 1950 (1)).

A volume table may be calculated by starting with equation (9f) and then adding terms until a further addition results in no significant increase in precision. This is the most objective and mathematically correct method but, if the volume equation turns out to be complex, the computational work may be considerable.

An alternative method of using the volume-basal area line is to prepare the volume table in four stages as follows :

1. Within each height class determine (graphically or by calculation) the linear regression of *Y* on *X*.
2. Using the values of *b* found in these regressions of *Y* on *X* determine the regression of *b* on *H*.
3. Similarly determine the regression of *a* on *H*.
4. The adjusted values of *b* and *a* obtained in stages (2) and (3) can now be used to construct the volume table, either by direct calculation or by reading the appropriate volumes from the replotted regression lines.

A slight refinement which has proved useful may be interposed between stages (2) and (3) : it is to recalculate the values of *a* in the equations : $Y = a + bX$, by inserting in these equations the adjusted values of *b* obtained in stage (2). These recalculated values of *a* are then used in determining the regression of *a* on *H*. The advantage of this refinement is that, with the recalculated values of *a*, the regression of *a* on *H* follows a more clearly defined trend than if the original *a* values are used.

General volume tables for six species were compiled by this method. In two species all stages were calculated, while in the other four species one or more of the stages were dealt with graphically.

The details of procedure, which were varied for each species to suit the material, are described in Chapter 3. Five of these general volume tables have been published as Forest Records. (See Appendix II, and Hummel, Irvine and Jeffers 1950 (2), (3) ; 1951 (1), (2), (3)). The sixth table, the one for Sitka spruce, was found to be unsatisfactory for reasons which will be discussed in Chapter 3 ; it was therefore not published.

Chapter 3

EXAMPLES OF THE PREPARATION OF GENERAL VOLUME TABLES FROM THE VOLUME-BASAL AREA LINE

GENERAL volume tables were prepared from the volume-basal area line for European larch (*Larix decidua* Mill.), Norway spruce (*Picea abies* (L.) Karst), Corsican pine (*Pinus nigra* var. *calabrica* (Loud.) Schneid.), Sitka spruce (*Picea sitchensis* (Bong.) Carr.), Douglas fir (*Pseudotsuga taxifolia* (Poir.) Rehder), and Japanese larch (*Larix leptolepis* (Sieb. and Zucc.) Murr.). The sequence indicated above is that in which the tables were prepared, and in which they will be dealt with below.

The general volume table for Scots pine (*Pinus sylvestris* L.) (Hummel, Irvine and Jeffers 1950 (1)) was prepared by a slightly different but related method which has been described in Chapter 1.

In each species the volumes were arranged by $\frac{1}{8}$ inch breast-height quarter girth classes, and 10-foot height classes. The mean girth and mean height in each girth-height group were actually calculated, as these means may differ slightly from the class means as has been shown in Table 1 for Scots pine.

For Sitka spruce, a summary of the number of trees by girth and height is given in Table 9. Similar tables for the other six species are to be found in the published volume tables (Hummel, Irvine, and Jeffers 1950 (1), (2), (3) ; 1951 (1), (2), (3)).

The preparation of the European larch volume table, which was the first to be completed, will be described in some detail while, in the other species, discussion will be confined mainly to a study of the volume-basal area lines, and to those points of procedure which differed from that adopted for European larch.

European Larch

The volume table for European larch was based on 4,903 trees.

Within each 10-foot height class the regression of volume on basal area was calculated from the equation :

$$Y = a + bX$$

where Y = volume ;
X = basal area ;
a = regression constant ;
b = regression coefficient.

The calculated values of the regression constants *a* and regression coefficients *b* in these equations are given in Table 4 while Figure 2, which has been discussed in Chapter 2, represents graphically the regressions for the 30-foot, 50-foot and 70-foot height classes. In order to illustrate the details of procedure, the calculation of the regression equation for the 30-foot height class is given in full in Table 5. The calculated value of the regression constant *a* in each height class is plotted over height in Figure 9. Similarly, the regression coefficients *b* are plotted over height in Figure 8.

TABLE 4

EUROPEAN LARCH. VALUES OF REGRESSION COEFFICIENT AND REGRESSION CONSTANT FOR EACH 10-FOOT HEIGHT CLASS

Height class (feet)	Regression coefficient *b*	Regression constant *a*
30	16·40574	−0·28254
40	20·11561	−0·28436
50	22·71879	+0·10569
60	25·54871	+0·96050
70	26·99653	+2·90121
80	28·40631	+6·21069
90	30·48808	+11·06037
100	34·65266	+12·85592

In both these figures it will be observed that the points follow an extremely well-defined trend, and it was easy to draw smooth curves from which adjusted values of *a*, termed a_1, and of *b*, termed b_1 were read. Owing to the extremely good fit of the curves, these graphically adjusted values of the regression constants and regression coefficients were almost identical with the unadjusted values, except in the 110-foot height class, in which the data were in any case inadequate.

In each 10-foot height class, the regression of volume on basal area was then recalculated, these graphically adjusted values a_1 and b_1 being used instead of the unadjusted *a* and *b* values. As it was desired that the volume tables should show volumes by one-foot height classes, instead of 10-foot height classes, the appropriate intermediate values of a_1 and b_1 were read from Figures 8 and 9, and the volume-basal area regressions for these intermediate height classes calculated accordingly.

For the smallest girths in each height class, especially in the lower height classes, it was found that the volumes calculated from the regressions : $Y = a + bX$, were too high. This is believed to be mainly attributable to the fact that volumes are

TABLE 5

CALCULATION OF THE REGRESSION EQUATION FOR THE THIRTY-FOOT HEIGHT CLASS IN EUROPEAN LARCH

f No. of trees	True girth inches	X Basal area square feet	fX	Y Volume hoppus feet	fY	Height feet
5	10	·043	·215	·34	1·70	28½
8	11	·053	·424	·58	4·64	30
2	11½	·057	·114	1·19	2·38	31
12	12	·063	·756	·77	9·24	30½
6	12½	·068	·408	·74	4·44	30
9	13	·073	·657	·92	8·28	30
9	13½	·079	·711	1·06	9·54	31
8	14	·085	·680	1·08	8·64	30½
6	14½	·091	·546	1·38	8·28	33
9	15	·098	·882	1·28	11·52	32
5	15½	·104	·520	1·56	7·80	31½
7	16	·111	·777	1·81	12·67	33
5	16½	·118	·590	1·60	8·00	31
5	17	·125	·625	2·01	10·05	33½
6	17½	·133	·798	1·83	10·98	32½
1	18	·141	·141	2·08	2·08	32½
3	19	·157	·471	2·43	7·29	34
2	19½	·165	·330	2·22	4·44	30½
1	20½	·182	·182	2·66	2·66	34
1	21½	·201	·201	3·06	3·06	32½
1	23	·230	·230	2·89	2·89	34½
1	31	·417	·417	6·16	6·16	34
62	11½	·056	3·472	·55	34·10	30
4	12½	·067	·268	1·04	4·16	32
11	15	·101	1·111	1·52	16·72	31
8	13	·073	·584	·94	7·52	33
20	15½	·104	2·080	1·40	28·00	32
3	18	·143	·429	2·02	6·06	30½

$n = 220$

$\Sigma X = 3{\cdot}338$ $\quad \Sigma fX = 18{\cdot}619$ $\quad \Sigma fX^2 = 1{\cdot}899335$ $\quad \Sigma fXY = 25{\cdot}89941$

$\Sigma Y = 47{\cdot}12$ $\quad \Sigma fY = 243{\cdot}30$ $\quad \Sigma fY^2 = 359{\cdot}5702$

$$\Sigma x^2 = \Sigma fX^2 - \frac{(\Sigma fX)^2}{n} = 0{\cdot}323576$$

$$\Sigma xy = \Sigma fXY - \frac{(\Sigma fx)\,(\Sigma fy)}{n} = 5{\cdot}30849$$

$$b = \frac{\Sigma xy}{\Sigma x^2} = 16{\cdot}40574$$

$$Y = \bar{y} + b\,(X - \bar{x}) = \frac{\Sigma fY}{n} + b\left(X - \frac{\Sigma fX}{n}\right)$$

$$= 16{\cdot}40574X - 0{\cdot}28254$$

where Σ = the sum of
f = number of trees in each $\frac{1}{8}$-inch quarter girth class
X = basal area of a tree
Y = volume of a tree
$\bar{x}$ = mean basal area
$\bar{y}$ = mean volume
n = total number of trees
$x = X - \bar{x}$
$y = Y - \bar{y}$

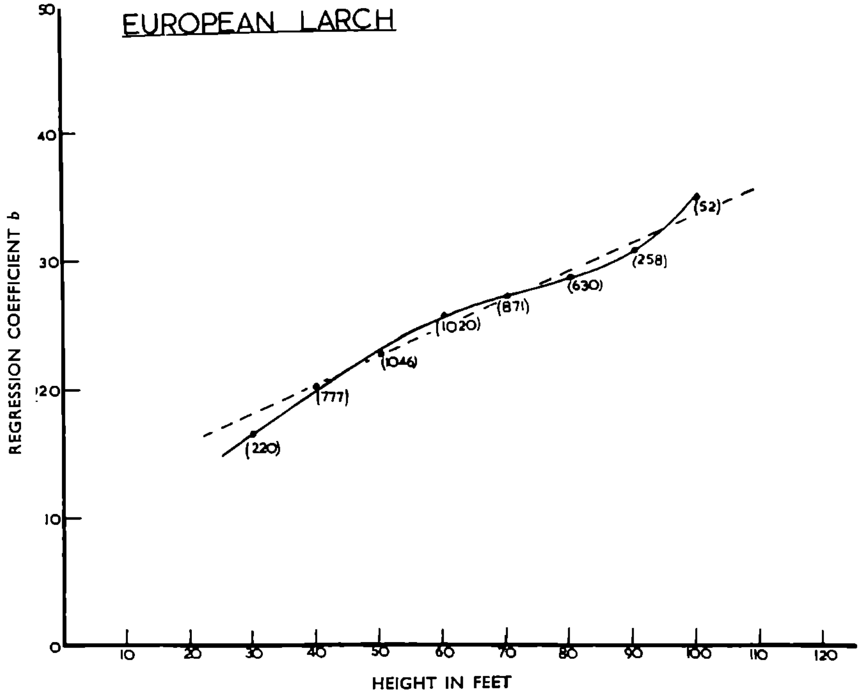

FIGURE 8. European larch. Relationship between regression coefficient *b* and height.

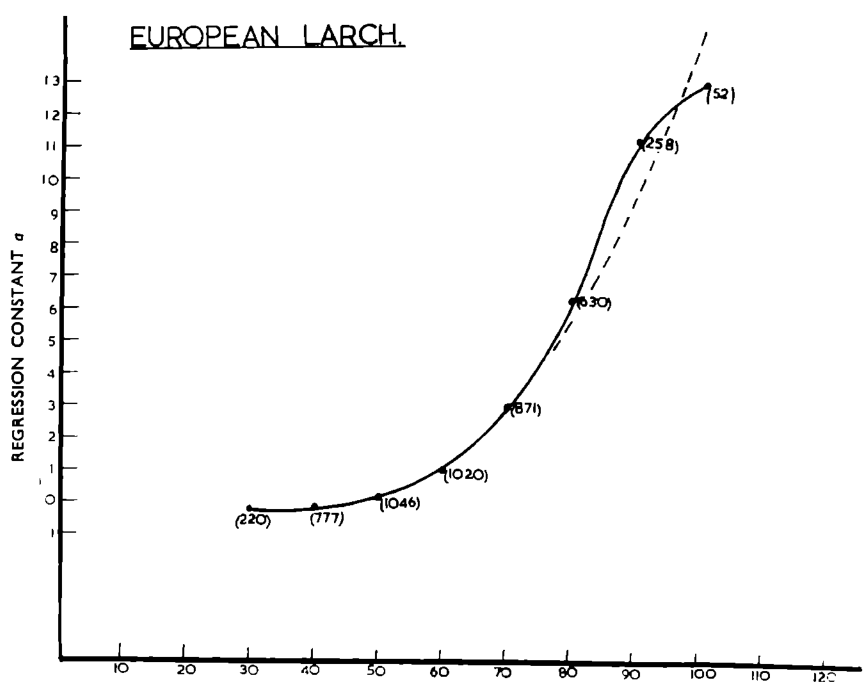

FIGURE 9. European larch. Relationship between regression constant *a* and height.

recorded to a 3-inch diameter limit, instead of to the tip of the tree. This difficulty, which was also encountered in the other five species, was overcome by adjusting by eye the lower end of the calculated volume-basal area regression lines, so as to fit the plotted points for the smallest girth classes. This crude graphical adjustment, while improving the estimates for the smallest girth classes, must have introduced a slight bias into the volume tables as a whole. But the various tests to which the tables were subjected on completion, and which are described below, suggest that this bias was too small to be harmful, or even to be detected by these tests.

For the first test, a random sample of ninety trees, stratified by height and girth, was drawn from the data from which the volume table had been compiled. The sample consisted of ten trees in each 10-foot height class, starting at 30 feet; within each height class the sample was stratified by girth. The total volume for the ninety trees according to the volume table was 3,351 hoppus feet; this was an underestimate of 0·9 per cent compared with the measured volume of 3,380 hoppus feet; but the difference was not significant statistically and is therefore no indication of bias.

Analysis of the difference (volume table volume minus measured volume) for each tree in the sample gave a standard deviation of 7·4 hoppus feet; this is 20 per cent of the measured volume of the mean tree of the sample, which was 37 hoppus feet. In a second analysis the difference (volume table volume minus measured volume) for each tree was expressed as a percentage of the measured volume; the standard deviation calculated from these percentages was 18 per cent.

Another test was carried out on the forty-four permanent European larch sample plots of the Forestry Commission. In each of these, the volume corresponding to the mean girth and mean height of the plot was read from the volume table, and multiplied by the number of trees in the plot. The aggregate volume thus calculated for the forty-four plots was 0·25 per cent more than the volume as determined by measurement according to Forestry Commission sample plot procedure. In thirty-one of the plots the volumes estimated from the volume table were within ±5 per cent, and in the remaining thirteen plots between ±5 and ±10 per cent, of the measured volumes.

At this point a slight digression is called for. It relates to the fact, which is demonstrated in Figure 8, that in European larch the regression of b on height was not linear. As, in all the other species except Sitka spruce, the regression of b on height was found to be linear, it was decided, as a matter of interest, to examine what difference it would have made to the European larch volume table if the regression of b on height had been assumed to be linear. The dotted line in Figure 8 is this calculated straight line, in which each point was weighted according to the number of trees on which it was based. The equation was:

$$b_1' = 11{\cdot}51642 + 0{\cdot}21999H \qquad (10)$$

By inserting the adjusted values of the regression coefficient (b_1') in the equations: $Y = a + bX$ in each 10-foot height class, adjusted values of a termed a_1' were obtained. A fourth degree equation, which is represented as a dotted line in Figure 9, was then calculated for the regression of a_1' on

TABLE 6

EUROPEAN LARCH: COMPARISON BETWEEN PUBLISHED VOLUME TABLE AND TABLE BASED ON THE ASSUMPTION THAT THE REGRESSION OF b ON H IS LINEAR

Height (feet)	Breast-height Quarter girth (inches)	Volume in hoppus feet		
		Published table	b/h linear	difference %
30	3	0·76	0·71	−6·6
..	5	2·57	2·72	−5·8
..	8	7·00	7·62	8·9
60	4	(3·80)	4·04	−6·3
..	11	22·4	22·1	−1·3
..	18	58·4	56·9	−2·6
90	8	(24·6)	23·8	−3·3
..	17	72·2	72·8	−0·8
..	26	154	157	−1·9

Note: The two figures in brackets differ from the published volume table because the volume adjustment for the smallest girths in each height class (see p. 15) have not been made: with the adjustment, the figures in column 3 in table 6 would not have been comparable with the corresponding figures in column 4.

height ; this gave adjusted values of a_1' which are termed a_2'. The equation was :

$$a_2' = 1{\cdot}9371 + 1{\cdot}7879T + 0{\cdot}4661T^2 + 0{\cdot}0330T^3 - 0{\cdot}0029T^4 \qquad (11)$$

$$\text{where } T = \frac{H - 65}{10}$$

(*T* was used instead of *H* for ease of computation.)

It will be observed that these values of a_2' differ from the original values of *a* mainly in the height classes above 70 feet, where the regression of a_2' on *H* does not have the inflection in curvature exhibited by the regression of *a* on *H*.

The European larch volume table was then recalculated with the values of b_1' and a_2' obtained from equations (10) and (11). The effect on the volume table of assuming the regression of *b* on *H* to be linear is shown in Table 6, in which the published volume table is compared with the volume table which would have resulted from the assumption of the regression of *b* on *H* being linear. The comparison is confined to the largest, medium and smallest girths given in the published table, for the 30-foot, 60-foot and 90-foot height classes.

The differences between the two tables are small, except in the smallest height class, and this suggests that, in preparing general volume tables from the volume-basal area line, the regression of *b* on *H* may be assumed to be linear unless there is strong evidence to the contrary.

Norway Spruce

The Norway spruce volume table was based on 2,617 trees. For each 10-foot height class the regression of volume on basal area was calculated, and the resulting values of the regression coefficients *b* and regression constants *a* are given in Table 7.

TABLE 7

NORWAY SPRUCE : VALUES OF REGRESSION COEFFICIENT AND REGRESSION CONSTANT FOR EACH 10-FOOT HEIGHT CLASS

Height class (feet)	Regression coefficient *b*	Regression constant *a*
20	16·1660	−0·4378
30	18·1143	−0·4025
40	21·2124	−0·2957
50	20·6823	1·0647
60	24·7474	1·7176
70	26·9562	3·1376
80	29·8556	3·5986
90	29·3206	9·4791
100	33·8659	5·7948

The values of *b* and *a* given in this table are plotted over height in Figures 10 and 11.

The straight line in Figure 10 represents the calculated linear regression of *b* on height, the equation for which was :

$$b_1 = 11{\cdot}3602 + 0{\cdot}2202\,H \qquad (12)$$

The curve drawn through the points in Figure 11, on the other hand, was drawn by eye. The volume table was calculated direct from the values of b_1 and a_1 taken from Figures 10 and 11. No attempt was made to use the values of b_1 calculated from equation (12) in order to adjust the values of a_1, as was done in some of the other volume tables.

The volumes of ninety-six trees, distributed equally over all height classes, were determined from the volume tables, and compared with the measured volumes of these trees. The volume table was found to give an average over-estimate of 0·56 ±0·71 hoppus feet (1·56 per cent of the mean actual volume). The standard deviation of individual tree volumes from the values given in the volume table was 19 per cent of the mean volume of the sample.

A test of accuracy was also carried out on all the thirty-six permanent Norway spruce sample plots of the Forestry Commission. In each of these the volume corresponding to the mean girth and height of the plot was obtained from the volume table and multiplied by the number of trees in the plot. The aggregate volume thus calculated for the thirty-six plots was 0·45 per cent more than the volume as determined by direct measurement according to the standard sample plot procedure. In twenty-four of the thirty-six plots, the volumes estimated from the volume tables were within ±5 per cent ; in ten plots within ±10 per cent ; and in the remaining two plots within ±20 per cent, of the measured volumes.

Corsican Pine

The Corsican pine volume table was based on 1,108 trees. The calculated values of the regression coefficients *b* and regression constants *a* are given in Table 8.

TABLE 8

CORSICAN PINE : VALUES OF REGRESSION COEFFICIENT AND REGRESSION CONSTANT FOR EACH 10-FOOT HEIGHT CLASS

Height class (feet)	Regression coefficient *b*	Regression constant *a*
20	14·555	−0·336
30	17·482	−0·438
40	20·820	−0·311
50	23·979	+0·208
60	29·969	+0·174
70	35·268	−0·205
80	37·773	+0·699
90	41·819	−0·798
100	37·696	+14·174

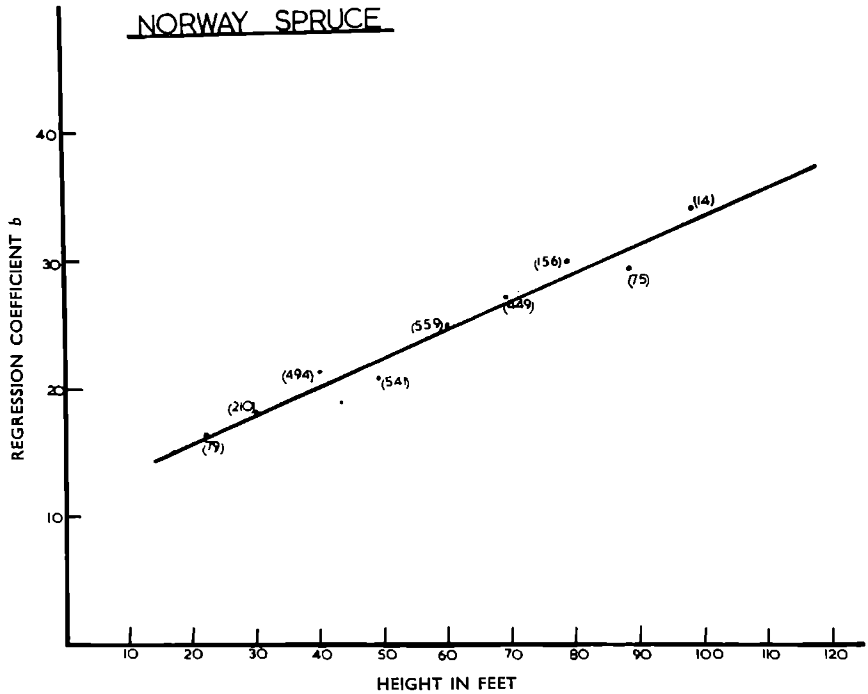

FIGURE 10. Norway spruce. Relationship between regression coefficient *b* and height.

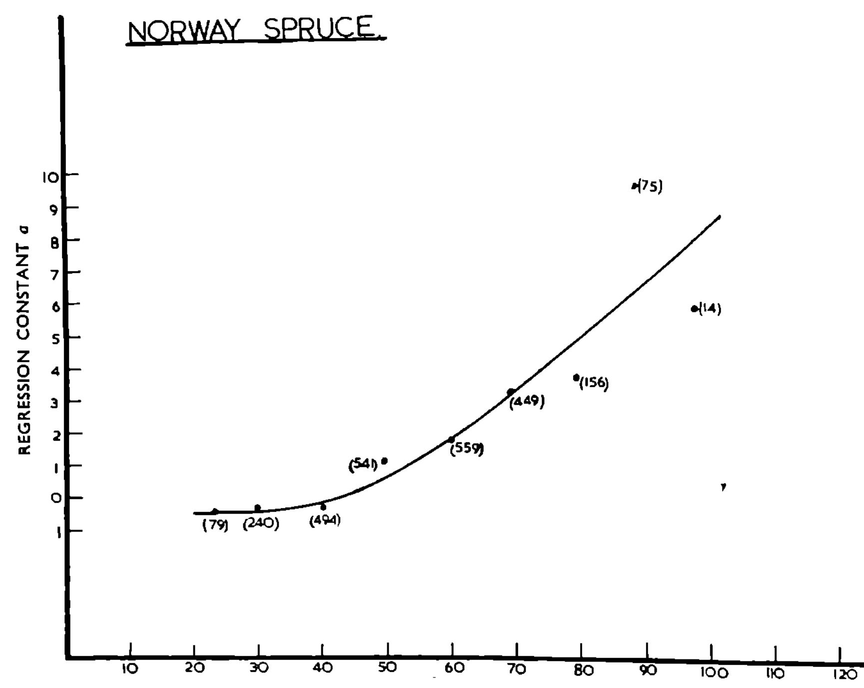

FIGURE 11. Norway spruce. Relationship between regression constant *a* and height.

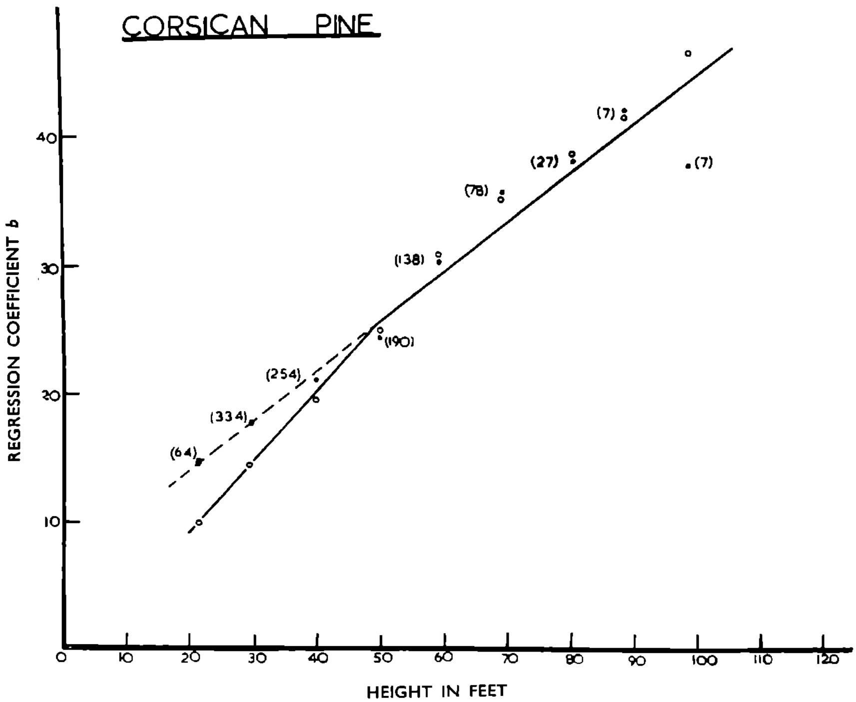

FIGURE 12. Corsican pine. Relationship between regression coefficient *b* and height.

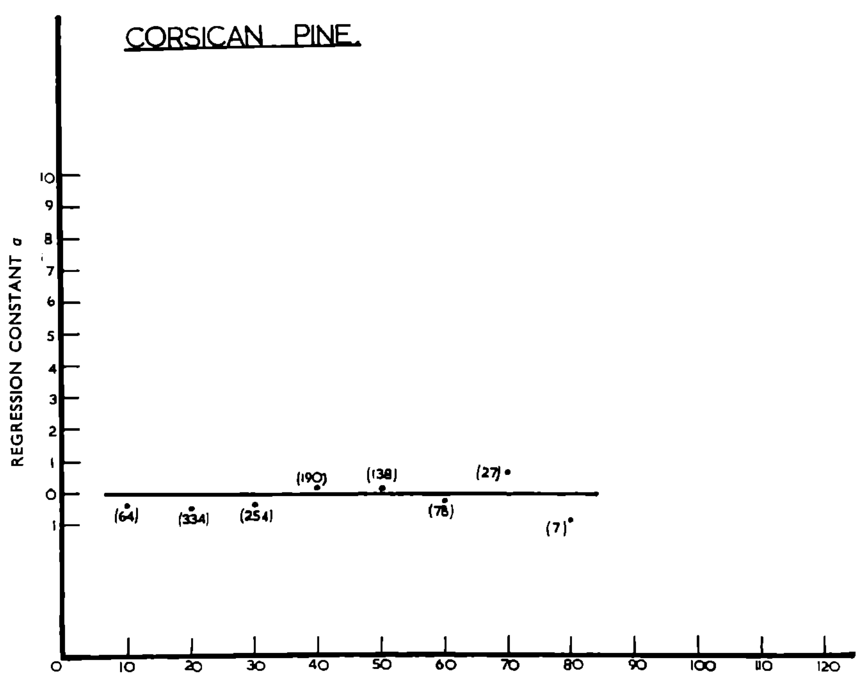

FIGURE 13. Corsican pine. Relationship between regression constant *a* and height.

In Figure 12, the values of b are plotted over heights as dots (.), and a linear regression was calculated for these points :

$$b_1 = 5{\cdot}3893 + 0{\cdot}3975\,H \qquad (13)$$

Similarly, in Figure 13, the values of a are plotted over height ; they are seen to be scattered fairly closely around zero for all heights.

A volume table was then prepared taking : $a = 0$ for all heights and using the values of b_1 from the above equation (13).

When tested on a stratified random sample of ninety-three trees, this table was found to be satisfactory for the height classes of 50 feet and above, but not for the 20-foot, 30-foot and 40-foot height classes.

Adjusted values of b_1 were then calculated by taking : $a = 0$, in the equation : $Y = a + bX$. These values b_2 are shown as circles (o) in Figure 12. The volume table for the height classes of 50 feet and above was left unaltered, but for the 20-foot, 30-foot and 40-foot height classes it was recalculated by using these values of b_2.

From the experience gained with the calculation of volume tables for other species, it now appears that the calculation of adjusted values of a from the regression of b on height would have been preferable to the reverse procedure actually adopted.

The standard deviation of the differences between the ninety-three individual tree volumes in the sample, and the values given in the volume table, was 19 per cent of the mean volume of the sample.

A test for accuracy was carried out against all the fifty-four permanent Corsican pine sample plots of the Forestry Commission. For each of these plots the volume corresponding to the mean girth and height of the plot was obtained from the volume table and multiplied by the number of trees in the plot. The aggregate volume thus calculated for the fifty-four plots was 0·3 per cent more than the volume as determined from direct measurement by the standard sample plot procedure. The variation coefficient of the differences between the measured volumes and the volumes estimated by means of the volume tables, was about 8 per cent of the mean measured volume.

Sitka Spruce

The Sitka spruce data consisted of 907 trees ; their distribution by height classes is given in Table 9.

The regression : $Y = a + bX$ was calculated for each 10-foot height class and the values of the regression coefficients b and regression constants a are shown in Table 10, while in Figures 14 and 15 respectively they are plotted over height.

TABLE 9

SITKA SPRUCE : NUMBERS OF TREES IN EACH GIRTH/HEIGHT CLASS USED IN PREPARING VOLUME TABLE

Breast height Quarter girth (inches)	Height class (feet)											Total
	20	30	40	50	60	70	80	90	100	110	120	
3	7	51	18	2								78
4	2	28	60	21	4							115
5	3	9	63	49	24	1						149
6		—	26	43	27	18	2					116
7		—	12	38	32	43	3					128
8		2	3	3	21	39	10	2				80
9			1	1	13	30	25	4				74
10			—		3	23	31	6				63
11			1			7	22	13	1			44
12						6	10	8	2			26
13						—	8	13	7			28
14						1	3	7	7			18
15							1	2	5	1		9
16 and over								2	2	1	13	18
Totals	12	90	184	157	124	168	115	57	24	2	13	946

TABLE 10

SITKA SPRUCE : VALUES OF REGRESSION COEFFICIENT AND REGRESSION CONSTANT FOR EACH 10-FOOT HEIGHT CLASS

Height class (feet)	Regression coefficient *b*	Regression constant *a*
20	12·87 ± 0·365	−0·319
30	14·13 ± 0·214	−0·169
40	16·14 ± 0·317	0·242
50	24·99 ± 0·267	−0·246
60	27·41 ± 0·264	0·374
70	32·73 ± 0·378	0·256
80	31·20 ± 0·688	2·942
90	22·19 ± 0·904	15·835
100	24·28 ± 1·211	24·359

The regression of *b* on height (Figure 14) seems to follow no well-defined trend, and it is not even possible to fit a simple curve to pass within the fiducial limits of the individual points. The 5 per cent fiducial limits are indicated by the vertical lines through the points.

It is particularly disturbing that the value of *b* in the largest two height classes is less than in the preceding three height classes. This means that for very large girths an increase in height would be accompanied by a reduction in volume, which is most unlikely. The most probable explanation seems to be that the data in the different height classes are not drawn from a single population. It may be that the inherent genetic characteristics of the trees in each height class, as well as the sites on which they grew, and the thinning treatments to which they were subjected, differed somewhat ; in each height class these factors may have influenced the amount of buttressing at breast height. Where trees are buttressed an increase in breast height girth will be accompanied by a smaller increase in volume, than when there is no buttressing.

The regression constant *a* (Figure 15) was found to approximate to zero up to a height of 60 feet, but above that height the value of *a* increases considerably. Up to 60 feet *a* was therefore assumed to be zero, and through the five points from 60 feet and upward (taking : a = 0 for H = 60) the following fourth degree equation was calculated.

$$a_1 = 2{\cdot}9421 + 8{\cdot}3561\,T + 6{\cdot}0345T^2 - 0{\cdot}5666T^3 - 0{\cdot}9313T^4 \qquad (14)$$

$$\text{where } T = \left(\frac{H - 80}{10}\right)$$

By recalculating the equation : $Y = a + bX$, for the height classes 20, 30, 40, 50 and 60 feet, taking $a = 0$, adjusted values of b, termed b_1, were obtained. These are shown as circles (O) in Figure 14. The fourth degree equation calculated for a_1 for the larger height classes actually goes through the originally calculated points a, so that $a_1 = a$ in the 70, 80, 90 and 100 foot classes in which therefore $b_1 = b$.

The curve drawn in Figure 14 was calculated from the values of b_1 (O). It represents the equation :

$$b_2 = 27{\cdot}7838 + 2{\cdot}0775T - 0{\cdot}7959T^2 \qquad (15)$$

$$\text{where } T = \left(\frac{H - 60}{10}\right)$$

A volume table was prepared from the values of a_1 and b_2 thus obtained. This table was tested, first on single trees, and then on whole plots as follows :

1. The volumes of seventy-two trees, distributed over six 10-foot height classes, were determined from both the Sitka spruce and the Norway spruce volume tables, and then compared with the measured volumes of these trees. The results are summarised in Table 11.

For the Sitka spruce volume table, the overall mean difference between tabulated and actual volumes was —0·90 hoppus feet (6·4 per cent of the actual mean volume of the sample, which was 14 hoppus feet) with a standard error of ±0·23 hoppus feet. The standard deviation of the differences between individual tree volumes, and the tabulated values, was ±1·93 hoppus feet, or 13·8 per cent of the mean actual volume.

For the Norway spruce volume table, the mean difference between tabulated and actual volumes was —0·38 hoppus feet (2·7 per cent of the actual mean volume of the sample) with a standard error of ±0·23 hoppus feet. The standard deviation of the differences between individual tree volumes and the tabulated values, was ±1·91 hoppus feet (13·5 per cent).

2. The volume tables for both species were then used to estimate the standing volumes of fifty-six permanent Sitka spruce sample plots. Two tests were applied :

(i) For the first test, the volume table volume for the mean girth and mean height of the plot was multiplied by the number of trees per acre. This volume was compared with the volume measured by the standard Forestry Commission sample plot procedure.

For the Sitka spruce volume table, the mean difference (volume table volume *minus* measured volume) was 81·8 hoppus feet, i.e. 3·10 per cent of the mean measured volume, with a standard error of ±33·6 hoppus feet (1·27 per cent). There was thus a significant positive bias in using this volume table. The standard deviation was ±255·5 hoppus feet (9·67 per cent). Expressed as percentage differences, the mean difference was 3·82 per cent, with a standard error of ±1·05 per cent and a standard deviation of ±8·00 per cent.

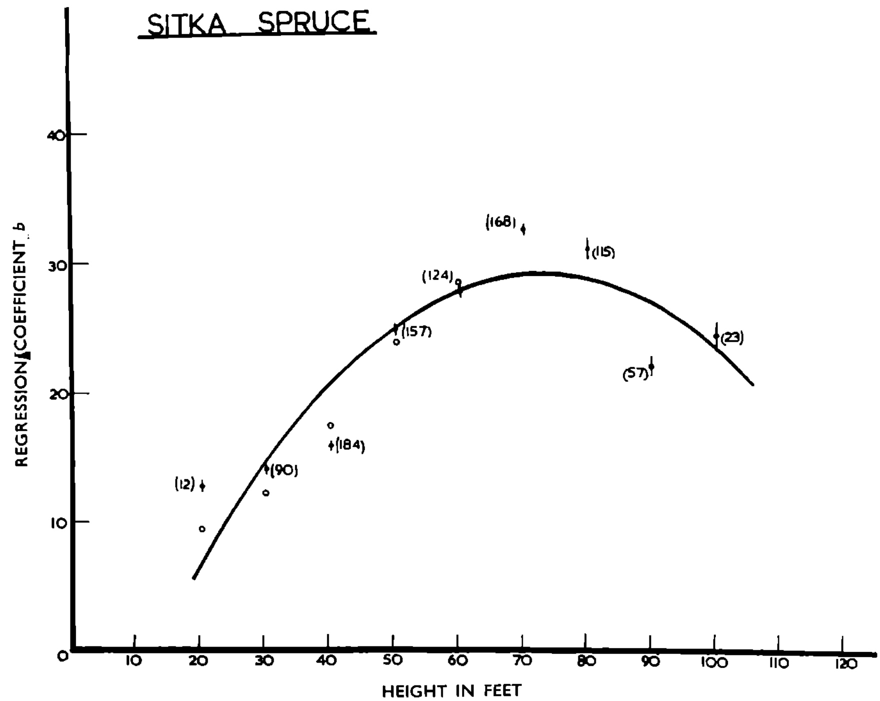

FIGURE 14. Sitka spruce. Relationship between regression coefficient *b* and height.

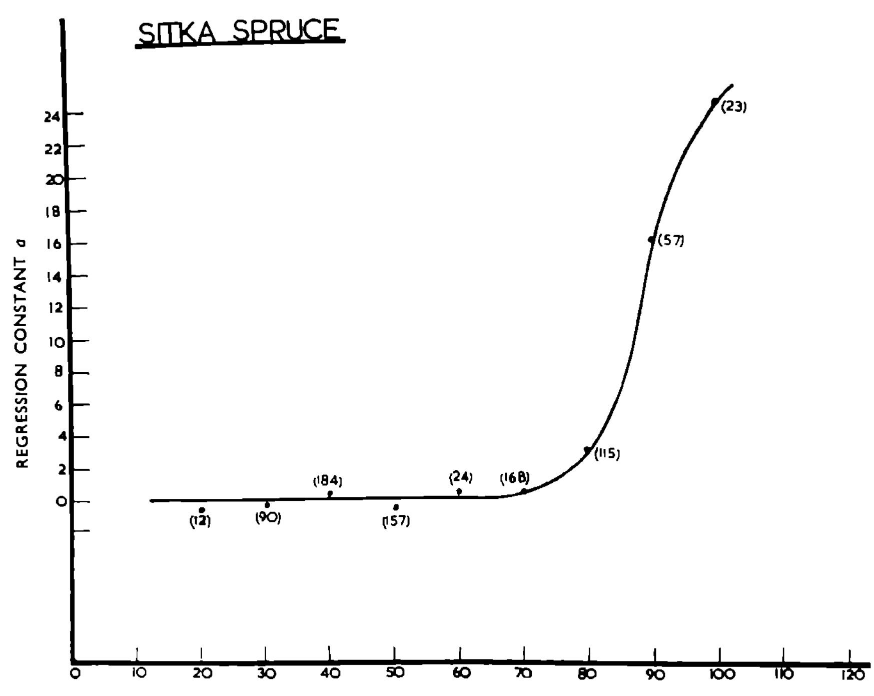

FIGURE 15. Sitka spruce. Relationship between regression constant *a* and height.

TABLE 11

COMPARISON OF THE KNOWN VOLUMES OF 72 RANDOMLY SELECTED SITKA SPRUCE TREES WITH THE VOLUMES ESTIMATED FOR THESE TREES FROM THE SITKA SPRUCE AND NORWAY SPRUCE VOLUME TABLES

Height class (feet)	No. of trees	Sitka Spruce Table: Average differences (hoppus feet)		Sitka Spruce Table: %	Norway Spruce Table: Average differences (hoppus feet)		Norway Spruce Table: %
30	3	0·43	±0·574	17·6	−0·14	±0·576	−5·1
40	19	0·77		17·2	0·30		7·5
50	9	0·04		0·5	0·49		5·5
60	7	−0·53		−4·6	0·07		−0·6
70	20	−3·08		−14·9	1·82		8·8
80	14	−1·12		−4·6	0·67		2·8
Differences for significance	5%	1·63		—	1·63		—
	1%	2·16		—	2·16		—

Note : The "Differences" in the above table refer to volume table volumes *minus* measured volumes.

For the Norway spruce volume table, the mean difference was 106·6 hoppus feet, i.e. 3·97 per cent of the mean estimated volume, with a standard error of ±25·6 hoppus feet (0·95 per cent). There was thus also a significant positive bias in using this volume table. The standard deviation was ±191·5 hoppus feet (7·14 per cent). Expressed as percentage differences, the mean difference was 5·09 per cent, with a standard error of ±0·94 per cent, and a standard deviation of ±7·02 per cent.

(ii) For the second test, use was made of the fact that, in permanent sample plots, the trees are grouped by girth classes, and the volume of each group is determined separately. The volume table volume of the mean tree of each such group was multiplied by the number of trees in the group, and the sum of these group volumes compared with the measured volume of the plot.

For the Sitka spruce volume table, the mean difference (volume table volume *minus* measured volume) was 7·44 hoppus feet ; i.e. 1·09 per cent of the mean measured volume, with a standard error of ±8·96 hoppus feet (1·31 per cent). There was thus no significant bias in the results obtained by the use of this volume table. The standard deviation was ±37·98 hoppus feet (5·56 per cent). Expressed as percentage differences, the mean difference was 1·34 per cent, with a standard error of ±1·36 per cent, and a standard deviation of ±5·77 per cent.

Also the Norway spruce volume table gave no significant bias : the mean difference was 11·46 hoppus feet ; i.e. 1·40 per cent of the mean measured volume, with a standard error of ±12·04 hoppus feet (1·48 per cent). Expressed as percentage differences, the mean difference was 2·12 per cent, with a standard error of ±1·55 per cent, and a standard deviation of ±5·59 per cent.

In view of these results, which suggested that, for estimating the volumes of Sitka spruce trees, the Norway spruce volume table could be relied upon to give as good or as bad estimates as the Sitka spruce volume table, it was decided not to proceed with the publication of the latter until considerably more material becomes available.

Douglas Fir

The Douglas fir volume table was prepared from 1,472 trees. The calculated values of the regression constants *a* and regression coefficients *b* in the equation : $Y = a + bX$, for each 10-foot height class are given in Table 12.

The regression of *b* on height was linear, as is evident from Figure 16. The regression equation was :

$$b_1 = 6{\cdot}1918 + 0{\cdot}2493H \qquad (16)$$

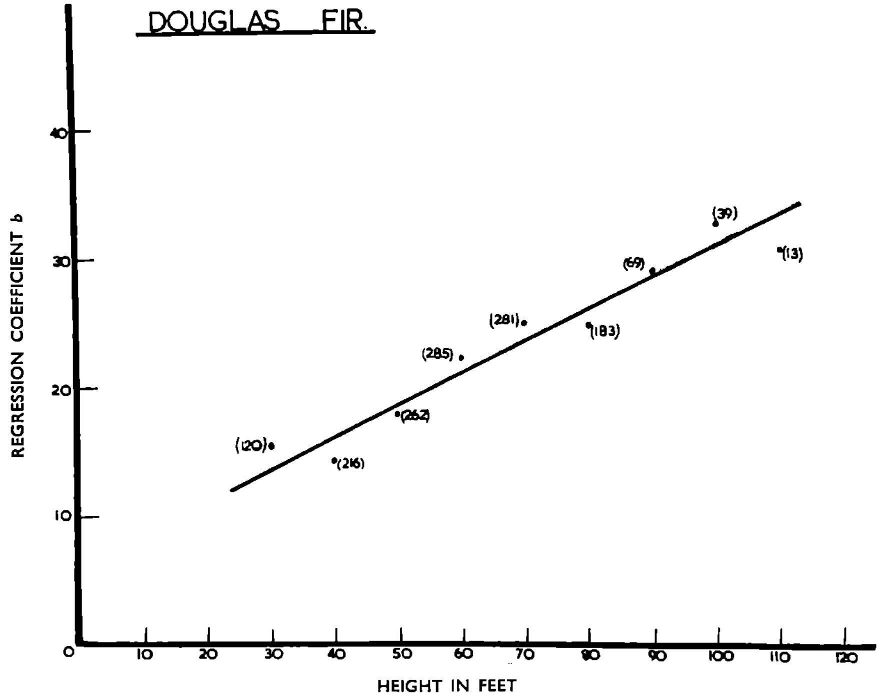

FIGURE 16. Douglas fir. Relationship between regression coefficient *b* and height.

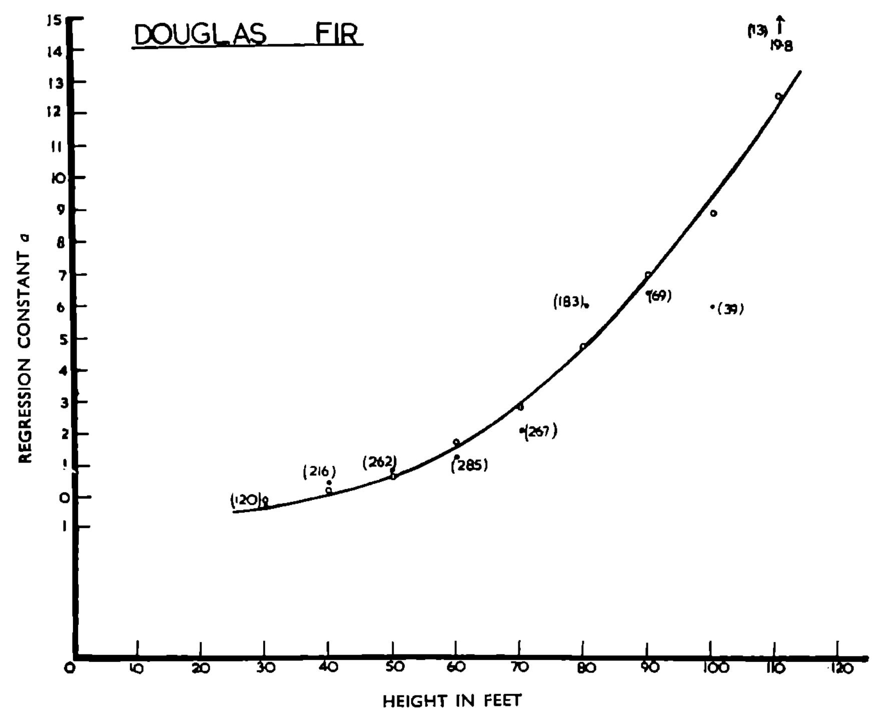

FIGURE 17. Douglas fir. Relationship between regression constant *a* and height.

TABLE 12

DOUGLAS FIR : VALUES OF REGRESSION COEFFICIENT AND REGRESSION CONSTANT FOR EACH 10-FOOT HEIGHT CLASS

Height class (feet)	Regression coefficient *b*	Regression constant *a*
30	15·4899	−0·2568
40	14·3167	0·4388
50	17·8569	0·8380
60	22·2423	1·2465
70	24·9122	2·1259
80	24·6941	6·0125
90	29·0341	6·3918
100	32·8148	6·0423
110	30·6934	19·8334

The values of *a* plotted over height are shown in Figure 17 as dots (.). From the adjusted values b_1 adjusted values of *a*, termed a_1, were calculated. These are shown as circles (O) in Figure 17. It will be observed that these values of a_1 follow a smoother trend than the unadjusted values of *a*. A second degree equation was then calculated through the points a_1 :

$$a_2 = 2{\cdot}9306 + 1{\cdot}5443T + 0{\cdot}2013T^2 \qquad (17)$$

$$\text{where } T = \left(\frac{H - 70}{10}\right)$$

The volume table for each height was then calculated from the equation $Y = a_2 + b_1X$. There have thus been two adjustments to the originally calculated values of *a* and one adjustment to those of *b*.

A random sample of ninety-six trees, stratified by girth and height, was drawn from the volume table data. The sample gave an average over-estimate of 0·34 hoppus feet, or 1·0 per cent of the mean volume of the sample. The standard deviation of individual tree volumes from the values given in the volume table was found to be 29 per cent of the mean volume of the sample.

A further test of accuracy was carried out against all the sixty-three permanent Douglas fir sample plots of the Forestry Commission. For each of these plots the volume corresponding to the mean girth and height of the plot was obtained from the volume table and multiplied by the number of trees in the plot. The aggregate volume thus calculated for the sixty-three plots was 3·09 per cent. more than the volume as determined by direct measurement according to the standard sample plot procedure. The standard deviation of the differences between measured volumes and the volumes estimated by means of the volume tables was about 8 per cent of the mean measured volume.

Japanese Larch

The volume table for Japanese larch was based on 1,389 trees. Within each 10-foot height class, the regression : $Y = a + bX$, was calculated ; the values of the regression coefficients *b* and regression constants *a* are given in Table 13.

TABLE 13

JAPANESE LARCH : VALUES OF REGRESSION COEFFICIENT AND REGRESSION CONSTANT FOR EACH 10-FOOT HEIGHT CLASS

Height class (feet)	Regression coefficient *b*	Regression constant *a*
20	15·2203	−0·3844
30	15·0680	−0·2081
40	20·3500	−0·3062
50	23·1668	0·0715
60	27·8723	0·2763
70	31·9436	0·7216

In Figures 18 and 19 the calculated values of *b* and *a* respectively are plotted over height as dots (.).

A linear regression was calculated through the points in Figure 18. Adjusted values b_1 were thus obtained, which were inserted instead of *b* in the original equations : $Y = a + bX$, in order to obtain adjusted values a_1. These are shown as circles (O) in Figure 19. A second degree equation was then calculated through the points a_1 :

$$a_2 = -0{\cdot}1417 + 0{\cdot}2009T + 0{\cdot}0726T^2 \qquad (18)$$

$$\text{where } T = \frac{H - 45}{10}$$

The volume table was calculated by inserting the appropriate values of a_2 and b_1 into the regression : $Y = a + bX$, in each height class. As in the case of Douglas fir, there have thus been two adjustments to the originally calculated values of *a*, and one to those of *b*.

A random sample of forty-five trees, stratified by girth and height, was drawn from the volume table data. The sample gave an average underestimate of 0·04 hoppus feet, or 0·6 per cent of the mean volume of the sample. The standard deviation of individual tree volumes from the values given in the volume table was about 11 per cent of the mean volume of the sample, which is the lowest standard deviation for any of the tables that were prepared.

A test of accuracy was also carried out on all the forty-five permanent Japanese larch sample plots of the Forestry Commission. For each of these plots, the volume corresponding to the mean girth and height of the plot was obtained from the volume table, and multiplied by the number of trees in the plot. The aggregate volume thus calculated for the forty-five plots was 0·5 per cent less than the

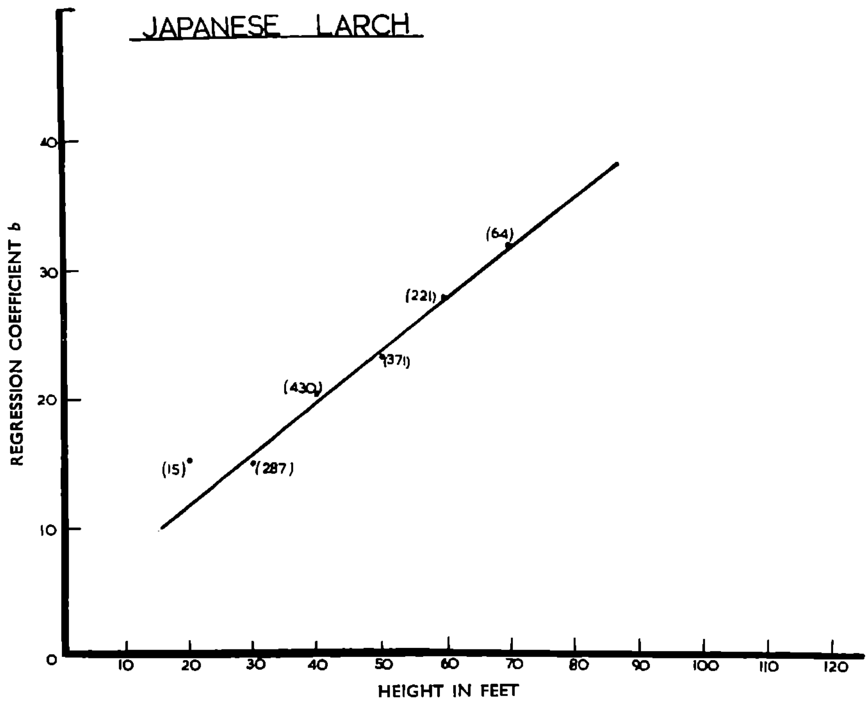

FIGURE 18. Japanese larch. Relationship between regression coefficient *b* and height.

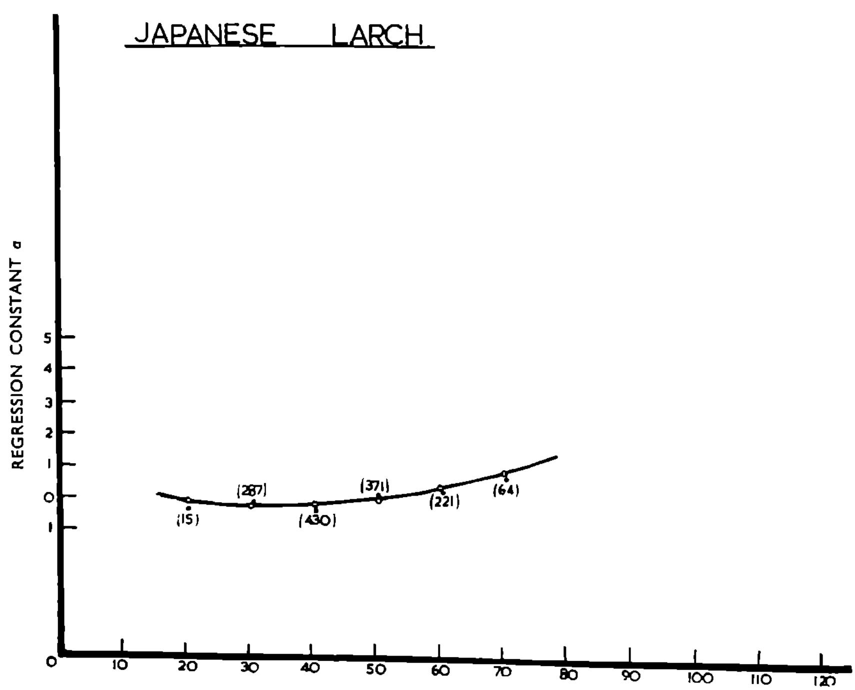

FIGURE 19. Japanese larch. Relationship between regression constant *a* and height.

volume as determined by direct measurement according to the standard sample plot procedure. The standard deviation of the differences between measured volumes and the volumes estimated by means of the volume tables, was about 6 per cent of the measured volume.

Discussion and Conclusions

The regression of b on H was linear in Norway spruce, Corsican pine, Douglas fir and Japanese larch ; it was nearly linear in European larch, where the assumption of linearity was found to have little effect on the resulting volume table ; and only in Sitka spruce did this regression show a marked departure from linearity. Even in this species, the assumption of linearity which is implied in testing the data on the Norway spruce volume table, gave no worse results than the volume table based on the actual values of b in Sitka spruce. The available evidence in the six species examined therefore suggests that the regression of b on H is either linear, or if it is not linear, that the assumption of linearity will not appreciably detract from the precision of the resulting volume table.

The regression of a on H was more variable. In Norway spruce, Douglas fir, and Japanese larch it was a concave curve ; in European larch and Sitka spruce the lower portion of the curve was also concave, but there was a change in the direction of curvature, the upper part of the curve being convex ; and in Corsican pine all the a values were around zero. The recalculation of a, by inserting in the equations : $Y = a + bX$ the values of b_1 obtained from the linear regression of b on H, removed the inflection of curvature in the case of European larch. A similar recalculation of a in Corsican pine, which was, however, not made use of in the construction of the volume tables, gave a regression of a on H resembling in shape the original sigmoid curve of a on H in European larch. No recalculation of a was attempted in Sitka spruce. The general rule thus seems to be that the regression of a on H takes a concave form in the lower height classes ; and that near the upper limit of the height range there may, in some species, be a point of inflection.

Thus a study of the material examined suggests that the form, which the general volume equation (9) will normally take, is likely to be :

$$Y = a_1 + a_2H + a_3H^2 + b_1X + b_2XH \quad (9b)$$

but there may be additional terms with : a_4H^3, and : a_5H^4.

The question arises, whether it was preferable to prepare the volume tables by the method described, or whether it would have been better to prepare the tables direct from equation (9b), with a probable minimum of 5 terms and a probable maximum of 7 terms. Direct calculation would have been quite impracticable on account of the coding required, unless the data had been summarised into very broad girth groups with a resulting loss in precision. Had this been done, it is difficult to say whether or not there would have been a saving in time. The advantage would have been complete objectivity ; the main disadvantage, that the insight into the data given by the various graphs would have been lost. It is also impossible to say whether anything was gained by not depending entirely on graphical solutions, once the regressions : $Y = a + bX$ in each height class had been calculated. It would appear, however, that when the trends of the regressions of a and b on H are as clearly defined as they were in European larch, Norway spruce, Douglas fir and Japanese larch, little is to be gained by calculating these regressions, or by recalculating the values of a from the adjusted values of b. If, however, as was the case in Corsican pine and Sitka spruce, there is some doubt about the trends, it appears best to assume the regression of b on H to be linear, and to recalculate the values of a accordingly.

PART II. THE VOLUME-BASAL AREA LINE WITHIN A STAND

Chapter 4

PRECISION

PART I of this paper has dealt with the regression of volume on basal area when individual trees of a species are taken from a wide range of sites and ages, and are treated as one population. In each of seven coniferous species, it was found that *for a given height* the regression of tree volume on basal area can be adequately approximated by a straight line, i.e. :

$$Y = a + bX$$

where Y = volume of tree,
X = basal area of tree.

It was also found that *a* and *b* in the above equation are functions of height. These relationships form the basis of the method of preparing general volume tables which has been described in Part I.

Part II deals with the regression of tree volume on basal area *within one stand*, and more particularly within even-aged coniferous stands of a single species. It has been known for some time that this relationship is usually linear or nearly linear. The important difference between the volume-basal area line relating to heterogeneous data, as described in Part I, and the line derived from data relating to one stand, is that, with heterogeneous data, the regression of volume on basal area may be regarded as linear *for a given height ;* whereas if all the trees are from a single stand, the regression can be adequately approximated by a straight line *irrespective* of height. Several publications, among the more recent of which are those by Krenn (1944), Jolly (1950), Prodan (1951), and Loetsch 1952), have dealt with the regression of volume on basal area within a stand, and have also described to what extent departures from linearity may occur under certain conditions. But it appears that no thorough study has yet been made of how the volume-basal area line changes with species, site, age and thinning treatment. It is the main object of the investigation now to be described to fill this gap in our knowledge, and to discover whether the information thus obtained can be put to practical use. The most important facts that have emerged are that :

(i) Apart from a few exceptions, the point where the regression line cuts the *X* (horizontal) axis is more or less the same for all sites and all the seven species examined, and does not change with either age or thinning treatment.

(ii) Within a species, the slope of the regression line (i.e. the regression coefficient) increases with the top height of a stand, but is not appreciably affected by thinning treatment or site.

The practical application of these findings to the solution of mensurational problems in the forest will be discussed in Chapter 6.

Material

The material for this investigation was provided by the permanent sample plot records of the Forestry Commission, but only plots in comparative thinning series which had been remeasured at least three times, and some single plots which had been remeasured at least four times, were examined. It was felt that data from plots with fewer remeasurements would contribute too little additional information to make their examination worth while. The only exceptions to this general rule were four plots in which the volumes of all trees were determined after the plots had been clear felled ; these plots supplied information not available from the other data. The number of plots examined and the total number of remeasurements are shown in Table 14. Further particulars are given in Appendix V.

In contrast to Part I, where, in conformity with current practice, all volumes are given in *over* bark measure, all volumes in Part II are given *under* bark, except the volumes relating to the four clear felled plots mentioned above : the original records and computations of volumes in the sample plot files are nearly all in *under* bark measure ; and there are various reasons, apart from the amount

TABLE 14

NUMBER OF SAMPLE PLOTS EXAMINED AND TOTAL NUMBER OF REMEASUREMENTS

Species	Single plots: Number of Plots	Single plots: Number of Remeasurements	Thinning series: Number of Series	Thinning series: Number of Plots	Thinning series: Number of Remeasurements
		Permanent Sample Plots			
Scots pine	2	10	10	25	125
Corsican pine	—	—	5	10	49
European larch	3	20	12	32	149
Japanese larch	3	12	7	16	69
Norway spruce	—	—	7	29	140
Sitka spruce	5	28	3	8	24
Douglas fir	5	26	5	10	46
TOTALS	18	96	49	130	602
		Felled Plots			
Corsican pine	—	—	1	2	—
Douglas fir	2	—	—	—	—

of work involved, which would have made it difficult to convert the original data to *over* bark. Conversely it was not practicable to achieve uniformity in Part II by converting the volumes for the four clear felled plots to *under* bark, because insufficient measurements of bark thickness had been taken for carrying out the conversion with accuracy.

Basal areas, in Part II, are given *over* bark, as in Part I.

In accordance with Forestry Commission sample plot procedure (Macdonald 1931) the sample plots are normally thinned and remeasured at intervals of three to six years, depending on their rate of growth. At each remeasurement, the volumes of about eight sample trees are determined by measuring the volume of each tree in 10-foot sections to the point where the over-bark diameter is 3 inches. These sample trees are distributed over the range of girth in the plot, and each tree must satisfy the condition that its height is within 2 feet of the average height for the particular girth, that average height being determined by plotting the heights of twenty to thirty trees over their breast-height girths, and drawing a smooth curve. Within the limits imposed by these conditions, the selection of the sample trees is subjective, the aim being to take trees which, in stem form and taper, appear to be "representative" of the crop at the time of measurement. The method of selecting the sample trees is intended to increase the precision of the volume estimate, but it precludes the calculation of a valid estimate of that precision. This point will be discussed more fully later.*

* A more objective method of selecting sample trees has since been introduced.

No attempt is made to select the identical sample trees at consecutive remeasurements. Until a few years ago, the volumes of the sample trees were plotted over their basal areas, and the line was drawn by eye through the points. But more recently it has become customary to calculate the regression, although a graph is still prepared showing the calculated line and the points on which it is based. This is done as check on the calculations, and in order to find out whether there is any evidence of a straight line being inadequate to describe the relationship. On the whole, the Forestry Commission data confirm that departures from linearity are not frequent ; and where they occur they are usually within the limits attributable to sampling. Bearing in mind the other errors to which the volume estimate is subject, these departures are sufficiently small to be disregarded in determining the total volume of timber in a plot, which is more important than the volume of any particular girth class. In the present investigation, however, the sampling errors must be considered in some detail, because they may influence the apparent changes in the volume-basal area line with species, age, site and thinning treatment.

Errors of Volume-Basal Area Line

If the relationship between volume and basal area in a stand is linear, tl. regression is described by the equation :

$$Y = \alpha + \beta X$$

where Y = volume of tree,
X = basal area of tree,
α = true regression constant,
β = true regression coefficient.

If the regression is calculated from a sample, instead of from all trees in a stand, *estimates*, instead of the true values, of α and β will be obtained. As these estimates are subject to error, they will be referred to as *a* and *b* in order to distinguish them from the true regression constant α and the true regression coefficient β.

The estimate *a* of α is obtained from the equation :

$$a = \frac{\Sigma Y - b\,\Sigma X}{n} \qquad (19)$$

and the estimate *b* of β from the equation :

$$b = \frac{n\,\Sigma XY - (\Sigma X)(\Sigma Y)}{n\,\Sigma X^2 - (\Sigma X)^2} \qquad (20)$$

where n = number of sample trees, and
Σ denotes summation.

The error of *a* is partly dependent on that of *b*, and is partly due to the fact that the estimated mean volume ($\bar{y}'$) of the stand may not be the same as the true mean volume ($\bar{y}$).

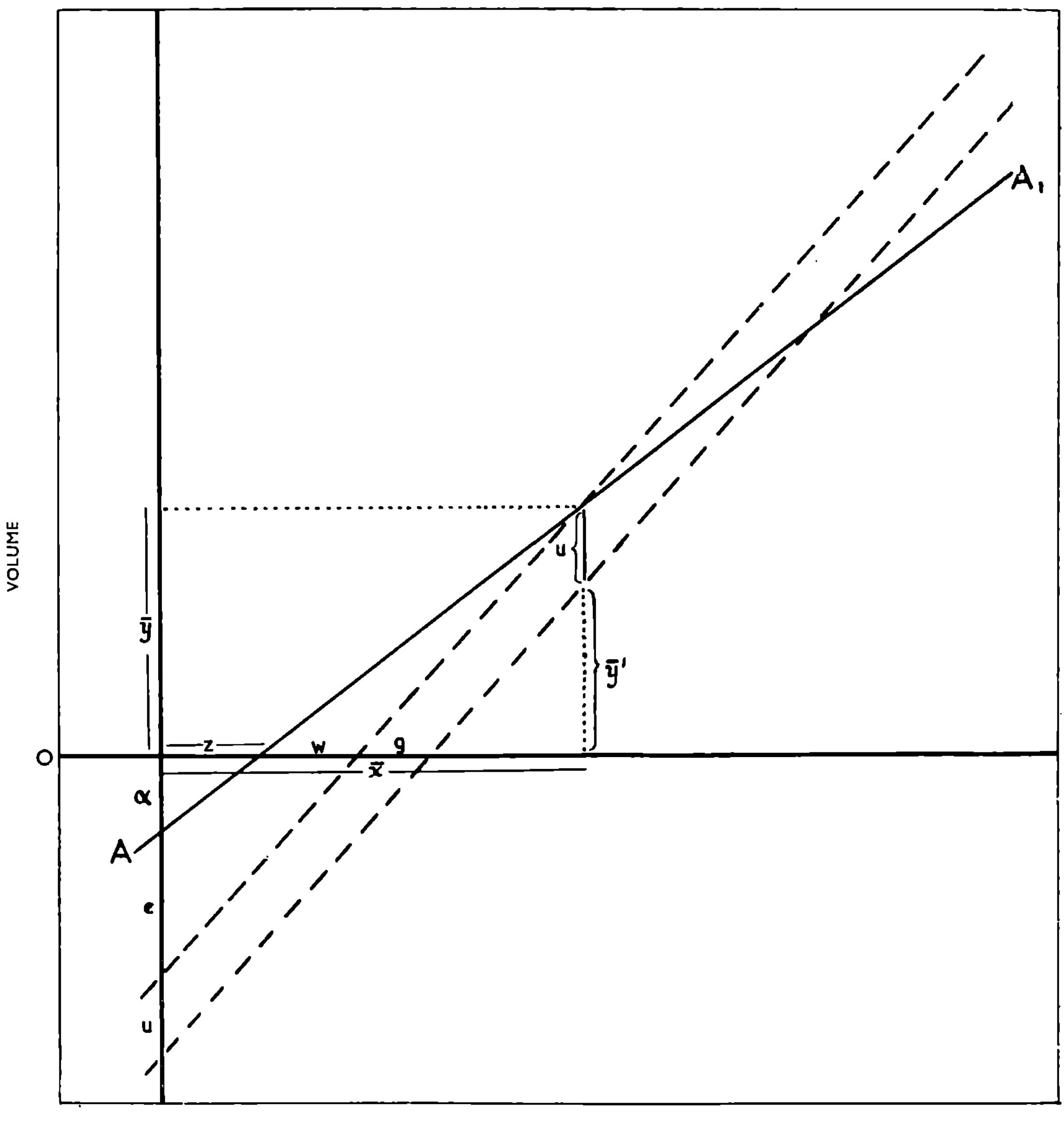

FIGURE 20. The relationship between the errors of $\bar{y}$, β, α and z.

These relationships are illustrated in Figure 20. The symbols used in its discussion are :

AA_1 = true regression line $Y = \alpha + \beta X$;
$\bar{x}$ = mean basal area of stand;
$\bar{y}$ = true volume at $\bar{x}$, i.e. mean volume of stand;
$\bar{y}'$ = the estimated volume at $\bar{x}$;
$\sigma_{\bar{y}}'$ = standard error of $\bar{y}'$;
$s_{\bar{y}}'$ = estimate of $\sigma_{\bar{y}}'$;
α = true regression constant ;
a = estimate of α ;
σ_a = standard error of *a* ;
s_a = estimate of σ_a ;
β = true regression coefficient ;
b = estimate of β ;
σ_b = standard error of *b* ;
s_b = estimate of σ_b ;
e = error in *a* if regression coefficient is estimated to be *b* instead of β ;
u = error in *a* if the volume at $\bar{x}$ is estimated to be $\bar{y}'$ instead of $\bar{y}$;
z = basal area at which Y = O ;
z′ = estimate of *z* ; the reason for introducing *z* will be explained later.
w = error in *z* if the regression coefficient is estimated to be *b* instead of β ;
g = error in *z* if the volume at $\bar{x}$ is estimated to be $\bar{y}'$ instead of $\bar{y}$.

The following relationships which are relevant to the argument may be deduced from Figure 20.

1. $-e = \bar{x}(b - \beta)$ (21)

because

$$\beta = \frac{\bar{y} - \alpha}{\bar{x}} \text{ (note in Figure 20 } \alpha \text{ is negative)}$$

$$b = \frac{\bar{y} - \alpha - e}{\bar{x}}$$

by subtraction :

$$b - \beta = \frac{-e}{\bar{x}}$$

$$-e = \bar{x}(b - \beta) \quad (21)$$

whence the standard error of *e* is :

$$\sigma_e = \bar{x}\sigma_b \quad (21a)$$

2. $u = \bar{y} - \bar{y}'$

whence the standard error of *u* is :

$$\sigma_u = \sigma_{\bar{y}}' \quad (22a)$$

3. $-\frac{\alpha}{z} = \beta$ (23)

whence

$$z = -\frac{\alpha}{\beta} \quad (23a)$$

and :

$$z' = -\frac{a}{b} \quad (23b)$$

4. The error *u* may be either in the same direction as, or in the opposite direction to, the error *e*. Estimating :

$s_e = \bar{x}s_b$ (from 21a)

and : $s_u = s_{\bar{y}}'$ (from 22a)

and assuming these two standard errors independent, as is the case when the regression line is calculated from a random sample of trees, then :

$$s_a^2 = s_e^2 + s_u^2 = \bar{x}^2 s_b^2 + s^2_{\bar{y}}' \quad (24)$$

5. $w = (\bar{x} - z)\left(\frac{b - \beta}{b}\right)$ (27)

because :

$$\beta = \frac{\bar{y}}{\bar{x} - z} \text{ and } \bar{y} = \beta(\bar{x} - z) \quad (25)$$

$$b = \frac{\bar{y}}{\bar{x} - z - w} \quad (26)$$

by substitution of (25) in (26) :

$$b = \frac{\beta(\bar{x} - z)}{\bar{x} - z - w}$$

$$\bar{x} - z - w = \frac{\beta(\bar{x} - z)}{b}$$

$$w = (\bar{x} - z) - \frac{\beta(\bar{x} - z)}{b}$$

$$w = \frac{(\bar{x} - z)(b - \beta)}{b} \quad (27)$$

6. $g = \frac{\bar{y} - \bar{y}'}{b}$ (28)

7. $z' = z + w + g$ (29)

and by substituting (27) and (28) in (29)

$$z' - z = \frac{(\bar{x} - z)(b - \beta) - (\bar{y}' - \bar{y})}{b} \quad (29a)$$

then, if the error in *b* is small compared with *b* itself (here the ratio is normally less than 10 per cent), we can, as a first approximation, replace *b* in the denominator by β ; then squaring and taking expected values we get :

$$\sigma^2_{z}' = \frac{(\bar{x} - z)^2 \sigma_b^2 + \sigma^2_{\bar{y}}'}{\beta^2} \quad (29b)$$

(since *b* and $\bar{y}'$ are uncorrelated).

For practical purposes β and the population variances may be replaced by their estimates b, s_b^2 and $s_{\bar{y}}'$ giving :

$$s^2_{z}' = \frac{1}{b^2}((\bar{x} - z)^2 (s_b)^2 + s^2_{\bar{y}}') \quad (30)$$

Therefore, if both $\bar{x}$ and *z* are positive, (since $\bar{x}$ is always > z) :

$$s^2_{z}' < \frac{1}{b^2}[\bar{x}^2 (s_b)^2 + s^2_{\bar{y}}']$$

and as :

$$\bar{x}^2 (s_b)^2 + s^2_{\bar{y}}' = s_a^2, \quad (24)$$

$$s^2_{z}' < \frac{s_a^2}{b^2} \text{ and :}$$

$$s_z' < \frac{s_a}{b}. \quad (31)$$

But if $\bar{x}$ is large in relation to z :

$$s_z' \approx \frac{s_a}{b} \qquad (31a)$$

Errors in the estimates of the regression constant a or the regression coefficient b, or of both these factors, may be caused by :

(i) errors in the actual measurements,

(ii) errors due to drawing the regression line by eye instead of calculating it.

(iii) errors arising from the fact that the regression is estimated from a sample of trees instead of being determined from all trees in a plot ; in addition to the normal sampling errors there may be an element of bias because of the subjective selection of the sample trees.

(i) *Errors in Measurement*

From various routine checks in the measuring procedure the errors in actual measurement are known to be too small to be of consequence, but the errors under (ii) and (iii) may be more serious and require attention.

(ii) *Errors in Drawing Graph*

It would not have been practicable to calculate all the regression lines which had previously been drawn by eye, but they were calculated for the Norway spruce sample plots at Bowmont (Hummel 1947) where there are four thinning grades replicated in a latin square. Particulars of these plots are given in Appendix V. The plots have been measured at five-yearly or six-yearly intervals, starting from 1930, so that there are 4 × 4 × 5 = 80 regression lines. In each of these, the graphically obtained values of b and a were compared with the corresponding calculated values. The results are shown in Table 15.

a is given in hoppus feet, while the numerical values of b represent the volume increase in hoppus feet for every 1-square-foot increase in basal area ; e.g. if $b = 20$, the volume of a tree with a basal area of 2 square feet would be 20 hoppus feet *more* than the volume of a tree having a basal area of only 1 square foot. These units for a and b apply throughout Part II, unless there is a specific statement to the contrary.

Each figure in the main body of Table 15 is the average for the four plots of one thinning grade in one year. In order to give some idea of the importance of the differences resulting from the two methods of determining b and a, the calculated mean values of these factors are also shown in Table 15. The table indicates that drawing the regression lines by eye has not introduced any bias into the estimates of b, but there is a suggestion of a slight negative bias in a at the first two measurements. This bias, if real, is fortunately too small to be of importance.

The errors in b and a which stand out as being particularly large are those for the D grade plots

TABLE 15

THE DIFFERENCES BETWEEN THE GRAPHED AND CALCULATED VALUES OF *b* AND *a* IN THE BOWMONT NORWAY SPRUCE PLOTS

Plot number and Thinning grade	Age (20 years)	(25 years)	(30 years)	(35 years)	(40 years)	Mean difference
			b			
S.85 B	0·4	−0·5	0·2	−0·3	0·9	+0.14
S.86 C	0·1	0·1	−0·3	0·9	−0·5	+0·06
S.87 D	−0·1	0·3	−2·5	−0·4	0·9	−0·36
S.88 L.C.	0·2	0·0	−0·8	0·4	−0·3	−0·10
Mean difference	+0·15	−0·025	−0·85	+0·15	+0·25	−0·065
Average calculated value of *b*	13·6	16·1	18·4	20·0	21·5	—
			a			
S.85 B	−0·037	−0·007	−0·087	+0·030	−0·210	−0·062
S.86 C	−0·022	−0·024	−0·028	−0·163	+0·094	−0·029
S.87 D	−0·007	−0·062	+0·464	+0·031	−0·160	+0·053
S.88 L.C.	−0·024	−0·020	+0·011	−0·076	+0·050	−0·012
Mean difference	−0·023	−0·028	+0·090	−0·044	−0·057	−0·012
Average calculated value of *a*	−0·358	−0·401	−0·395	−0·307	−0·239	—

at 30 years of age. Inspection of the records of these plots shows that the regression line in one of them, Plot 87 (3), clearly does not follow the trend of the points on which it is based. It appears that whoever drew the line must, for some reason, have considered the sample trees "unrepresentative" and drawn the line "as he thought it should go". In all other instances at Bowmont the difference between the calculated and graphed values of b and a are so small that they do not obscure the changes of these factors with age and thinning treatment. There are, however, a few instances elsewhere where the line drawn by eye does not fit the points. The last two measurements in Plot S.48, discussed later, provide an example.

(iii) *Sampling Errors*

The material available for examining the sampling errors is more restricted. It was not possible to use the sample tree measurements for obtaining unbiased estimates of the standard deviations of b, s_b, and of a, s_a, because the sample trees were chosen not at random, but subjectively by the method described earlier in this chapter. The only plots in which the effects of sampling and of the subjective selection of sample trees can be examined, are plots in which the volumes of all trees have been measured, so that the actual total volume of the plot is known. These measurements are available only in a few plots which have been clear felled.

Four of these felled plots are listed in Table 14, and particulars relating to them are given in Table 16. In each of these four plots, two sets of sample trees were selected from the records, one by the standard procedure, and the other by dividing the girth range into a number of equal classes and then taking one sample tree at random in each class.

In contrast to the other data discussed in Part II, all volumes recorded for these four plots are in hoppus feet *over* bark instead of hoppus feet *under* bark.

The values of b, a, s_b and s_a, as determined from the two sets of sample trees, are shown in Table 17. Table 18 gives the volumes calculated from these values of b and a, together with the actual volumes, and also the standard errors of the volume estimates, which were calculated from the formula :

$$s_{vol} = Ns_b\sqrt{\Sigma x^2\left(\frac{1}{n}+\frac{(\bar{x}_p-\bar{x}_s)^2}{\Sigma x^2}\right)} \qquad (32)$$

where :

s_{vol} = standard error of volume estimate of plot ;
N = number of trees in plot ;
n = number of trees in sample ;
$\bar{x}_p$ = mean basal area of plot ;
$\bar{x}_s$ = mean basal area of sample ;
Σx^2 = sum of squared deviations of the individual basal areas in the sample from their mean, i.e. :

$$\Sigma(X-\bar{x}_s)^2$$

Tables 17 and 18 show that the drawing of the volume-basal area line by eye has, as is usual, not appreciably affected the volume estimates in any of the plots. The tables also suggest, contrary to expectation, that the apparent precision of the estimates of a and b, and also those of the volume, are increased by very little, if at all, by the customary subjective method of selecting sample trees. The standard errors of the volume estimates are seen to range between 2·45 per cent and 4·18 per cent, for the subjective sets of sample trees ; and between 1·49 per cent and 6·29 per cent, for the randomly chosen sets. The main object of the subjective selection is to increase the precision of the volume estimate by endeavouring to find sample trees which

TABLE 16

PARTICULARS OF THE FOUR FELLED PLOTS

Plot number and locality	Area of Plot (Acres)	Thinning Grade	Species	Age (Years)	Number of trees per Plot	Top Height (feet)	Mean true Girth at breast height (inches)	Felled volume of plot hoppus ft. over bark
E.59 Highclere, Hants.	0·378	B	Corsican pine	37	322	72½	24	2,579
E.60 Highclere, Hants.	0·374	D	Corsican pine	37	106	73	34	1,907
E27 Lake Vyrnwy, Mont.	0·408	D	Douglas fir	29	166	61	26	1,315
S.32 Kildrummy, Aberdeen	0·317	C	Douglas fir	40	82	75½	37	1,536

TABLE 17

DETAILS OF b, a, s_b AND s_a IN THE FOUR FELLED PLOTS

Plot Number	Sample trees									
	Subjectively chosen					Randomly chosen				
	No. of sample trees	b	s_b	a	s_a	No. of sample trees	b	s_b	a	s_a
E.59	8	41·8	1·76	−2.28	1·08	8	39·7	2·20	−1.95	1·06
E.60	8	41·3	4·53	−3·39	2·94	8	36·0	1·55	−0·44	1·12
E.27	8	27·5	2·28	+0·02	1·08	8	26·8	3·05	−0·06	1·26
S.32	9	32·8	2·26	−0·63	2·03	9	32·5	4·83	−1·51	4·78

appear to be about average in volume for their particular breast-height quarter girth ; and it was surprising to find that, in these four plots at least, this object has not been achieved.

On the other hand, there is no evidence of any bias being introduced by this subjective selection of sample trees. However, on the evidence from four plots alone, the possibility of bias in all the other plots cannot be excluded, and it is desirable to consider what effects such a bias would have. If a bias were correlated with thinning treatment and age the matter would be serious ; because the bias would obscure the true changes of b and a with these factors, but if, as is more probable, any bias that may exist is not correlated with either of these factors, but is consistent throughout or varies only with the observer, then the results of the study would not be distorted, although there will be a loss of precision.

The results from the four plots suggest that, if s_b and s_a, as calculated from the subjectively selected sample trees, do not give entirely correct estimates of σ_b and σ_a, they give at least some indication of what these true standard deviations are likely to be. It was therefore considered reasonable to calculate the values of s_b and s_a from the subjectively selected sample trees in each of the Norway spruce plots at Bowmont. The resulting estimates of σ_b and σ_a

TABLE 18

THE VOLUME ESTIMATES IN THE FOUR FELLED PLOTS

Volumes in hoppus feet over bark

Plot number	Actual volume	Volume estimates by sample tree methods							
		Sample trees subjectively selected				Sample trees randomly selected			
		Regression calculated			Regression drawn by eye	Regression calculated			Regression drawn by eye
		Volume	Standard error: Hoppus feet	Standard error: %		Volume	Standard error: Hoppus feet	Standard error: %	
E.59	2,579	2,586	±108	4·18	2,595	2,524	±98	3·79	2,523
Difference from actual volume		+0·27%			+0·62%	−2·13%			−2·17%
E.60	1,907	1,872	±58	3·04	1,859	1,896	±28	1·49	1,893
Difference from actual volume		−1·84%			−2·52%	−0·58%			−0·73%
E.27	1,315	1,337	±46	3·46	1,332	1,297	±51	3·85	1,294
Difference from actual volume		+1·67%			+1·29%	−1·37%			−1·60%
S.32	1,536	1,539	±38	2·45	1,550	1,453	±97	6·29	1,480
Difference from actual volume		+0·20%			+0·91%	−5·40%			−3·65%

are given in the first half of Table 19, and they are represented graphically in the upper part of Figure 21 ; each number in the body of the table, and each point along the dotted lines on the graph, represents the mean of the four plots of the same treatment at a given remeasurement.

The treatment means for s_b range between 0·77 and 2·5, while the extreme values for individual plots, which are not shown in the table, are 0·16 and 4·78.

The values of s_b are of the same order as those in the four felled plots referred to in Tables 16, 17 and 18, where they were between 1·76 and 4·53 for the subjectively selected sets of sample trees, and between 1·55 and 4·83 for the randomly chosen sets.

The values of s_a are shown in the lower halves of Table 19 and Figure 21. The treatment means range between 0·066 and 1·059, while the extreme values of individual plots, not shown in Table 19, are 0·015 and 1·488. Even allowing for the fact that the volumes at Bowmont are under bark, these values of s_a are rather smaller than those found in the four felled plots, where they were 1·08, 1·08, 2·03 and 2·94 for the subjective sets of sample trees, and 1·06, 1·12, 1·26 and 4·78 for the randomly selected sets. It has already been remarked that s_a is proportional to the mean girth, and the larger values of s_a in the felled plots may in part be due to the fact that the mean breast-height quarter girths in the two plots with the largest values of

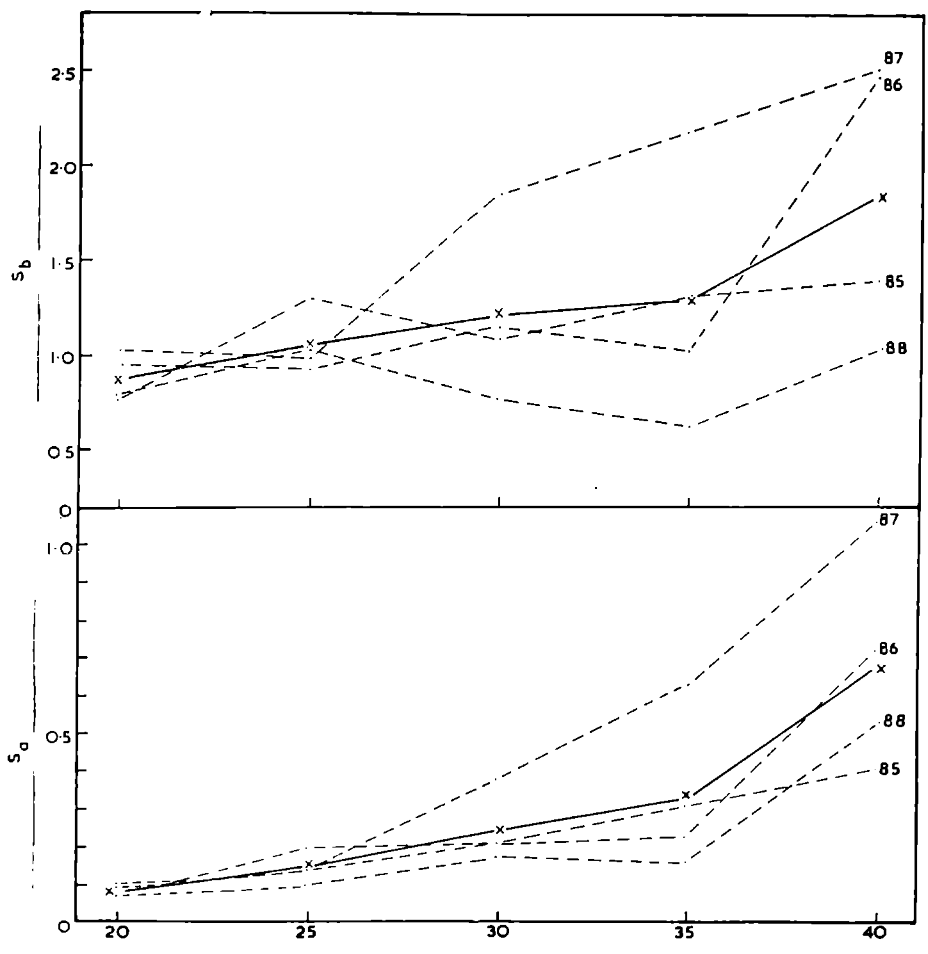

FIGURE 21. Bowmont Norway spruce plots. The changes of s_b and s_a with age. Each point is the mean of the four plots of the same treatment ; the crosses (X) and solid lines indicate means of all treatments.

TABLE 19

THE VALUES OF s_b AND s_a IN THE BOWMONT NORWAY SPRUCE PLOTS

Treatment number and thinning grade	Age (20 years)	(25 years)	(30 years)	(35 years)	(40 years)
			s_b		
S.85 B	0·77	1·30	1·09	1·31	1·40
S.86 C	0·95	0·93	1·15	1·03	2·47
S.87 D	1·02	0·98	1·85	2·18	2·50
S.88 L.C.	0·79	1·02	0·77	0·63	1·02
Average	0·88	1·06	1·21	1·29	1·85
Average calculated value of *b*	13·6	16·1	18·4	20·0	21·5
			s_a		
S.85 B	0·071	0·192	0·204	0·305	0·407
S.86 C	0·084	0·132	0·201	0·227	0·716
S.87 D	0·090	0·136	0·376	0·624	1·059
S.88 L.C.	0·066	0·096	0·169	0·156	0·519
Average	0·078	0·139	0·238	0·328	0·675
Average calculated value of *a*	−0·358	−0·401	−0·395	−0·301	−0·239

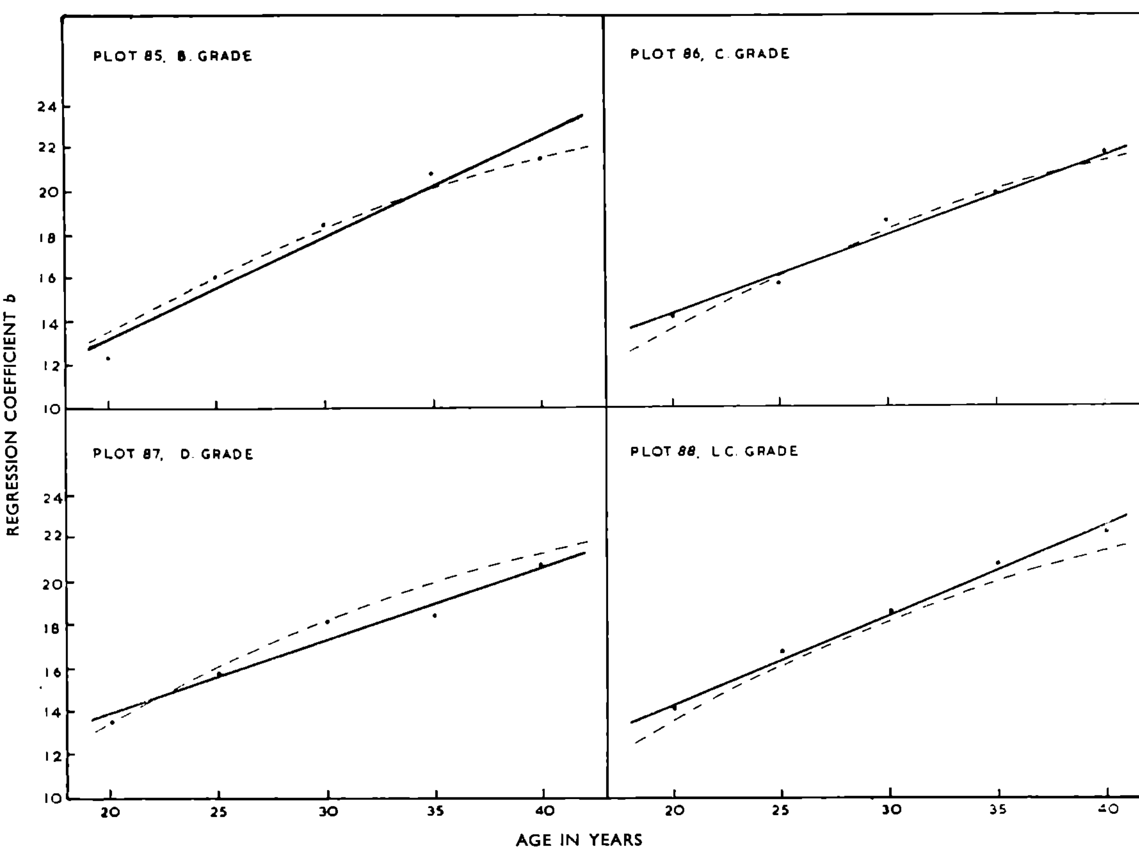

FIGURE 22. Bowmont Norway spruce plots. Changes of *b* with age by thinning treatments. The continuous lines are the calculated regression for each thinning treatment; the broken line is the calculated regression for the four treatments combined.

s_a, are 8½ inches and 9½ inches respectively ; while in the Bowmont plots the mean breast-height quarter girth never exceeds 8 inches. Bark, and these differences in girth, however, cannot account completely for the larger values of s_a in the four clear felled plots.

Combined Effect of Errors in Drawing Graph and Sampling Errors

The evidence so far suggests that the subjective selection of sample trees does not greatly affect the sampling errors. It also suggests that the standard errors in *b*, (s_b), which are caused by the sampling, may go up to over 4 hoppus feet per square foot, but are usually nearer half that amount ; and also that these sampling errors are usually greater than the additional errors introduced by drawing the volume-basal area line by eye, instead of calculating it. Therefore, a combined error in the estimates of *b* from those two sources may as a rule be expected to be slightly, but not very much, greater than the sampling errors alone.

In respect of *a*, the evidence is rather less conclusive, but it appears that the sampling errors may reach a maximum of at least 6 hoppus feet. As in the case of *b*, the errors caused by sampling appear usually to be greater than those caused by drawing the regression line by eye.

Some idea of the combined effect of the two sources of error to which the estimates of *b* and *a* are subject, may be obtained by comparing the values of these factors as determined at successive remeasurements in a sample plot. If *b* and *a* are plotted over age, or height (which is very closely correlated with age), the points are seen to follow a definite trend around which the individual values of *b* and *a* are scattered. This scatter must be due either to erratic changes in tree form from remeasurement to remeasurement, or to the errors discussed above, or to a combination of these two causes. From all that is known of the development of stands, erratic changes in tree form are most unlikely, even if the actual rate of growth fluctuates from year to year ; most of the scatter must therefore be attributed to the errors in determining *a* and *b*.

The scatter of individual *b* values around the regression of *b* on top height is shown in Figures 23, 25, 27, 29, 31, 33, and 35, each figure representing the sample plots of one species. The points relating to each plot are connected by lines, the type of symbol and line indicating the thinning treatment. These figures will be discussed more fully in Chapter 5 and only the scatter of the points will be considered here. This scatter is usually less than 3 units of *b*, but there are some plots in which

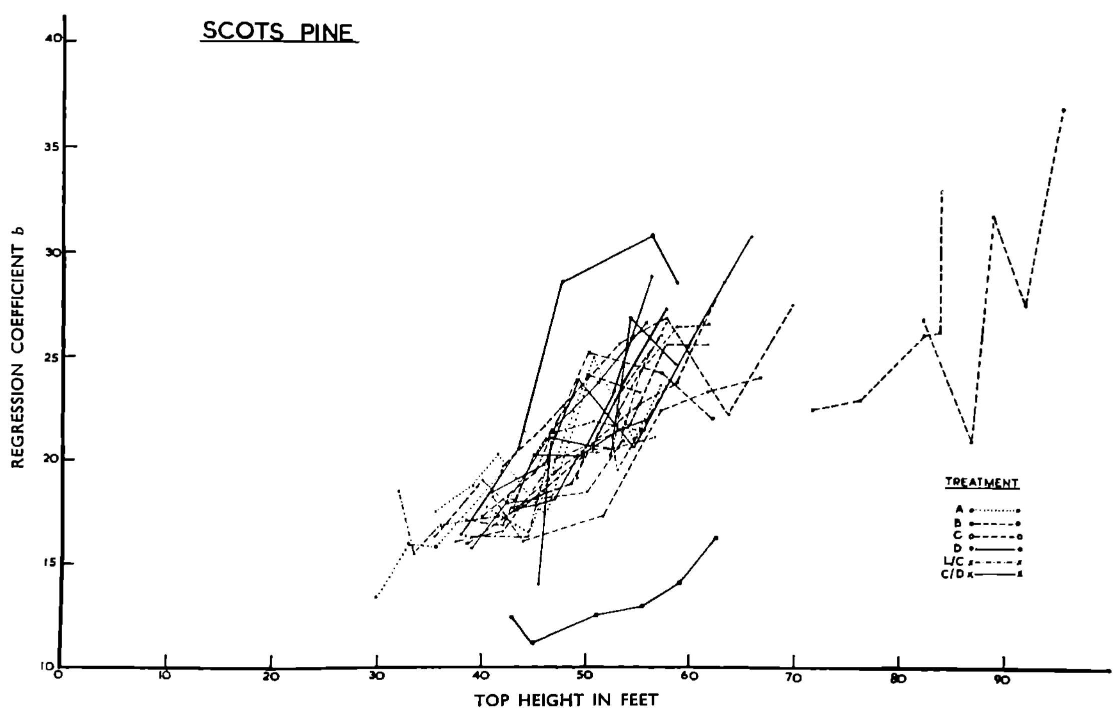

FIGURE 23. Scots pine. Relationship between regression coefficient *b* and height.

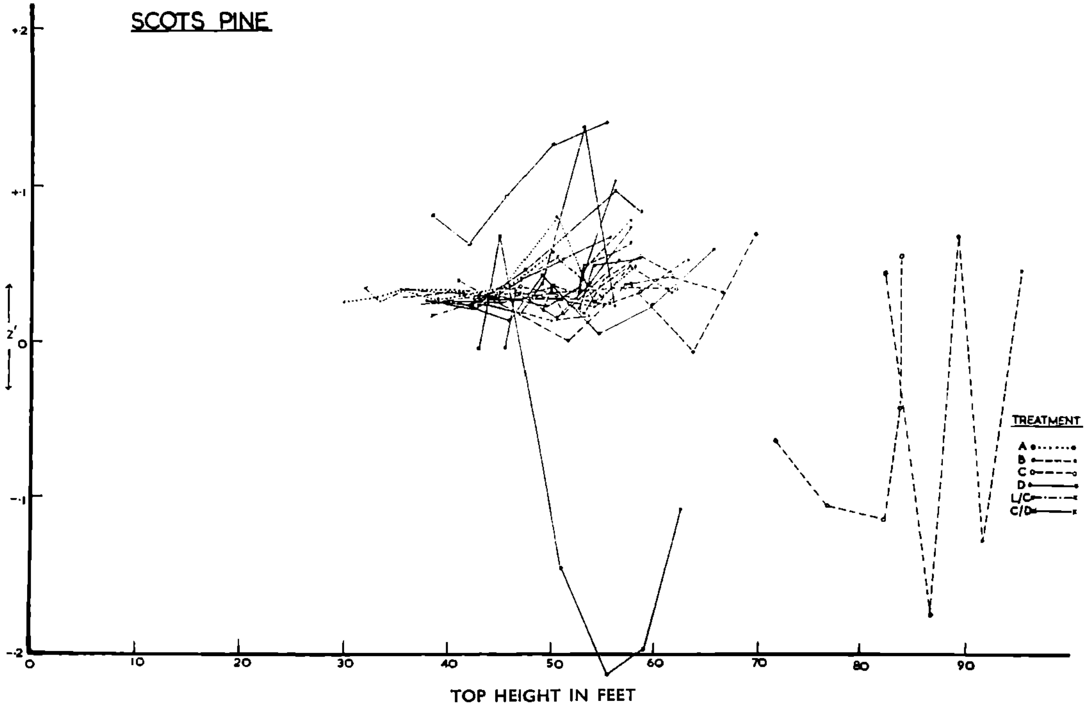

FIGURE 24. Scots pine. Relationship between z' and height.

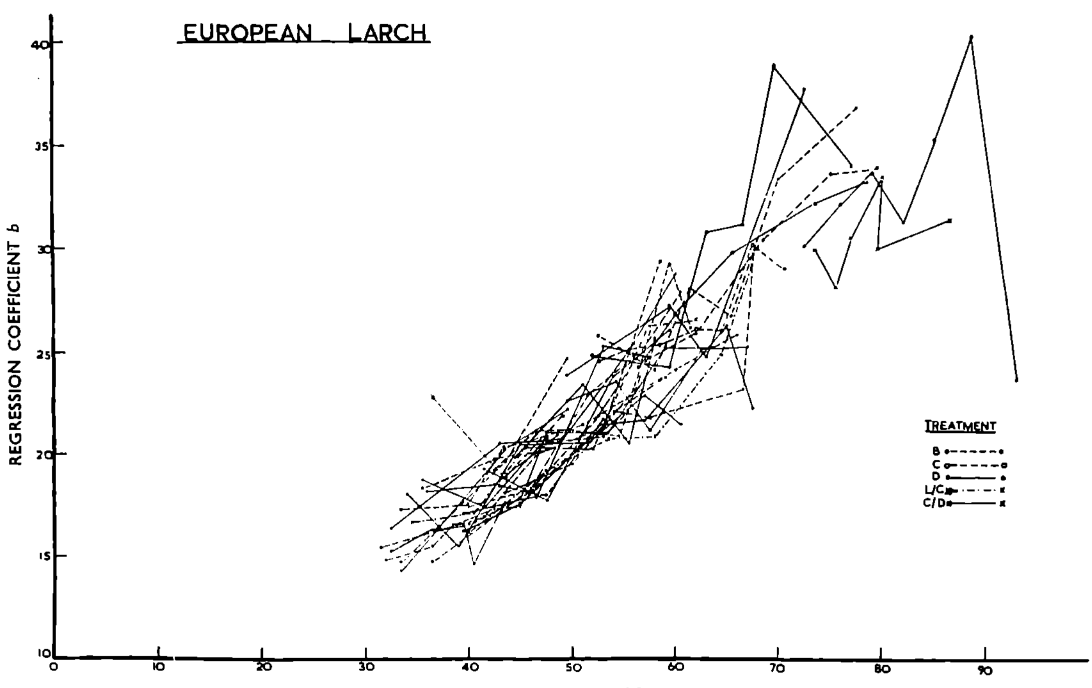

FIGURE 25. European larch. Relationship between regression coefficient b and height.

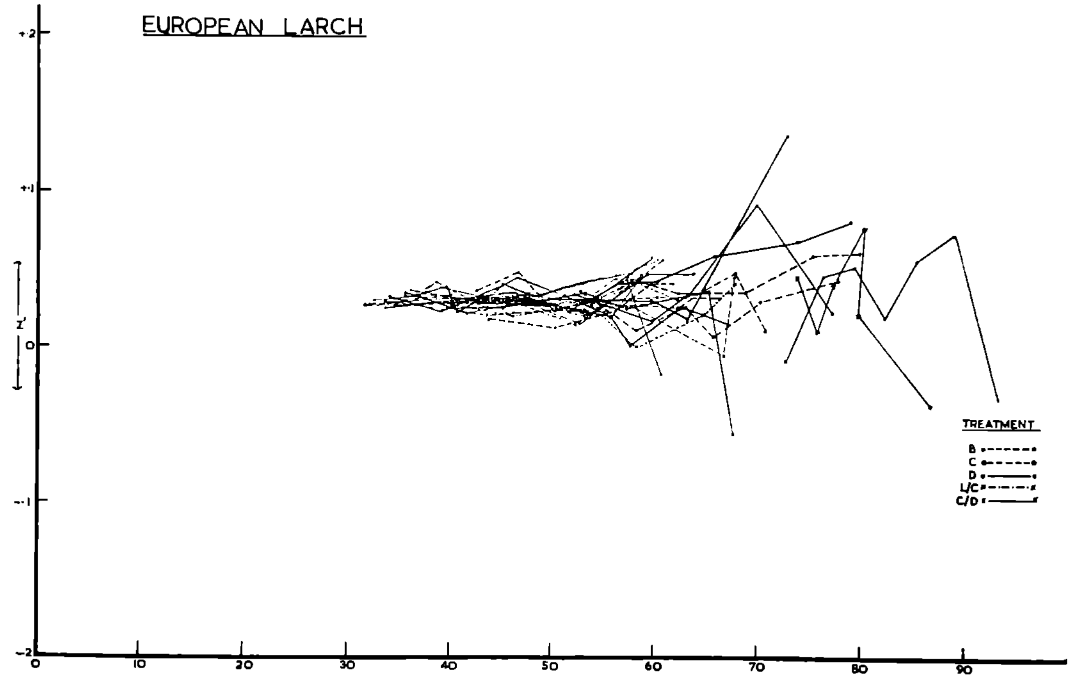

FIGURE 26. European larch. Relationship between z' and height.

deviations of as much as 5 units from the mean trend of a particular plot are found. The estimates of s_b given previously, which were based on the internal evidence of the sample trees from which the values of b and s_b were calculated, would lead one to expect deviations of the magnitude shown in these diagrams.

The values of a could have been plotted over top height, as was done for b, but for reasons that will be explained in Chapter 5, it was found preferable to plot z' over top height instead of a, z' being the basal area at which : $Y = 0$ (see Figure 20). The relationship between z' and top height is shown by species in Figures 24, 26, 28, 30, 32, 34, and 36.

In order to deduce the variability of a from these graphs, it is necessary to recall the relationship between s_z' and s_a. It has been demonstrated that if z is positive, then :

$$s_z' < \frac{s_a}{b} \qquad (31)$$

and that, if z' is small in relation to $\bar{x}$, then :

$$s_z' \approx \frac{s_a}{b} \text{ and :} \qquad (31a)$$

$$s_a \approx b s_z'$$

An inspection of the graphs for the seven species indicates that the average value of z' remains constant at about 0·03 square feet throughout the range of height, and that individual z' values may deviate from the regression of z' on height in a plot by as much as 0·2 square feet. The maximum values of s_z' are likely to be rather less, probably between 0·1 and 0·15 square feet.

The scatter of z' values is greatest near the upper limit of the height range, where the values of b are seen to be about 40 hoppus feet per square foot and $\bar{x}$ is known to be large in relation to z', because tall trees have large basal areas. The product bs_z' may therefore be expected to give only a slight overestimate of s_a (Equation 31a) and the maximum values of s_a are likely to lie between 4 and 8 hoppus feet. This agrees with the results, already discussed, for those plots in which s_a was obtained by direct calculation.

To sum up, the regression constants a and the regression coefficients b are subject to two main sources of error : first the drawing of the regression line by eye, and, secondly, the errors due to sampling. The resulting standard errors of b are usually less than 2 hoppus feet per square foot, but may be as much as 5 hoppus feet per square foot ; the larger values of s_b tend to occur where b itself is large and, expressed as a percentage, s_b is nearly always less than 10 per cent of b.

s_a is usually less than 0·5 hoppus feet, but may be as much as 8 hoppus feet, and s_z' is usually less

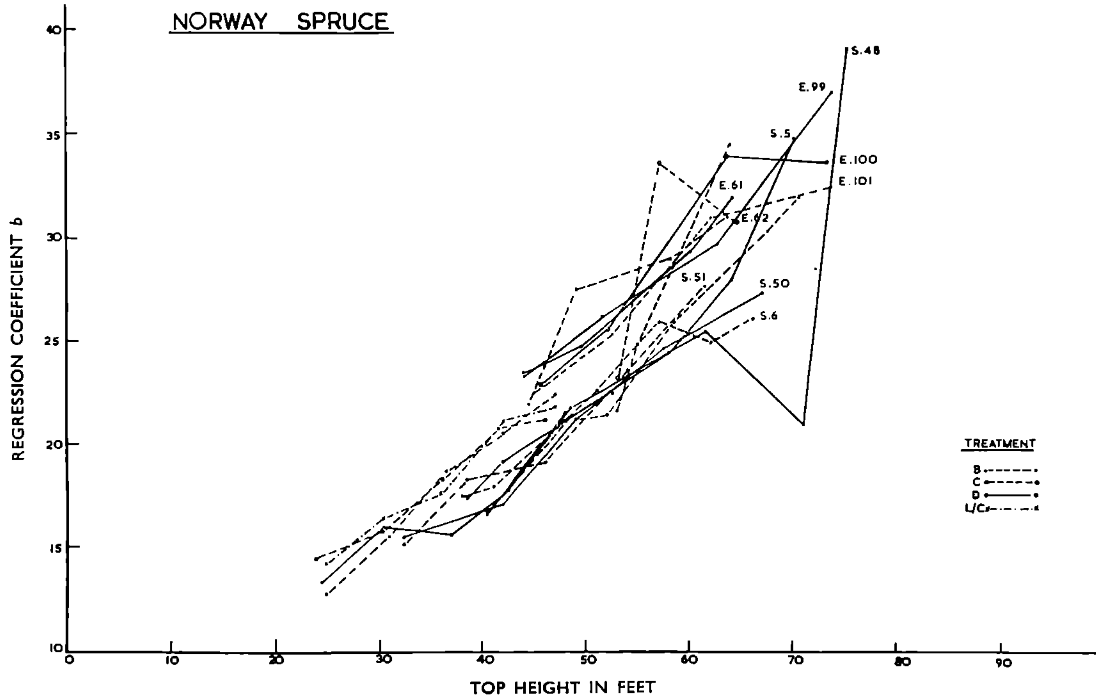

FIGURE 27. Norway spruce. Relationship between regression coefficient *b* and height.

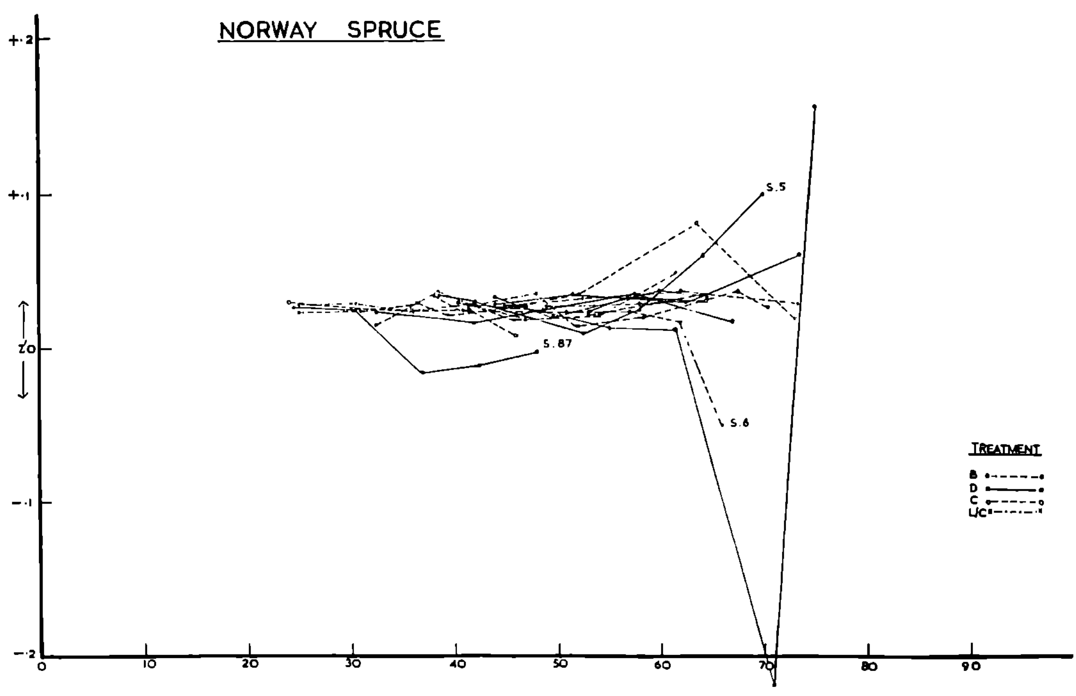

FIGURE 28. Norway spruce. Relationship between z' and height.

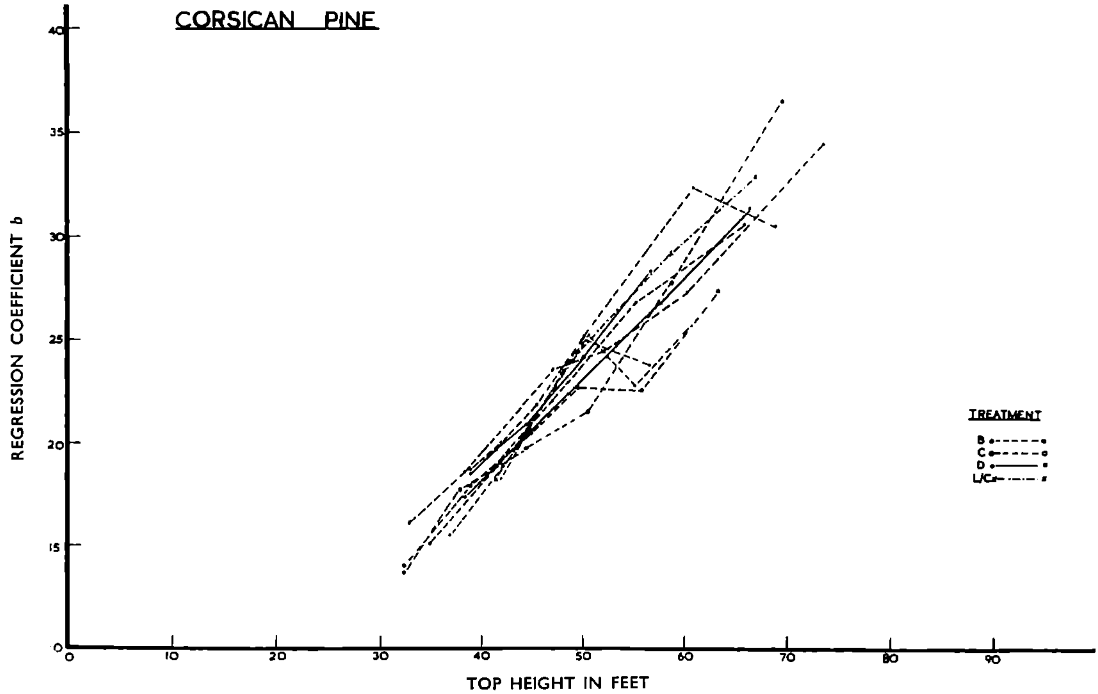

FIGURE 29. Corsican pine. Relationship between regression coefficient b and height.

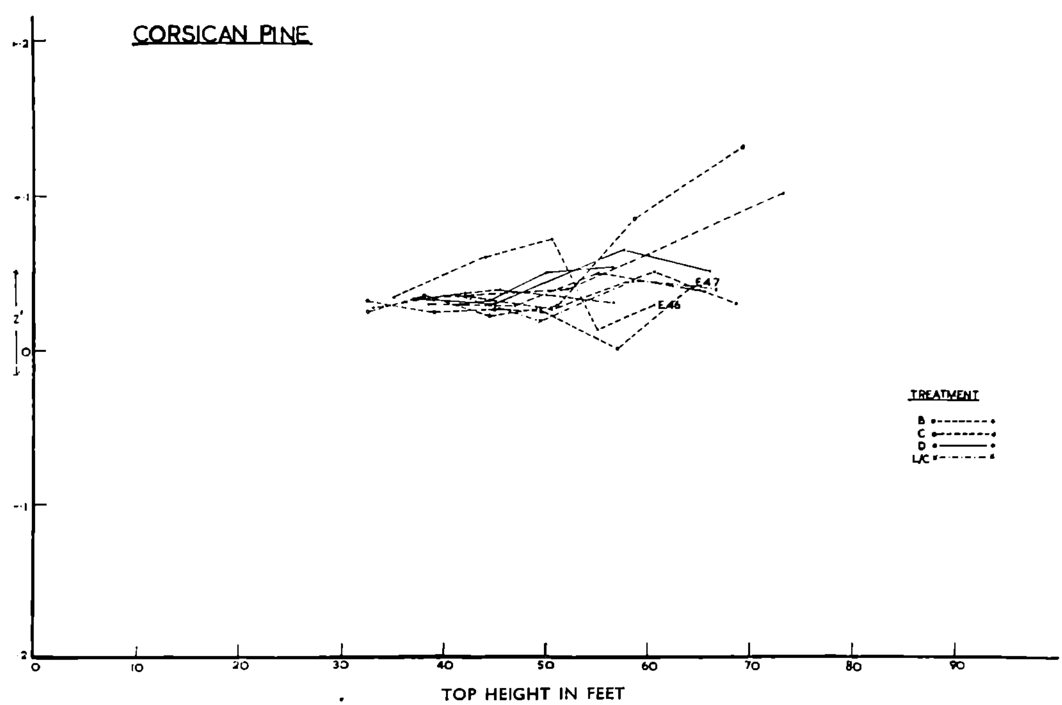

FIGURE 30. Corsican pine. Relationship between z' and height.

than 0·03 square feet but may be as much as 0·2 square feet. Maximum errors of this order would be very serious in volume estimates, for plots with trees of small girth, but these large errors are normally found in the older plots, where trees of small girth do not occur.

Effect of these Errors on Volume Estimates

It is, however, desirable to examine more closely the actual effects of the combined errors in *a* and *b* on the volume estimates in sample plots. In Table 17, the values of b, s_b, a and s_a have been given for the four felled plots, and Table 18 shows that the corresponding standard errors of the volume estimates vary between 6·29 per cent and 1·49 per cent of the measured volumes.

This degree of precision is probably typical of most sample plots, but it is also desirable to get some idea of the maximum errors that are likely to occur in the volume estimate under the existing sample plot procedure. In order to search for abnormal values of *a* and *b*, a visual inspection of Figures 23 to 36 was carried out. This presented no difficulty, although *z′* and not *a* had been plotted over height in these figures ; for it has been shown that a simple relationship exists between these two factors, viz. :

$$z' = \frac{-a}{b} \qquad (23b)$$

Two Norway spruce plots S.5 and S.6, which are marked on Figures 27 and 28, were selected for further examination. These two plots are in the same stand and, in spite of the contrasting thinning treatments (D and B grades), the values of *a* and *b* in the two plots have always been similar, except at the most recent remeasurement. It seemed likely that the apparent divergence at that remeasurement might be due to error.

Table 20 gives both the graphically derived and the calculated values of *b*, *a* and *z′* in each of the two plots ; the table also gives the means of the graphically derived values of these factors for the two plots combined.

It will be observed that the differences between the graphically derived values, and the calculated values, of *a*, *b* and *z′*, are very slight compared with the differences between the two plots ; this means that the apparent differences between the plots must be either real or due to sampling ; they cannot be due to the drawing of the regression lines by eye.

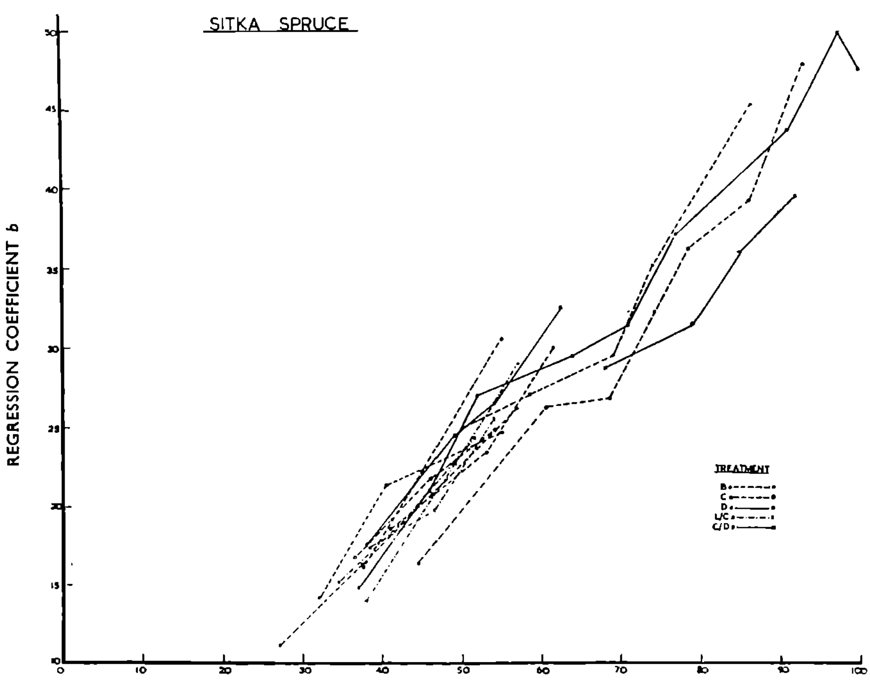

FIGURE 31. Sitka spruce. Relationship between regression coefficient *b* and height.

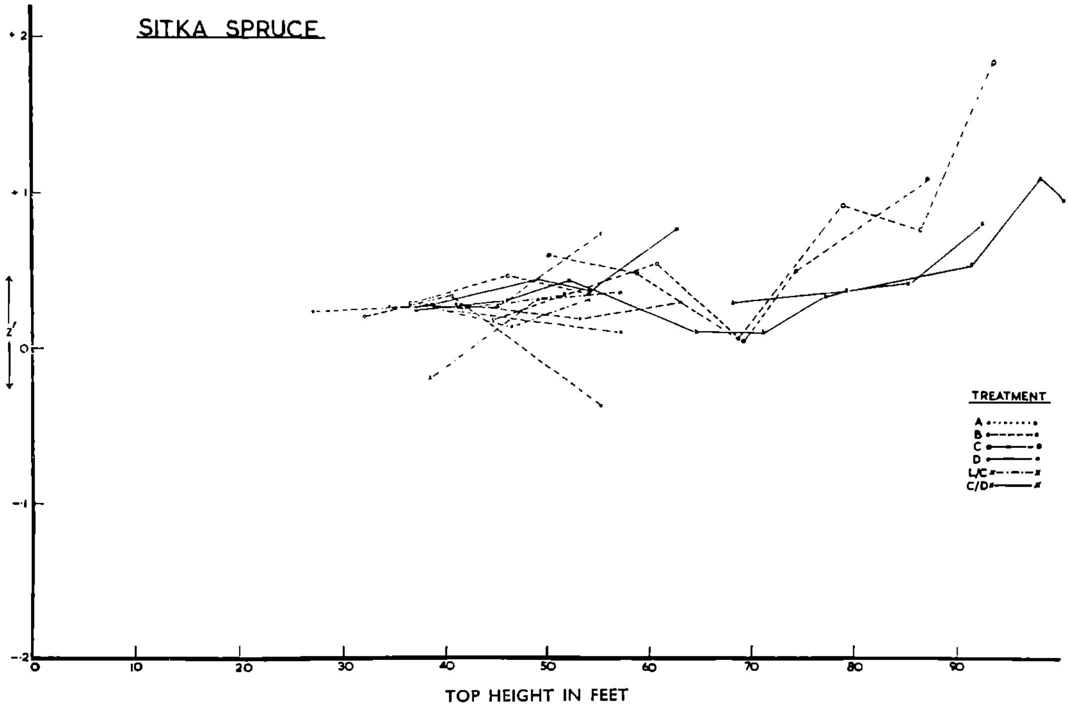

FIGURE 32. Sitka spruce. Relationship between z' and height.

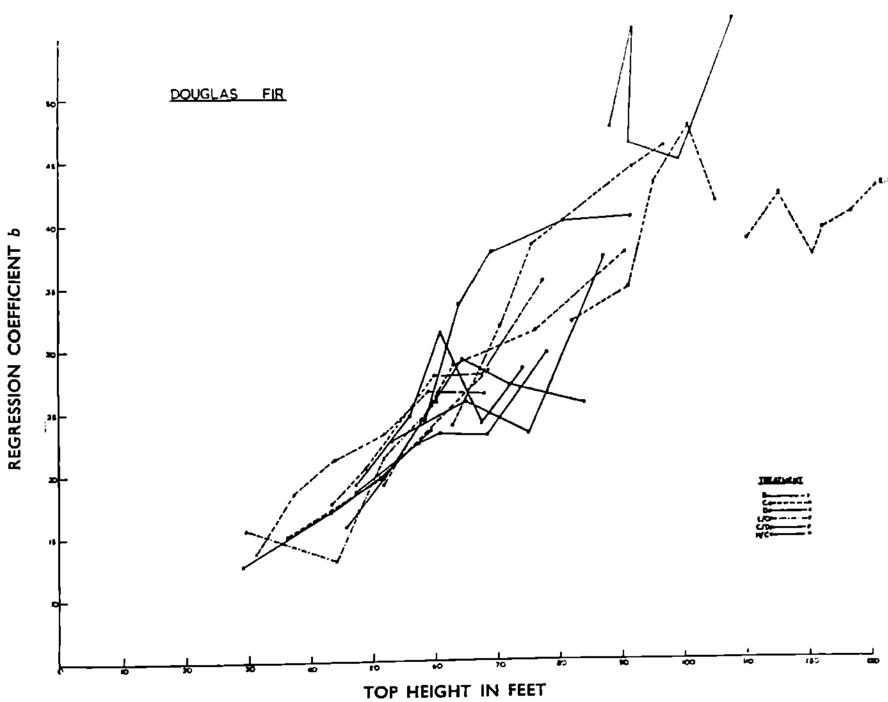

FIGURE 33. Douglas fir. Relationship between regression coefficient b and height.

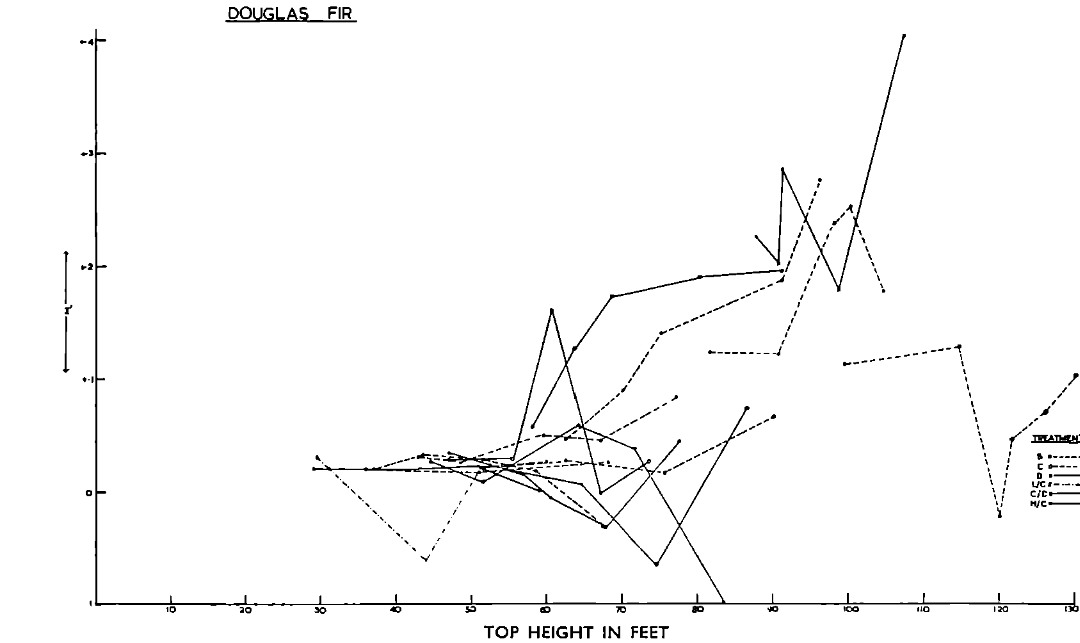

FIGURE 34. Douglas fir. Relationship between z' and height.

The values of the standard errors s_b and s_a, which are also shown in Table 20, confirm that the differences between the regression lines for the two plots are not significant at the 5 per cent probability level, and may well be due to sampling ; the mean values of a and b, given in Column 4 of Table 20, are within the 5 per cent fiducial limits of the calculated values for each plot.

Table 21 compares by groups, arranged in descending order of girth, the volumes, as determined from the original lines drawn by eye, with the volumes that would have been obtained by assuming a common regression of values on basal area in the two plots using the mean values of b and a given in Column 4 of Table 20.

In plot S.5 the mean line gives an overestimate of 119·3 hoppus feet, i.e. 6·30 per cent of the estimate derived from the original line ; and in plot S.6 the mean line gives an underestimate of 163·7 hoppus feet or 6·45 per cent ; but the differences in the volume estimates for individual girth groups reach +14 per cent in plot S·5, and —20 per cent

TABLE 20

NORWAY SPRUCE PLOTS S.5 AND S.6 1943. GRAPHICAL AND CALCULATED VALUES OF b, a AND z'

	Plot number and thinning grade		Mean
	S.5 (D. Grade)	S.6 (B. Grade)	
b graphical calculated	34·9 33·3 ± 4·158	26·2 26·8 ± 2·048	30·55 —
a graphical calculated	−2·77 −2·9 ± 3·17	+1·05 +0·76 ± 1·34	−0·86 —
z' graphical calculated	+0·080 +0·087	−0·040 −0·028	+0·028* —

Note: *This figure is calculated from the mean values of a and b above, and is therefore not the arithmetical mean of the two z' values.

TABLE 21

NORWAY SPRUCE PLOTS S.5 AND S.6. COMPARISON BY GIRTH GROUPS OF THE ORIGINAL AND MEAN LINE ESTIMATES IN 1943

Volumes and basal areas are in hoppus feet

Number of trees	Average basal area	Total under bark volumes per group from		Difference		Form factors	
		Original estimate	Mean line estimate	Hoppus feet	%	Original Estimates	Mean line Estimates
			Plot No. S.5				
20	0·809	490·1	477·1	−13·0	2·65	·424	·418
20	0·633	353·4	369·5	+16·1	4·56	·415	·426
20	0·558	301·0	323·5	+22·5	7·48	·411	·430
20	0·489	256·0	281·8	+25·8	10·08	·406	·437
56	0·356	492·6	560·5	+67·9	13·78	·397	·454
136	—	1893·1	2012·4	+119·3	6·30	·408	·434
			Plot No. S.6				
20	0·694	390·3	407·0	+16·7	4·28	·420	·438
20	0·525	303·2	303·8	+0·6	0·20	·437	·445
20	0·478	278·3	274·6	−3·7	1·33	·449	·446
20	0·426	251·6	242·8	−8·8	3·50	·458	·449
40	0·371	445·8	418·5	−27·3	6·12	·479	·452
40	0·297	368·5	328·3	−40·2	10·91	·508	·455
73	0·207	500·5	399·5	−101·0	20·18	·552	·455
233	—	2538·2	2374·5	−163·7	6·45	·479	·448

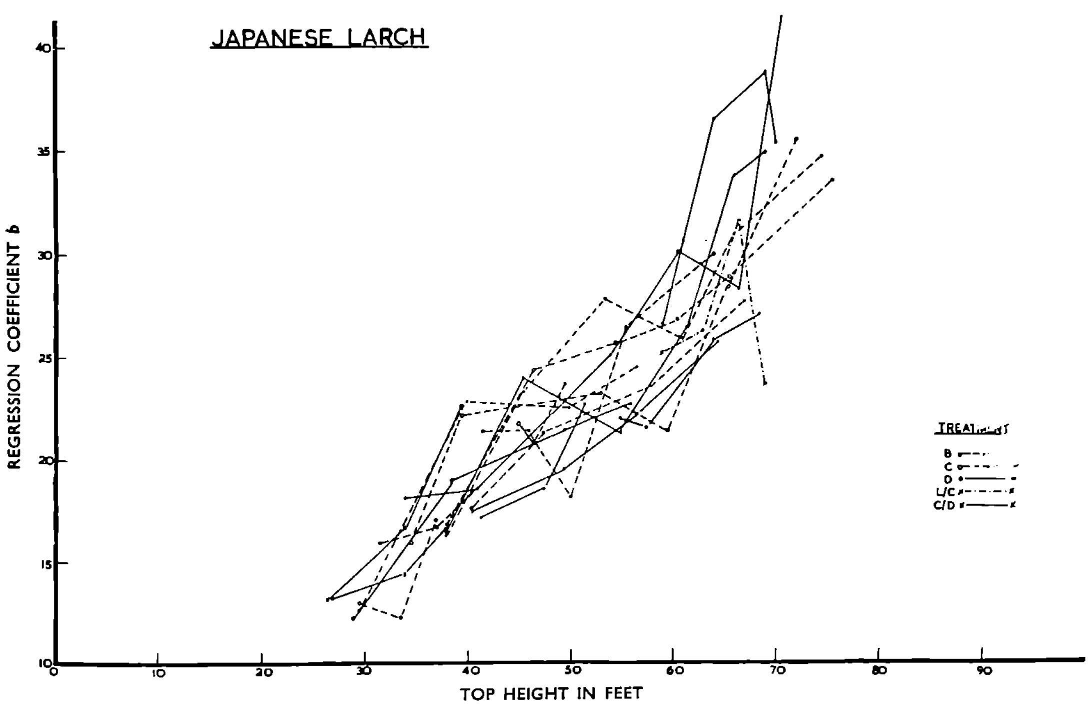

FIGURE 35. Japanese larch. Relationship between regression coefficient *b* and height.

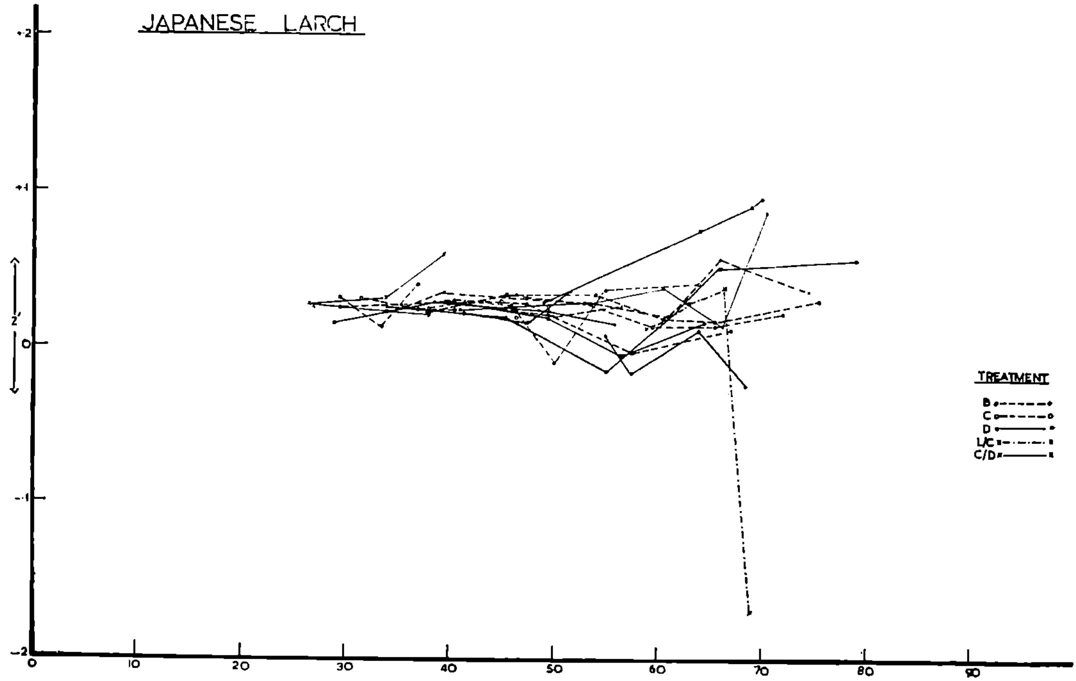

FIGURE 36. Japanese larch. Relationship between z' and height.

in plot S.6. These figures give some indication of the maximum errors in the volume estimates, which may result from the existing procedure ; it is not possible to decide with certainty whether the mean line gives a better or a worse estimate than the volume lines drawn separately for each of the two plots. There is, however, some evidence that the mean line estimate is better, at any rate for plot S.6, because the form factors implied in the separate line, as shown in Table 21, are abnormally high in the smallest girth classes. That the form factors are not correct is suggested by the fact that those recorded for the larger girth classes are quite normal, as are the form factors recorded for all girth classes in the 1938 remeasurement and shown in Table 22. The evidence relating to plot S.5 is less conclusive, but there is nothing to suggest that the mean line has not given as good an estimate of volume for each girth group as the original line for that plot.

These results will be further discussed in Chapter 6.

TABLE 22

NORWAY SPRUCE PLOTS S5 AND S6. COMPARISON BY GIRTH GROUPS OF THE VOLUMES AND FORM FACTORS IN 1938

Number of trees	Average basal area (square feet)	Volume under bark (hoppus feet)	Form factor	Number of trees	Average basal area (square feet)	Volume under bark (hoppus feet)	Form factor
	Plot S.5				Plot S.6		
20	0·671	345·6	·396	20	0·589	288·6	·397
20	0·529	259·2	·391	20	0·448	221·0	·412
20	0·470	226·2	·388	20	0·407	200·7	·413
20	0·418	194·8	·385	20	0·358	173·9	·414
40	0·337	302·5	·381	40	0·315	307·9	·414
45	0·218	199·6	·374	40	0·262	250·4	·414
				40	0·221	208·0	·414
				80	0·164	299·7	·414
				63	0·108	138·7	·414
165	—	1527·9	·388	343	—	2088·9	·411

Chapter 5

CHANGES WITH AGE, HEIGHT AND THINNING TREATMENT

THE changes of the volume-basal area line with age were examined separately in each of the seven species covered by this investigation. In Norway spruce, the data from the Bowmont plots were at first studied by themselves, and then together with the other data of that species. There were two reasons for this. The first is that the replication of each of four thinning treatments in a latin square permitted a more detailed analysis to be carried out for the Bowmont data than was possible elsewhere. The second reason is that, at Bowmont, all the regressions of sample tree volumes on basal areas had been calculated, while in most of the other plots they had only been drawn by eye. One of the sources of error discussed in the previous section was thus absent at Bowmont.

Studies on the Norway Spruce Plots at Bowmont

The first step was to examine the variation with age and thinning treatment of the calculated values of b. The results are shown in Table 23.

The data for each remeasurement were at first analysed separately. Bartlett's test for heterogeneity of variance (Snedecor 1946) was then applied in order to test whether there were significant differences between the error variances at successive remeasurements. There were no significant differences, and the standard error of b for individual plots which was calculated from the pooled error variances, was found to be 1·30 hoppus feet per square foot. This estimate of the standard error, referred to as s_p, therefore applies to all remeasurements.

Each figure in the body of Table 23 is the mean of the four plots of the same thinning treatment, and is therefore subject to a standard error of $\frac{1 \cdot 30}{\sqrt{4}} = 0 \cdot 65$. The estimated plot standard error ($s_p = 1 \cdot 30$) has two components: the first, s_b, results from the fact that b in each plot is determined from a sample instead of from all trees. This sampling error was examined in Chapter 4 and was shown, in Table 19, to vary between 0·77 and 2·50, with an overall mean of 1·26. The second component of s_p, which will be referred to as s_d, is due to actual differences in b between plots. If these two components are independent of one another, and there is nothing to suggest that they are not, then the following relationship exists:

$$s_p^2 = s_b^2 + s_d^2 \qquad (33)$$

From this equation and the estimates of s_p and s_b given above, it appears that s_d must be very small and that, as is to be expected, the true value of b has not varied significantly between the plots of the same treatment.

Table 23 suggests that b has also not varied between plots subjected to different thinning treatments. There has, however, been a very significant increase in b with age.

In 1930, when the plots were 20 years old and had a top height of about 25 feet, the values of b ranged between 12·4 for the mean of the B grade plots and 14·1 for the mean of the L.C. grade plots. At an age of 40 years, when the top heights had increased to between 46 and 48 feet, the range of b was between 20·7 for the D grade and 22·2 for

TABLE 23

THE REGRESSION COEFFICIENT b IN THE BOWMONT NORWAY SPRUCE PLOTS

Thinning grade	Treatment number	Age in years				
		20	25	30	35	40
				b		
B	S.85	12·4	16·1	18·5	20·9	21·6
C	S.86	14·4	15·8	18·7	19·9	21·8
D	S.87	13·5	15·8	18·2	18·4	20·7
L/C	S.89	14·1	16·8	18·5 ±0·65	20·8	22·2
All grades		13·6	16·1	18·4 ±0·32	20·0	21·5

TABLE 24

AVERAGE HEIGHT OF THE 100 LARGEST TREES PER ACRE IN THE BOWMONT NORWAY SPRUCE PLOTS

Thinning grade	Treatment number	Age in years 20	25	30	35	40
		Heights in feet				
B	S.85	25	31	36½	42	47
C	S.86	24	30½	36	41½	46
D	S.87	24½	30½	37	42½	48
L/C	S.88	25	30½	36	42	47

the L.C. grade. The trend of the increase in *b* with age is presented graphically in Figure 22. In each thinning treatment the regression of *b* on age suggests a slight curve, but in no treatment was the departure from linearity significant. If, however, all the treatments are taken together, the departure does become significant, and the dotted line represents this combined curve. If *b* is plotted over top height instead of age, the results are similar (age and top height being very closely correlated on any one site) ; but the curvature, although not entirely eliminated, is reduced, because height growth has dropped off slightly during the 20-year period under observation. This slowing down in height growth is indicated in Table 24.

The next step was to examine whether *a* had varied with age and thinning treatment. Table 25 gives the means for each treatment at each remeasurement.

The estimated plot standard errors for *a* (s_{pa}), which are shown in the bottom line of Table 25, were found to vary considerably from remeasurement to remeasurement, the actual figures being 0·074, 0·066, 0·085, 0·645 and 0·304. The differences between these standard errors were found to be significant, and they could therefore not be pooled as had been done in the case of *b*. Separate treatment standard errors are therefore shown for each remeasurement. As there are four plots in each treatment, these treatment standard errors are equal to one-half of the plot standard errors.

The plot standard errors (s_{pa}) have two components : the first (s_a), results from the fact that *a* in each plot has been determined from a sample of trees instead of from all trees. This component (s_a) has been discussed in Chapter 4, and was shown in Table 19 to average 0·078, 0·139, 0·238, 0·328 and 0·675 respectively at the five measurements. The

TABLE 25

THE REGRESSION CONSTANTS *a* IN THE BOWMONT NORWAY SPRUCE PLOTS

Thinning grade	Treatment No.	Age in years 20		25		30		35		40	
		a									
B	S.85	−0·26	±0·037	−0·38	±0·033	−0·48	±0·042	−0·48	±0·323	−0·39	±0·152
C	S.86	−0·41		−0·39		0·43		−0·37		−0·30	
D	S.87	−0·37		−0·36		0·22		+0·17		+0·20	
L.C.	S.88	−0·39		−0·47		−0·45		−0·55		−0·47	
Mean all grades		−0·36±0·018		−0·40±0·016		−0·40±0·021		−0·30±0·164		−0·24±0·076	
Plot standard errors		±0·074		±0·066		±0·085		±0·645		±0·304	

second component of s_{pa}, which will be referred to as s_{da}, is caused by actual differences in *a* between plots of the same treatment. If these two components are independent of one another, as they are likely to be, then the following relationship exists :

$$s_{pa}^2 = s_a^2 + s_{da}^2 \quad (34)$$

If s_{da} is very small, then the values of s_a and s_{pa} should be of the same order ; but they will, in practice, not be identical, and s_a may in fact even exceed s_{pa}, for at any given remeasurement the errors in the estimates which are due to sampling may be in the same direction in a majority of plots. If this happens, then an analysis of the differences between plots will give an estimate of σ_{pa} which is smaller than the mean of the estimates of σ_a. This appears to have happened at the second, third and fifth remeasurements, where the average values of s_a are larger than the values of s_{pa}.

From equation (34), and the figures for s_{pa} and s_a quoted above, it thus appears that s_{da} must be very small, i.e. the true value of α is very similar in the plots of the same treatment at any one remeasurement.

There are also no significant differences in *a* between thinning treatments or between remeasurements, the overall mean being —0·34 hoppus feet. It is, however, perhaps worth noting that, in the D grade, *a* has risen from —0·37 at 20 years to +0·20 at 40 years, while the *a* values in the other thinning treatments are all between —0·26 and —0·55.

Studies on other Norway Spruce Plots

After having examined the Norway spruce plots at Bowmont, all the plots in the seven species covered by this investigation were also examined ; but instead of studying the regressions of *b* and *a* on age, it was found preferable to study the regressions of *b* and *z'*, which has been shown to equal : $\frac{-a}{b}$ (23). The use of *z'* was preferred to *a*, because a preliminary inspection of the data suggested that, although the magnitude of *a* remained more or less constant with age (and height), at Bowmont, in most other localities it increased with age, while *z'* appeared to remain constant.

Top height was chosen as the independent variable instead of age for two reasons. First, the Bowmont plots suggested that the regression of *b* on *top height* is more likely to be linear than the regression of *b* on *age* ; secondly, it has been found that, when comparing crops from different sites and with widely differing rates of growth, most crop characteristics are more closely correlated with height than with age, and it was thought likely that the same would apply to *b*. For example, the total volume production of a Norway spruce stand with a top height of 80 feet is approximately 10,000 hoppus feet over bark per acre, irrespective of whether it has taken the stand fifty years or eighty years to reach that height. The results have confirmed the assumption that *b* is more closely correlated with top height than with age. At Bowmont this disadvantage of age did not apply, because all the replications are on a uniform site, so that age and height are very closely correlated ; age was chosen as being more convenient for analysis, because there were equal intervals of time between remeasurements, while the height intervals varied somewhat because of the gradual slowing down of height growth.

When dealing with the other Norway spruce data, the Bowmont plots were again included so that their behaviour could be compared with that of plots on different sites ; but in order to bring the Bowmont data into line with the others, the graphically obtained values of *b* and *z'* were used instead of the calculated values. In addition to the sixteen Bowmont plots, there were available for study six thinning series comprising thirteen plots. These plots are listed in Appendix V.

In Figure 27, the value *b* of each plot at each remeasurement has been plotted over the top height. The successive values of *b* in each plot are connected by straight lines, thinning treatments being differentiated by the type of line and symbols used. There are several points that emerge from this figure :

(i) *b* is not affected by thinning treatment.

(ii) *b* averages about 15, at 30 feet of height, and increases with height at a rate of about 4 units for every 10 feet increase in top height. The regression appears to be linear ; this does not contradict the very slight and not significant curvature found at Bowmont, which is almost too small to be detected in Figure 27.

(iii) The scatter of the points increases with top height. At a top height of 30 feet, the values of *b* range between 14 and 17 ; and, at 70 feet, between 26 and 36, if one omits Plot S.48. This omission is justified because inspection of the file of Plot S.48 showed that, at the last two remeasurements, the volume-basal area line had been drawn with obvious disregard for the points on which it was based.

The scatter of points appears to be due mainly to the erratic variations in *b* from remeasurement to remeasurement, which may be attributed to the various sources of error discussed previously. Nevertheless, the graph does suggest the possibility that, for a given height, *b* may vary slightly with locality. For example, at top heights of between 40 and 50 feet, plots E.61, 62, 99, 100, 101, have rather higher values of *b* than the other plots at that height. If this difference is genuine, it does not seem to be closely correlated with the rate of growth. These five plots all belong to the 1st and 2nd quality class, but plots S.50 and S.51, which belong to the group of plots where *b* appears to be lower, also belong to the 1st quality class (see Appendix V).

In Figure 28, in which z' has been plotted over top height in the same way as *b* was plotted in Figure 27, the following points may be observed :

Throughout the height range covered by the data, i.e. from a top height of about 25 feet to 75 feet, the average value of z' remains constant at about 0·03 square feet. In some plots the variation of z' from measurement to measurement is of appreciable magnitude ; but these variations, which tend to increase with height, are erratic, and are likely to be caused mainly by the sampling errors inherent in the estimation of z'. (The two aberrant z' values in plot S.48 are, however, mainly due to faulty drawing of the volume-basal area line.) Apart from the D grade at Bowmont, S.87, there are no plots where z' is consistently high or low. As regards S.87, no satisfactory explanation has been found for the three consecutive low values of z' : they cannot be attributed to site, because the other plots at Bowmont behave normally in this respect ; and they cannot be attributed to the thinning treatment, because in the D grade plots elsewhere z' is much the same as in the other thinning grades. Whatever their cause, the low z' values for treatment S.87 do not contradict the general evidence provided by Figure 28 that *z* has been unaffected by top height, locality or thinning treatment.

The observation that z' remains constant with height appears to contradict the finding at Bowmont that *a* has remained constant with height, for : $z' = \frac{-a}{b}$, and it has been shown that *b* increases with height. Therefore, if *a* remains constant with height, z' must decrease. However, by inspection of Figure 28 it becomes evident that this decrease is so small that the assumption of a constant value of : $z = 0{\cdot}03$ would introduce no serious error into the volume estimates for these plots.

Having established that *z* remains very nearly constant irrespective of height, thinning treatment or site, it is necessary to consider why the constant value should be approximately 0·03 square feet. A basal area of 0·03 square feet is equivalent to a breast-height quarter girth of 2⅛ inches, which in turn is equivalent to a stump diameter of about 3 inches. All volume measurements are to a diameter limit of 3 inches, any volume below that limit being ignored. A tree with a breast height quarter girth of 2⅛ inches or less will therefore, by definition, be shown as having no volume, while trees with a breast-height quarter girth of more than 2⅛ inches must have a volume. For this reason, in young plots where there are trees with breast height quarter girths down to 2⅛ inches, *z* must be approximately 0·03 square feet ; but it does not necessarily seem to follow that *z* must remain at 0·03 square feet when the smallest trees in a plot are well above the critical girth of 2⅛ inches, and the reason why *z* should then remain constant is not clearly understood.

Studies on other Species

The changes of *b* with top height in the species other than Norway spruce are shown as follows :

Figure 23 : Scots pine
Figure 25 : European larch
Figure 29 : Corsican pine
Figure 31 : Sitka spruce
Figure 33 : Douglas fir
Figure 35 : Japanese larch

A study of these graphs shows that, with a few modifications, the observations made in respect of Norway spruce also apply to the other species :

(i) *b* is not affected by thinning treatment. This suggests that *b* is probably not very closely correlated with girth, because girth is known to be greater in plots that have been thinned heavily over a period of years than in plots that have been thinned lightly.

(ii) The regression of *b* on height appears to be adequately linear to a top height of 80 feet, above which the scatter of the points is too great to indicate the trend very clearly.

(iii) The scatter of individual values of *b* from the mean value for a species at a particular height appears to be due mainly to the errors in determining *b* rather than to any variation in the true regression coefficient β with site. Nevertheless, there are a few plots with consistently high or low values of *b* at consecutive remeasurements, which suggests that, in these particular stands, β does differ genuinely from what it is elsewhere. The most conspicuous of these plots on the graphs are the Scots pine plot S.30, and the Douglas fir plot E.19, but no satisfactory explanation was found why they should differ from the other plots.

(iv) At any given height, the average value of *b* is similar in all the species examined, and it is not possible to say whether such differences between species as are evident on the graphs are genuine, or whether they are due solely to random variation between sites within a species, and between trees within sites.

In some species, the scatter of the points is greater than in others. It is low in Sitka spruce and Corsican pine, and great in Douglas fir.

The relationship between z' and top height, which for Norway spruce is shown in Figure 28, has been presented graphically for the other species as follows:

Figure 24 : Scots pine
Figure 26 : European larch
Figure 30 : Corsican pine
Figure 32 : Sitka spruce
Figure 34 : Douglas fir
Figure 36 : Japanese larch

Studying these graphs and comparing them with the graph for Norway spruce, which has already been discussed, it was observed that up to a top height of about 70 feet, the average value of z' remains constant at about 0·03 square feet in all species, on all sites and under all thinning treatments covered by the data ; and that there are very few individual stands where z' has deviated appreciably from this mean value of 0·03 square feet. Above a top height of 70 feet, z' becomes more variable, and although it has remained more or less constant in the majority of plots, there are some plots mainly in Douglas fir and Sitka spruce, where z' differs from 0·03 square feet by a greater amount than could safely be ascribed to error.

The practical applications of these findings will be discussed in the next chapter.

Chapter 6

APPLICATIONS TO FORESTRY PRACTICE

THE main points that have been revealed by the examination of the volume-basal area line within a stand may be summarised as follows :

(i) The regression of volume (Y) on basal area (X) can usually be regarded as linear :

$$Y = a + bX$$

(ii) The regression of b on top height is also adequately linear. The regression seems to be unaffected by thinning treatment, and the slight differences in b (at a given height) between species and localities are mostly within the limits attributable to error.

(iii) Up to a top height of about 70 feet, the average value of z' remains constant at about 0·03 square feet in all species, on all sites and under all thinning treatments covered by the data ; and there are very few stands where z' has deviated appreciably from this mean value of 0·03 square feet. Above a top height of 70 feet, z' becomes more variable, and although it has remained more or less constant in the majority of plots, there are some plots, mainly in Douglas fir and Sitka spruce, where z' differs from 0·03 square feet by a greater amount than could safely be ascribed to error.

(iv) With about eight sample trees, the errors in the determination of b and z' may lead to maximum errors of between five and ten per cent in the estimate of total volume in a plot, and up to twice that amount in the volume estimates for the extreme girth classes.

These findings appear to have two main applications to mensurational practice.

The first application is restricted to permanent sample plots in which the volume-basal area line has been determined at each measurement. In such plots it is possible to increase the precision of the volume estimates made at previous remeasurements by adopting the following procedure :

(i) Calculate or determine graphically the regressions of a and of b on top height.

(ii) Recalculate the plot volume at each remeasurement using these adjusted values of a and b in the regression : $Y = a + bX$.

In order to determine the volume-basal area line it has been necessary in the past, to determine, by sampling, both the slope and the mean point, i.e. the point whose co-ordinates are the mean basal area and the mean volume of the sample trees.

If, however, it is accepted that all such lines should pass through the point $z = 0{\cdot}03$ square feet, the line for a particular stand can be obtained from the mean point alone. This in itself saves a considerable amount of labour ; more important, however, is the fact that it opens the way to an even greater saving in labour, by making possible the construction of the so-called '*general tariff tables*', given in Appendix III, the use of which renders unnecessary the drawing of the volume-basal area line.

The term 'tariff' or 'tarif' has been applied in Switzerland, France and more recently in Germany, to volume tables based on breast-height girth (or diameter) alone, without differentiation of height classes. For want of a better English word, the term has been adopted here.

Tariff tables may be either local or general : those referred to in this paper are described as general tariff tables in that they may be applied to any site and to all coniferous species in Great Britain, provided that the top height of the stand is less than 80 feet. In being constructed from the volume-basal area line they differ somewhat from other general tariff tables in current use in Europe which have recently been described by Loetsch (1952).

These general tariff tables in Appendix III represent the tabulated values of a series of linear regression lines ; these lines have in common the point $z = 0{\cdot}03$ square feet, and the values of b are chosen so that the volume interval between successive lines is one hoppus foot at a basal area of one square foot, i.e. 12 inches breast-height quarter girth. Thus, tariff 18 shows a volume of 18 hoppus feet and tariff 19 a volume of 19 hoppus feet at a breast-height quarter girth of 12 inches.

The tariff appropriate to any particular stand at any particular time is then determined from the volumes of sample trees as follows :

1. Select an adequate number of sample trees by an objective sampling method, e.g. by taking every *n*th tree, or every tree in every *n*th row. A minimum of 20 sample trees is considered desirable.

2. For each of these sample trees determine the breast-height quarter girth (B.H.Q.G.) and the volume.

3. For each sample tree look in the tariff table for the B.H.Q.G. of the tree ; find the volume corresponding to that breast-height quarter girth which is nearest to the volume of the sample tree ; then record the number of the tariff in which that volume occurs : e.g. for a tree with a breast-height quarter girth of 5 inches and a volume of 4·77 hoppus feet the tariff number would be 32, the volume of 4·75 in that tariff being nearest to 4·77.

4. Add the tariff table numbers obtained under (3) and divide by the number of sample trees in order to arrive at the mean tariff number. The result is rounded up or down to the nearest whole number, and the tariff with that number is the one to be used.

Having determined which tariff table is applicable to an enumeration, the total volume for each breast-height quarter girth class in the stand is calculated by multiplying the number of trees in that class by the volume given for that girth in the appropriate tariff table.

In large enumerations it will normally suffice to estimate the number of trees in each girth class by girthing an objective sample, instead of girthing all trees, e.g. by girthing every *n*th tree or every tree in every *n*th row. It is, however, usually desirable to girth a minimum total of 200 trees.

Appendix III, in addition to giving the general tariff tables, also includes the following :

(i) A copy of the draft of a departmental instruction on the use of these tables for estimating the volumes of trees that have been marked for thinning but have not yet been felled.

(ii) A copy of a Forestry Commission form which was designed for recording all the necessary measurements and computations of volumes by the tariff table method, showing :

(iii) A worked example to illustrate the method and the use of the form.

Appendix IV shows a simple method of estimating the precision of volume estimates by the tariff table method.

Under certain conditions, it may be desirable to modify the procedures outlined above in some points of detail. For example, sampling by plots may be found preferable to sampling by trees or rows. Or an examination of the sampling errors may disclose that the sampling intensities for girthing or for felling sample trees should be increased or decreased. Also the method of finding the right tariff table may be varied, and two ways of doing this deserve special mention :

(i) Where the felling or climbing of sample trees is impracticable, the volumes of standing sample trees may be estimated by measuring their breast-height quarter girths and heights and applying the appropriate general volume table. This method is subject to the errors inherent in the use of general volume tables, and its use is confined to species for which general volume tables have been published. The method appears to be most useful for such purposes as national forest surveys, and it has actually been adopted in the Census of Woodlands in Britain.

(ii) Without the use of any sample trees, felled or standing, the tariff number which is appropriate to a stand may be estimated directly from the top height of the stand. This method of using the general tariff tables has not yet been worked out in detail or tested in practice, but it may well provide the key to the efficient working of the *Méthode du Contrôle* in even-aged high forest.

In applying the *Méthode du Contrôle* to uneven-aged high forest, it has been customary to prepare a local tariff table at the first enumeration, and it has been found that the same local tariff table can be applied at all subsequent enumerations, because, in uneven-aged high forest, the average height and average volume for a given girth always remain about the same.

In even-aged high forest, on the other hand, as has been shown in this paper, the average volume for any given girth increases with age because of height growth. Therefore, if the same tariff table is applied at successive enumerations, there will be a systematic underestimate of increment, and this underestimate will be greatest in young stands, where height growth is fast.

One way of eliminating the bias would be to prepare, at each enumeration, a new local tariff table or to select a tariff from the general tariff tables by means of sample trees, but the errors in the volume estimates at successive enumerations would then be uncorrelated, and the precision of the estimate of increment, although unbiased, correspondingly low. (Knuchel 1951.)

It seems likely, however, that an unbiased, and at the same time reasonably precise, estimate of increment could be obtained by using general tariff tables, but making the change from one tariff to another dependent on height growth. This is a point that merits further investigation.

It is appropriate to conclude this chapter by drawing attention to the main advantages and limitations of the general tariff tables as a means of estimating the volumes of standing trees.

The advantages are :

(i) An estimate is obtained not only of the total volume but also of its distribution by breast-height girth classes.

(ii) There is no need to choose the sample trees from any particular girth classes, although it is advisable to adopt an objective sampling procedure, e.g., every tree in every *n*th row, which will ensure that the sample trees are distributed over the whole range of girth in the stand or plot. If this is done, then any slight departure of z from 0·03 square feet will cause no great error in the total volume estimate, because an underestimate in volume at one end of the girth range will be countered, although perhaps not accurately balanced, by an overestimate at the opposite end.

(iii) There is no need to calculate the mean basal area of the stand or of the sample trees.

(iv) An approximate estimate of the precision of the volume estimate may be obtained without much difficulty provided that a suitable method of sampling is adopted.

(v) The general tariff tables may help in overcoming some of the difficulties encountered in applying the *Méthode du Contrôle* to even-aged high forest.

The main limitations of the method are :

(i) The method is not quite as easy to understand as some of the 'mean sample tree' methods. This is an important consideration when volumes are to be determined by junior personnel in the field.

(ii) Whether or not the general tariff tables are applicable under conditions other than those covered by this investigation, is not yet known. It seems probable, however, that they may safely be applied to all coniferous species in Great Britain, provided that the top height of the stand is not more than 80 feet, and preliminary tests suggest that these tables may also be used for young hardwood stands with mean breast-height quarter girths up to about 6 inches, but not to old hardwood stands where, in the few stands examined, z' departed very markedly from 0·03 square feet. It may be possible to prepare a different set of general tariff tables for old hardwood stands ; this is also a problem which requires further study.

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ditto (1951 (2)) *General Volume Tables for Douglas Fir in Great Britain*. Forest Record No. 15. H.M.S.O. 1s. 6d.

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Appendix I

DEFINITIONS AND CONVERSION FACTORS

Definitions

AGE. The age of a crop is reckoned from the year of planting, not from the year of sowing.

BASAL AREA. The sectional area of a tree at breast-height, i.e. 4 feet 3 inches above ground level.

All basal areas are given in square feet hoppus measure over bark ; i.e. the area is calculated by squaring the quarter girth in inches at the point of measurement, and dividing by 144 to get the result in square feet.

BREAST HEIGHT. This is taken to be at 4 feet inches (= 1·3 m above ground level, measured on the upper side of the tree on slopes.

HOPPUS FOOT. The cubic contents of round timber assessed by the method of multiplying the square of the quarter girth in inches, measured at the mid-point of the stem or log, by the length in feet, and dividing the result by 144. A log measuring 78·54 hoppus feet contains 100 cubic feet true measure.

QUALITY CLASS. Stands have been allocated to quality classes on the basis of the Revised Yield Tables for Conifers in Great Britain (Hummel & Christie, 1953).

QUARTER GIRTH. The girth of a tree divided by four, measured in inches.

TARIFF TABLES. The term *tarif* or *tariff* has been applied, in France, Switzerland and Germany, to volume tables giving volumes in terms of breast-height girth (or diameter) alone, without differentiation of height classes. For want of a better English word the term has been adopted here. Tariff tables, like other volume tables, may be either local or general : those given and discussed in this paper are general tariff tables. They consist of a series of 51 related volume tables ; the tariff which is applicable to a particular stand is determined from a limited number of sample trees.

TOP HEIGHT. Average total height of the 100 largest girthed trees per acre.

VOLUME. All volumes relate to stem wood measured from ground level to a 3-inch diameter limit over bark.

All volumes are given in hoppus feet.

In Part I all volumes are given *over* bark, in Part II volumes are *under* bark except where they are specifically stated to be *over* bark.

VOLUME-BASAL AREA LINE. This term is used to describe the relationship that exists between the volumes of trees and their sectional areas at breast height.

VOLUME TABLE. A table showing for a given species the average contents of trees for one or more given dimensions. The given dimensions in the general volume tables discussed in Part I are breast-height quarter girth (B.H.Q.G.) and total height.

Conversion Factors

The factors given below may be employed to convert the figures quoted in this paper from the quarter-girth system with British units of measurement to the Diameter system, true measure, with metric units. *The reciprocal factor is given in italics in each case.*

Feet to metres = feet × 0·3048

Metres to feet = metres × 3·2808

Inches quarter-girth to centimetres diameter = inches quarter-girth × 3·234

Centimetres diameter to inches quarter-girth = centimetres diameter × 0·3092

Number of stems per acre to number of stems per hectare = number of stems per acre × 2·471

Number of stems per hectare to number of stems per acre = number of stems per hectare × 0·4047

Square feet quarter-girth per acre to square metres per hectare = square feet quarter-girth per acre × 0·2922

Square metres per hectare to square feet quarter-girth per acre = square metres per hectare × 3·421

Cubic feet quarter-girth per acre to cubic metres per hectare = cubic feet quarter-girth per acre × 0·0891

Cubic metres per hectare to cubic feet quarter-girth per acre = cubic metres per hectare × 11·22

QUARTER-GIRTH TO TRUE MEASURE, BOTH IN BRITISH UNITS :

Cubic feet quarter-girth to cubic feet true measure = cubic feet quarter-girth × 1·273

Cubic feet true measure to cubic feet quarter-girth = cubic feet true measure × 0·7854

Appendix II

GENERAL VOLUME TABLES

In the interests of economy, only a sample page from these tables is reproduced here as Table 26. The full tables are published as Forest Records by H.M. Stationery Office, and may be obtained from the addresses appearing on the back cover at the prices set out below :

FORESTRY COMMISSION FOREST RECORDS

No. 8. General Volume Tables for Scots Pine in Great Britain.

1*s*. 6*d*. (1*s*. 7½*d*.)

No. 9. General Volume Tables for European Larch in Great Britain.

1*s*. 6*d*. (1*s*. 7½*d*.)

No. 10. General Volume Tables for Norway Spruce in Great Britain.*

1*s*. 0*d*. (1*s*. 1½*d*.)

No. 11. General Volume Tables for Corsican Pine in Great Britain.

1*s*. 6*d*. (1*s*. 7½*d*.)

No. 14. General Volume Tables for Japanese Larch in Great Britain.

9*d*. (10½*d*.)

No. 15. General Volume Tables for Douglas Fir in Great Britain.

1*s*. 6*d*. (1*s*. 7½*d*.)

* As discussed in this Bulletin (see pages 23-25) the Volume Table for Norway spruce may also be applied, with reasonable accuracy, to plantations of Sitka spruce.

TABLE 26

SAMPLE PAGE FROM FOREST RECORD NO. 9 : GENERAL VOLUME TABLES FOR EUROPEAN LARCH IN GREAT BRITAIN

Height 30 ft. to 40 ft.
B.H.Q.G. 2½ in. to 12½ in.

EUROPEAN LARCH

VOLUME TO 3in. TOP DIAMETER OVER BARK

B.H.Q.G. (inches)	Total Height (feet) 30	31	32	33	34	35	36	37	38	39	40	B.H.Q.G. (inches)
	Volumes in hoppus feet over bark											
2	—	—	—	—	—	—	—	—	—	—	—	2
¼	—	—	—	—	—	—	—	—	—	—	—	¼
½	·35	·35	·36	·36	·36	·36	·37	·37	·37	·37	·38	½
¾	·58	·58	·59	·59	·59	·60	·60	·61	·61	·61	·62	¾
3	·76	·78	·80	·82	·84	·85	·85	·85	·86	·87	·87	3
¼	·92	·94	·97	1·00	1·02	1·04	1·05	1·07	1·08	1·10	1·12	¼
½	1·11	1·15	1·18	1·21	1·24	1·26	1·29	1·31	1·34	1·36	1·39	½
¾	1·33	1·36	1·40	1·43	1·47	1·50	1·54	1·57	1·61	1·64	1·67	¾
4	1·54	1·58	1·62	1·66	1·70	1·74	1·78	1·82	1·86	1·89	1·93	4
¼	1·77	1·82	1·86	1·91	1·95	2·00	2·04	2·08	2·13	2·17	2·21	¼
½	2·03	2·08	2·14	2·19	2·24	2·29	2·34	2·38	2·43	2·48	2·52	½
¾	2·29	2·35	2·41	2·47	2·52	2·58	2·63	2·69	2·74	2·80	2·84	¾
5	2·57	2·64	2·70	2·76	2·83	2·89	2·95	3·01	3·07	3·13	3·18	5
¼	2·85	2·92	2·99	3·06	3·13	3·20	3·26	3·33	3·40	3·46	3·52	¼
½	3·16	3·24	3·32	3·39	3·47	3·54	3·62	3·69	3·76	3·83	3·90	½
¾	3·49	3·58	3·66	3·74	3·83	3·91	3·99	4·07	4·15	4·23	4·30	¾
6	3·82	3·91	4·00	4·09	4·18	4·27	4·36	4·45	4·53	4·62	4·70	6
¼	4·16	4·26	4·36	4·46	4·56	4·65	4·75	4·84	4·94	5·03	5·11	¼
½	4·52	4·63	4·74	4·84	4·95	5·05	5·16	5·26	5·36	5·46	5·55	½
¾	4·90	5·02	5·13	5·25	5·36	5·47	5·58	5·69	5·80	5·91	6·01	¾
7	5·30	5·42	5·54	5·67	5·79	5·91	6·03	6·15	6·27	6·38	6·49	7
¼	5·71	5·84	5·97	6·10	6·24	6·36	6·49	6·62	6·75	6·87	6·99	¼
½	6·13	6·28	6·42	6·56	6·70	6·84	6·97	7·11	7·25	7·38	7·50	½
¾	6·56	6·71	6·86	7·01	7·16	7·31	7·46	7·60	7·75	7·89	8·02	¾
8	7·00	7·17	7·33	7·49	7·65	7·80	7·96	8·11	8·27	8·42	8·56	8
¼	7·48	7·65	7·82	7·99	8·16	8·33	8·49	8·66	8·83	8·99	9·14	¼
½	7·95	8·14	8·32	8·50	8·68	8·86	9·03	9·21	9·38	9·55	9·72	½
¾	8·45	8·64	8·83	9·02	9·22	9·40	9·59	9·77	9·96	10·1	10·3	¾
9						9·97	10·2	10·4	10·6	10·7	10·9	9
¼						10·5	10·7	10·9	11·2	11·4	11·5	¼
½						11·1	11·4	11·6	11·8	12·0	12·2	½
¾						11·7	12·0	12·2	12·4	12·6	12·9	¾
10						12·4	12·6	12·8	13·1	13·3	13·5	10
¼						13·0	13·3	13·5	13·8	14·0	14·3	¼
½						13·7	13·9	14·2	14·5	14·7	15·0	½
¾						14·3	14·6	14·9	15·2	15·5	15·7	¾
11						15·0	15·3	15·6	15·9	16·2	16·4	11
¼						15·7	16·0	16·3	16·6	16·9	17·2	¼
½						16·4	16·7	17·1	17·4	17·7	18·0	½
¾						17·2	17·5	17·8	18·2	18·5	18·8	¾
12						17·9	18·3	18·6	19·0	19·3	19·6	12
¼						18·7	19·0	19·4	19·8	20·1	20·5	¼
½						19·5	19·8	20·2	20·6	21·0	21·3	½
¾						—	—	—	—	—	—	¾

Appendix III

THE TARIFF TABLE METHOD OF ESTIMATING STANDING VOLUMES AND GENERAL TARIFF TABLES

(This Appendix is reproduced from a departmental document (State Forest Memorandum 40) issued to Forestry Commission staff in June 1953.

The method is there applied to the estimation of the volumes of trees that have been marked for thinning, but have not yet been felled. The method, however, is equally applicable to the determination of the volume of the whole standing crop.)

The term 'tariff' has been applied by the Swiss to volume tables which are based on breast-height girth alone, without differentiation of height classes. For want of a suitable English word the term has been adopted here. Tariff tables may be either local or general : those referred to in this Appendix are general tariff tables because they may be applied to any site and to all coniferous species in Great Britain, provided that the top height of a stand is less than 80 feet (top height is the average height of the 100 largest trees per acre).

The reason why these general tariff tables (Table 27) are so widely applicable is that they are in fact not a single table but a large series of separate volume tables, the table which is appropriate to a particular stand being determined from the measurement of a limited number of sample trees. (See para. 7 below.)

The principle underlying the general tariff tables is this : in young, even-aged plantations, if the volume of each tree is plotted against its basal area, the points are normally scattered along a fairly clearly defined straight line. The slope of this line varies with species, age and site, but all such lines converge at the point on a graph where the basal area = 0·03 sq. ft. (hoppus measure) and the volume = zero. (See Figure 37.)

Each of the general tariff tables represents the tabulated values of one such line. These lines were drawn so that for a basal area of 1 sq. ft., i.e. 12 inches breast-height quarter girth (B.H.Q.G.), the volume intervals between successive lines are always 1 hoppus foot.

Each tariff occupies one column in the tables and is numbered according to the volume it gives for a B.H.Q.G. of 12 inches. This number is shown at the head of the column. For example, tariff 18 is the one which, at a B.H.Q.G. of 12 inches, shows a volume of 18 hoppus feet. There are 51 tariffs numbered consecutively from 10 to 60. Stands requiring tariffs beyond these limits are not likely to be encountered.

The volume estimate is arrived at as follows :

Field Work

1. Count every measurable tree marked for thinning (by species in mixtures), and record by the "gate" method in Part B of the thinning form (T.Y.5, specimen follows). Counting is best done at the time of marking. (Marked trees below measurable size, i.e. below 2½ inches B.H.Q.G., must, if required, also be counted, but kept separate from the others.)

2. Measure the quarter girth at breast height of every 10th measurable tree marked or, if sampling is done separately from marking, measure the quarter-girth of every measurable tree marked in every 10th row, or in lines running at right angles to the contour, or to the main fertility trend ; the lines to be spaced approximately to give a representative 10 per cent sample. If, however, fewer than 2,000 measurable trees are marked, the sampling fraction must be increased so that at least 200 trees are girthed.

The B.H.Q.G.s are entered by girth classes in Part C of the thinning form T.Y.5 (column 2).

In plantations of mixed species sampling should be done exactly on the same basis as for pure stands, except that each species is recorded separately; i.e. the quarter-girth samples will be every 10th measurable tree of each species, or the equivalent in line sampling. Usually a separate form will be required for each species.

3. Fell 1 per cent of the measurable trees marked for thinning : in small areas not fewer than 20 sample trees should be felled. The sample trees to be felled may be conveniently selected at the time of tallying and quarter-girthing, by a special marking, e.g. every 10th tree quarter-girthed could have

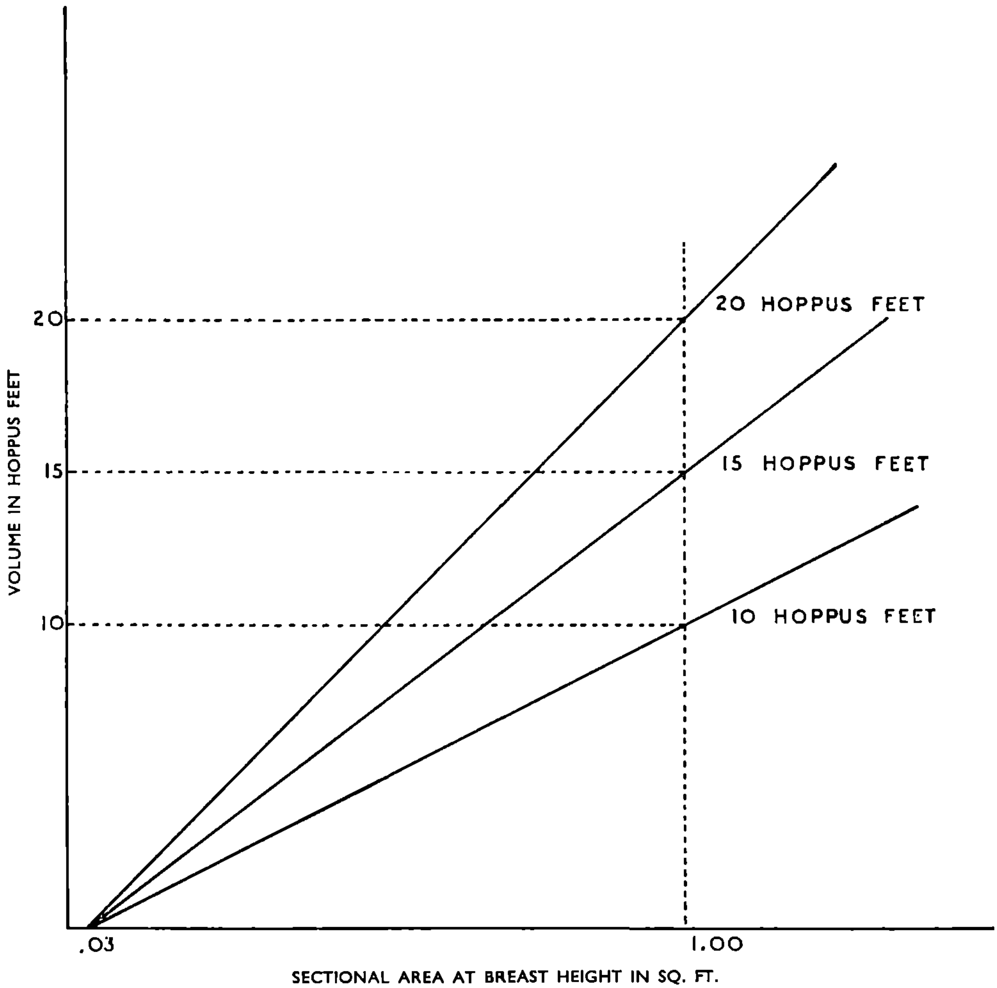


FIGURE 37. Graphical representation of tariffs Nos. 10, 15, and 20.

a distinctive scribe or other marking. Alternatively, sampling should be done in rows or lines on the same principle as described for quarter-girth sampling. In mixed stands the felled samples will be approximately 1 per cent of the trees of each individual species, a separate record being kept for each.

The following measurements are to be taken on the felled sample trees and recorded in Part D of the thinning form.

(i) Before felling—B.H.Q.G. (column 1).

(ii) After felling—length to 3 inches top diameter over bark (column 2).

(iii) After felling—mid quarter girth, i.e. quarter girth half-way to 3 inches diameter top (column 3).

Office Work

4. Calculate and record the number of trees girthed in each girth class (Part C, column 3).

5. Calculate and record the total number of trees in each girth class (Part C, column 4). If every 10th tree has been girthed, the total number is 10 times the number girthed : if quarter-girth sampling has been by lines, the multiplication factor is the total number of marked measurable trees over the total number of trees girthed ; e.g. if the total number of marked measurable trees is 2,980, and the total number of trees girthed is 327, the multiplication factor is $\frac{2980}{327} = 9{\cdot}1$.

6. Compute the volume of each felled sample tree by the hoppus method (i.e. mid Q.G. × length

to 3 inches diameter top) and record in Part D, Column 4.

7. Determine which tariff is applicable by finding the tariff that best fits the felled sample trees. This is done as follows :

(i) Look in the tariff table for the B.H.Q.G. of the sample tree ; find the volume corresponding to that B.H.Q.G., which is nearest to the volume of the sample tree ; then record the number of the tariff in which that volume occurs in Part D, Column 5 (e.g. for a tree with a B.H.Q.G. of 5 inches and a volume of 4·77 hoppus feet, the tariff number would be 32, the volume of 4·75 in that tariff being nearest to 4·77).

(ii) Add the tariff table numbers in column 5 and divide by the total number of entries (i.e. by the number of felled sample trees recorded on the form) rounding the result up or down to the nearest whole number in order to arrive at the *mean* tariff number. The tariff with this number is the one to be used. The volume given for each B.H.Q.G. in this tariff is entered in the appropriate line of column 5 in Part C of the thinning form.

8. The total volume in each B.H.Q.G. class is calculated by multiplying in Part C the entries in column 4 (compare para. 5) by the entries in column 5 ; the product is entered in column 6.

9. The volumes should be grouped : 6 inch B.H.Q.G. and under, and over 6 inch B.H.Q.G. (additional groupings may be required) and the totals for each group entered on the front of the form in Part A.

(*Note.* This particular grouping is required for departmental purposes ; other groupings could of course be used.)

N.B.—The tariff as determined above will only apply to the stand concerned at a particular stage of its growth. If, a few years later, it is again required to ascertain the volume of the thinnings marked in the stand, the procedure must be repeated and a fresh tariff calculated.

FORESTRY COMMISSION Form T.Y.5.

VOLUME ESTIMATE - STANDING THINNINGS (CONIFERS)

CONSERVANCY: WEST. FOREST: INVERLIEVER BEAT: — COMPT. NO. 185

SPECIES: SITKA SPRUCE AGE: 28 YRS NO. OF THINNING: 5 AREA TO BE THINNED: 18 (ACRES)

A

Species	2½" – 6" B.H.Q.G.		Over 6" B.H.Q.G.		TOTALS	
	No. of Trees	Vol. (H.Ft.U.B.)	No. of Trees	Vol. (H.Ft.U.B.)	No. of Trees	Vol. (H.Ft.U.B.)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
SITKA SPRUCE	1,580	7,723	600	6,298	2,180	14,021
TOTALS	1,580	7,723	600	6,298	2,180	14,021

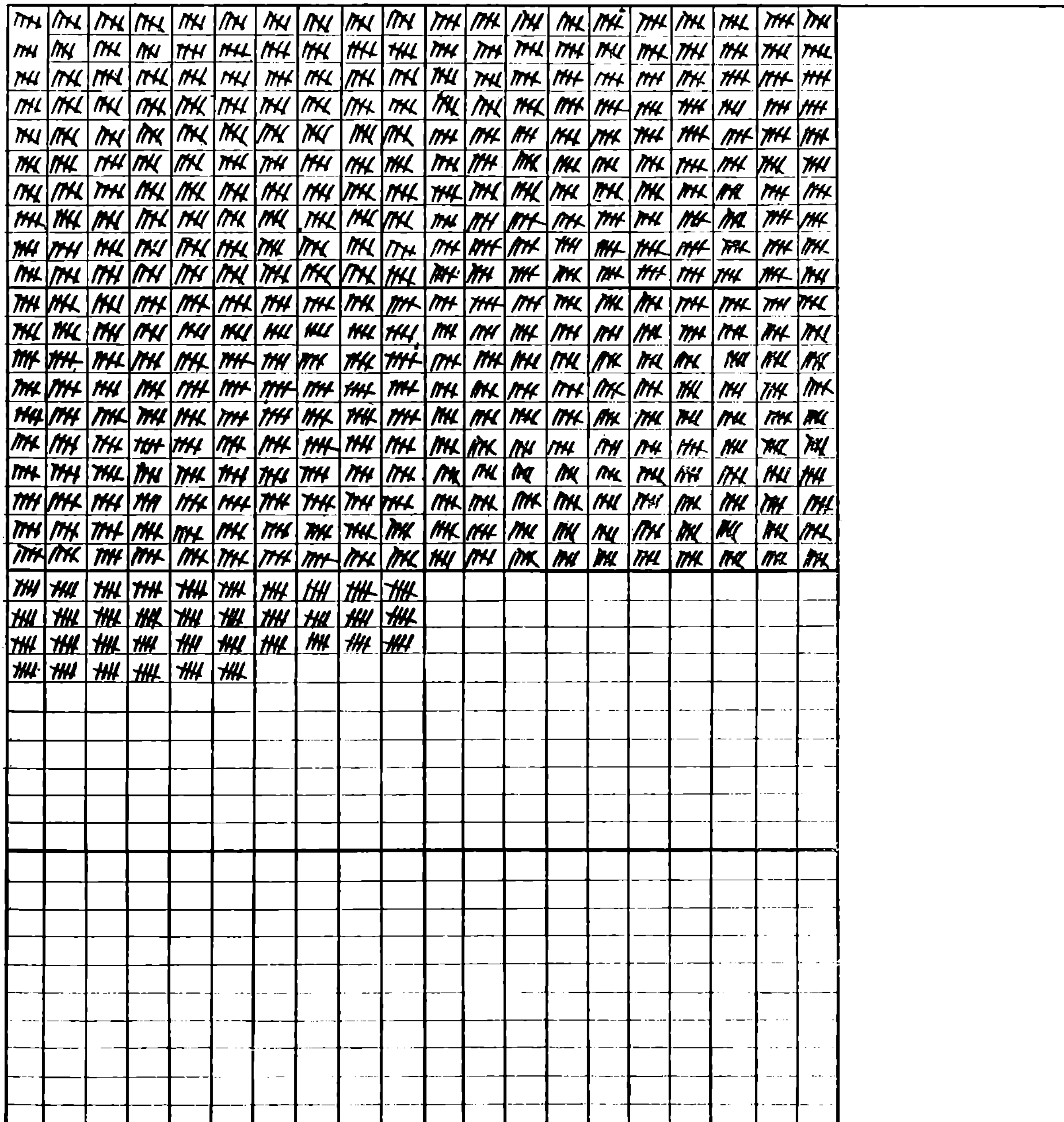
No. of unmeasurable trees (if tallied): —

B (continued from overleaf)

RECORD OF TREES MARKED

(to be used only if space provided overleaf is insufficient)

B. RECORD OF TREES MARKED (by Gate method - one gate to each square = 5 trees)



Enter number of trees marked, by Species and B.H.Q.G. classes in cols.2 and 4 of 'A' overleaf.

C. TREES GIRTHED

B.H. Q.G.	Number	Total	Stand Total	Vol. per Tree H.Ft.	Vol. of Class (Cols. 4 x 5) H.Ft.
(1)	(2)	(3)	(4)	(5)	(6)
3	||	2	20	1·09	22
3¼	||	2	20	1·42	28
3½	||	2	20	1·81	36
3¾	||||	4	40	2·24	90
4	卌 卌 ||||	14	140	2·67	374
4¼	||	2	20	3·13	63
4½	卌 卌 卌 卌 卌 |||	28	280	3·66	1,025
4¾	||||	4	40	4·19	168
5	卌 卌 卌 卌 卌 卌	30	300	4·75	1,425
5¼	卌 卌 ||	12	120	5·31	637
5½	卌 卌 卌 卌 ||||	24	240	5·94	1,426
5¾	卌 |	6	60	6·60	396
6	卌 卌 卌 卌 卌 |||	28	280	7·26	2,033
6¼	卌 |||	8	80	7·95	636
6½	卌 卌 ||||	14	140	8·68	1,215
6¾	卌 卌 ||	12	120	9·44	1,133
7	卌 |	6	60	10·20	612
7¼	||	2	20	11·10	222
7½	卌 |||	8	80	11·90	952
7¾	||	2	20	12·80	256
8	||||	4	40	13·70	548
8¼					
8½					
9	||	2	20	17·60	352
9¼	||	2	20	18·60	372
	Totals	218	2,180	—	14,021

Girth Sampling Fraction = $\frac{1}{10}$

D

DETAILS OF SAMPLE TREES FELLED AND MEASURED														
B.H. Q.G.	Length to 3" Top	Mid Q.G.	Volume (H.Ft.)	G.T.T. No.	B.H. Q.G.	Length to 3" Top	Mid Q.G.	Volume (H.Ft.)	G.T.T. No.	B.H. Q.G.	Length to 3" Top	Mid Q.G.	Volume (H.Ft.)	G.T.T. No.
(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
5	38	4¼	4·77	32										
4¼	30	3½	2·55	26										
6½	44	5¼	8·42	31										
4½	32	3¾	3·13	27										
4½	28	3¾	2·73	24										
4½	28	3¾	2·73	24										
6½	47	5¼	9·00	33										
4½	39	3¾	3·81	34										
7½	51	5¾	11·71	32										
5	39	4¼	4·89	33										
6	48	4¾	7·62	33										
7½	53	5¾	12·17	33										
9	60	7	20·40	37										
5½	43	4¾	6·74	36										
4½	39	3¾	3·81	34										
5¼	39	4¼	4·89	30										
4½	33	3¾	3·22	28										
7½	52	5¾	11·92	32										
6	43	4¾	6·74	30										
9¼	64	6¾	20·26	35										
6¾	54	5½	11·34	38										
6¾	50	5½	10·50	36										
			Total	698				Total					Total	

Mean tariff table No. $= \dfrac{\text{Total of G.T.T. Nos.}}{\text{No. of sample trees}} = \dfrac{698}{22} = 32$

TABLE 27

GENERAL TARIFF TABLES FOR CONIFERS IN GREAT BRITAIN

B.H.Q.G. (inches)	Volumes in hoppus feet over bark — Tariff Number 10	11	12	13	14	15	16	17	18	19	20	B.H.Q.G. (inches)
2	—	—	—	—	—	—	—	—	—	—	—	2
¼	·05	·06	·06	·07	·07	·08	·08	·09	·09	·10	·10	¼
½	·13	·15	·16	·17	·19	·20	·21	·23	·24	·25	·27	½
¾	·24	·26	·28	·31	·33	·36	·38	·40	·43	·45	·47	¾
3	·34	·37	·41	·44	·48	·51	·54	·58	·61	·65	·68	3
¼	·44	·49	·53	·58	·62	·66	·71	·75	·80	·84	·89	¼
½	·57	·62	·68	·74	·79	·85	·91	·96	1·02	1·08	1·13	½
¾	·70	·77	·84	·91	·98	1·05	1·12	1·19	1·26	1·33	1·40	¾
4	·84	·92	1·00	1·09	1·17	1·25	1·34	1·42	1·50	1·59	1·67	4
¼	·98	1·08	1·18	1·27	1·37	1·47	1·57	1·66	1·76	1·86	1·96	¼
½	1·14	1·26	1·37	1·49	1·60	1·72	1·83	1·95	2·06	2·17	2·29	½
¾	1·31	1·44	1·57	1·70	1·83	1·96	2·10	2·23	2·36	2·49	2·62	¾
5	1·48	1·63	1·78	1·93	2·08	2·23	2·38	2·52	2·67	2·82	2·97	5
¼	1·66	1·83	1·99	2·16	2·32	2·49	2·66	2·82	2·99	3·15	3·32	¼
½	1·86	2·04	2·23	2·41	2·60	2·78	2·97	3·15	3·34	3·53	3·71	½
¾	2·06	2·27	2·47	2·68	2·89	3·09	3·30	3·50	3·71	3·92	4·12	¾
6	2·27	2·50	2·72	2·95	3·17	3·40	3·63	3·86	4·08	4·31	4·54	6
¼	2·48	2·73	2·98	3·23	3·48	3·73	3·98	4·22	4·47	4·72	4·97	¼
½	2·71	2·98	3·25	3·52	3·80	4·07	4·34	4·61	4·88	5·15	5·42	½
¾	2·95	3·24	3·54	3·83	4·13	4·42	4·72	5·01	5·31	5·60	5·90	¾

Notes : (1) B.H.Q.G.=Breast-height quarter girth.
(2) All measurements are over bark.

TABLE 27—*cont.* TARIFF TABLES FOR CONIFERS

B.H.Q.G. (inches)	Volumes in hoppus feet over bark											B.H.Q.G. (inches)
	Tariff Number											
	10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	
7	3·20	3·52	3·83	4·15	4·47	4·79	5·12	5·43	5·75	6·07	6·39	7
¼	3·45	3·80	4·14	4·49	4·83	5·18	5·53	5·87	6·22	6·56	6·91	¼
½	3·72	4·09	4·47	4·84	5·21	5·58	5·96	6·33	6·70	7·07	7·44	½
¾	3·99	4·39	4·79	5·19	5·58	5·98	6·39	6·78	7·18	7·58	7·98	¾
8	4·27	4·69	5·12	5·55	5·97	6·40	6·83	7·26	7·68	8·11	8·54	8
¼	4·57	5·02	5·48	5·94	6·39	6·85	7·31	7·76	8·22	8·68	9·13	¼
½	4·87	5·35	5·84	6·33	6·81	7·30	7·79	8·27	8·76	9·25	9·73	½
¾	5·18	5·69	6·21	6·73	7·24	7·76	8·28	8·80	9·32	9·83	10·4	¾
9	5·50	6·04	6·59	7·14	7·69	8·24	8·79	9·34	9·89	10·4	11·0	9
¼	5·81	6·40	6·98	7·56	8·14	8·72	9·31	9·88	10·5	11·0	11·6	¼
½	6·16	6·77	7·38	8·00	8·61	9·23	9·85	10·5	11·1	11·7	12·3	½
¾	6·50	7·14	7·79	8·44	9·09	9·74	10·4	11·0	11·7	12·3	13·0	¾
10	6·85	7·53	8·21	8·90	9·58	10·3	11·0	11·6	12·3	13·0	13·7	10
¼	7·22	7·94	8·66	9·38	10·1	10·8	11·6	12·3	13·0	13·7	14·4	¼
½	7·59	8·35	9·10	9·86	10·6	11·4	12·1	12·9	13·7	14·4	15·2	½
¾	7·97	8·77	9·56	10·4	11·2	12·0	12·8	13·5	14·3	15·1	15·9	¾
11	8·35	9·19	10·0	10·9	11·7	12·5	13·4	14·2	15·0	15·9	16·7	11
¼	8·75	9·63	10·5	11·4	12·3	13·1	14·0	14·9	15·8	16·6	17·5	¼
½	9·16	10·1	11·0	11·9	12·8	13·7	14·7	15·6	16·5	17·4	18·3	½
¾	9·58	10·5	11·5	12·5	13·4	14·4	15·3	16·3	17·2	18·2	19·2	¾
12	10·0	11·0	12·0	13·0	14·0	15·0	16·0	17·0	18·0	19·0	20·0	12
¼												¼
½												½
¾												¾

TABLE 27—*cont.* TARIFF TABLES FOR CONIFERS

B.H.Q.G. (inches)	Volumes in hoppus feet over bark — Tariff Number 20	21	22	23	24	25	26	27	28	29	30	B.H.Q.G. (inches)
2	—	—	—	—	—	—	—	—	—	—	—	2
¼	—	—	—	—	—	—	—	—	—	—	—	¼
½	·27	·28	·29	·31	·32	·34	·35	·36	·38	·39	·40	½
¾	·47	·50	·52	·55	·57	·59	·62	·64	·66	·69	·71	¾
3	·68	·71	·75	·78	·82	·85	·88	·92	·95	·99	1·02	3
¼	·89	·93	·98	1·02	1·06	1·11	1·15	1·20	1·24	1·29	1·33	¼
½	1·13	1·19	1·25	1·30	1·36	1·42	1·47	1·53	1·59	1·64	1·70	½
¾	1·40	1·47	1·54	1·61	1·68	1·75	1·82	1·89	1·96	2·03	2·10	¾
4	1·67	1·75	1·84	1·92	2·00	2·09	2·17	2·25	2·34	2·42	2·51	4
¼	1·96	2·06	2·15	2·25	2·35	2·45	2·55	2·64	2·74	2·84	2·94	¼
½	2·29	2·40	2·52	2·63	2·75	2·86	2·98	3·09	3·20	3·32	3·43	½
¾	2·62	2·75	2·88	3·01	3·14	3·27	3·40	3·54	3·67	3·80	3·93	¾
5	2·97	3·12	3·27	3·41	3·56	3·71	3·86	4·01	4·16	4·31	4·45	5
¼	3·32	3·49	3·65	3·82	3·98	4·15	4·32	4·48	4·65	4·81	4·98	¼
½	3·71	3·90	4·08	4·27	4·46	4·64	4·83	5·01	5·20	5·38	5·57	½
¾	4·12	4·33	4·54	4·74	4·95	5·15	5·36	5·57	5·77	5·98	6·19	¾
6	4·54	4·76	4·99	5·22	5·45	5·67	5·90	6·12	6·35	6·58	6·80	6
¼	4·97	5·22	5·47	5·71	5·96	6·21	6·46	6·71	6·96	7·21	7·45	¼
½	5·42	5·69	5·96	6·24	6·51	6·78	7·05	7·32	7·59	7·86	8·13	½
¾	5·90	6·19	6·49	6·78	7·08	7·37	7·67	7·96	8·26	8·55	8·85	¾
7	6·39	6·71	7·03	7·35	7·67	7·99	8·31	8·63	8·95	9·27	9·59	7
¼	6·91	7·25	7·60	7·94	8·29	8·63	8·98	9·32	9·67	10·0	10·4	¼
½	7·44	7·82	8·19	8·56	8·93	9·30	9·68	10·0	10·4	10·8	11·2	½
¾	7·98	8·38	8·78	9·18	9·58	9·97	10·4	10·8	11·2	11·6	12·0	¾
8	8·54	8·96	9·39	9·82	10·2	10·7	11·1	11·5	12·0	12·4	12·8	8
¼	9·13	9·59	10·0	10·5	11·0	11·4	11·9	12·3	12·8	13·2	13·7	¼
½	9·73	10·2	10·7	11·2	11·7	12·2	12·7	13·1	13·6	14·1	14·6	½
¾	10·4	10·9	11·4	11·9	12·4	12·9	13·5	14·0	14·5	15·0	15·5	¾
9	11·0	11·5	12·1	12·6	13·2	13·7	14·3	14·8	15·4	15·9	16·5	9
¼	11·6	12·2	12·8	13·4	14·0	14·0	15·1	15·7	16·3	16·9	17·4	¼
½	12·3	12·9	13·5	14·2	14·8	15·4	16·0	16·6	17·2	17·8	18·5	½
¾	13·0	13·6	14·3	14·9	15·6	16·2	16·9	17·5	18·2	18·8	19·5	¾
10	13·7	14·4	15·1	15·7	16·4	17·1	17·8	18·5	19·2	19·9	20·5	10
¼	14·4	15·2	15·9	16·6	17·3	18·0	18·8	19·5	20·2	20·9	21·7	¼
½	15·2	15·9	16·7	17·5	18·2	19·0	19·7	20·5	21·2	22·0	22·8	½
¾	15·9	16·7	17·5	18·3	19·1	19·9	20·7	21·5	22·3	23·1	23·9	¾

TABLE 27—*cont.* TARIFF TABLES FOR CONIFERS

B.H.Q.G. (inches)	Volumes in hoppus feet over bark											B.H.Q.G. (inches)
	Tariff Number											
	20	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	
11	16·7	17·5	18·4	19·2	20·0	20·9	21·7	22·5	23·4	24·2	25·1	11
¼	17·5	18·4	19·3	20·1	21·0	21·9	22·8	23·6	24·5	25·4	26·3	¼
½	18·3	19·2	20·1	21·1	22·0	22·9	23·8	24·7	25·6	26·5	27·5	½
¾	19·2	20·1	21·1	22·0	23·0	23·9	24·9	25·9	26·8	27·8	28·7	¾
12	20·0	21·0	22·0	23·0	24·0	25·0	26·0	27·0	28·0	29·0	30·0	12
¼	20·9	21·9	23·0	24·0	25·0	26·1	27·1	28·2	29·2	30·3	31·3	¼
½	21·8	22·8	23·9	25·0	26·1	27·2	28·3	29·4	30·5	31·5	32·6	½
¾	22·7	23·8	24·9	26·1	27·2	28·3	29·5	30·6	31·7	32·9	34·0	¾
13	23·6	24·8	25·9	27·1	28·3	29·5	30·7	31·8	33·0	34·2	35·4	13
¼	24·5	25·7	27·0	28·2	29·4	30·6	31·9	33·1	34·3	35·5	36·8	¼
½	25·5	26·8	28·0	29·3	30·6	31·9	33·1	34·4	35·7	37·0	38·2	½
¾	26·5	27·8	29·1	30·4	31·8	33·1	34·4	35·7	37·0	38·4	39·7	¾
14	27·4	28·8	30·2	31·6	32·9	34·3	35·7	37·0	38·4	39·8	41.2	14
¼	28·5	29·9	31·3	32·7	34·2	35·6	37·0	38·4	39·8	41·3	42·7	¼
½	29·5	31·0	32·4	33·9	35·4	36·9	38·3	39·8	41·3	42·8	44·2	½
¾	30·5	32·1	33·6	35·1	36·7	38·2	39·7	41·2	42·8	44·3	45·8	¾
15	31·6	33·2	34·7	36·3	37·9	39·5	41·1	42·6	44·2	45·8	47·4	15
¼	32·7	34·3	35·9	37·6	39·2	40·9	42·5	44·1	45·8	47·4	49·0	¼
½	33·8	35·5	37·1	38·8	40·5	42·2	43·9	45·6	47·3	49·0	50·7	½
¾	34·9	36·7	38·4	40·1	41·9	43·6	45·4	47·1	48·9	50·6	52·4	¾
16	36·0	37·8	39·6	41·4	43·3	45·1	46·9	48·7	50·5	52·3	54·1	16
¼	37·2	39·1	40·9	42·8	44·6	46·5	48·4	50·2	52·1	53·9	55·8	¼
½	38·4	40·3	42·2	44·1	46·1	48·0	49·9	51·8	53·7	55·6	57·6	½
¾	39·5	41·5	43·5	45·5	47·5	49·4	51·4	53·4	55·4	57·3	59·3	¾
17	40·8	42·8	44·8	46·9	48·9	51·0	53·0	55·0	57·1	59·1	61·1	17
¼	42·0	44·1	46·2	48·3	50·4	52·5	54·6	56·7	58·8	60·9	63·0	¼
½	43·2	45·4	47·6	49·7	51·9	54·0	56·2	58·4	60·5	62·7	64·9	½
¾	44·5	46·7	48·9	51·2	53·4	55·6	57·9	60·1	62·3	64·5	66·7	¾
18	45·8	48·1	50·3	52·6	54·9	57·2	59·5	61·8	64·1	66·4	68·7	18
¼												¼
½												½
¾												¾

TABLE 27—*cont.* TARIFF TABLES FOR CONIFERS

B.H.Q.G. (inches)	Volumes in hoppus feet over bark											B.H.Q.G. (inches)
	Tariff Number											
	30	31	32	33	34	35	36	37	38	39	40	
2	—	—	—	—	—	—	—	—	—	—	—	2
¼	—	—	—	—	—	—	—	—	—	—	—	¼
½	·40	·42	·43	·44	·46	·47	·48	·50	·51	·52	·54	½
¾	·71	·73	·76	·78	·81	·83	·85	·88	·90	·92	·95	¾
3	1·02	1·05	1·09	1·12	1·16	1·18	1·22	1·26	1·29	1·33	1·36	3
¼	1·33	1·37	1·42	1·46	1·51	1·55	1·60	1·64	1·68	1·73	1·77	¼
½	1·70	1·76	1·81	1·87	1·93	1·98	2·04	2·10	2·15	2·21	2·27	½
¾	2·10	2·17	2·24	2·31	2·38	2·45	2·52	2·59	2·66	2·73	2·80	¾
4	2·51	2·59	2·67	2·76	2·84	2·92	3·01	3·09	3·17	3·26	3·34	4
¼	2·94	3·04	3·13	3·23	3·33	3·43	3·55	3·62	3·72	3·82	3·92	¼
½	3·43	3·55	3·66	3·78	3·89	4·01	4·12	4·23	4·35	4·46	4·58	½
¾	3·93	4·06	4·19	4·32	4·45	4·58	4·71	4·84	4·98	5·11	5·24	¾
5	4·45	4·60	4·75	4·90	5·05	5·20	5·35	5·49	5·64	5·79	5·94	5
¼	4·98	5·15	5·31	5·48	5·64	5·81	5·98	6·14	6·31	6·47	6·64	¼
½	5·57	5·75	5·94	6·12	6·31	6·50	6·68	6·87	7·05	7·24	7·42	½
¾	6·19	6·39	6·60	6·80	7·01	7·22	7·42	7·63	7·84	8·04	8·25	¾
6	6·80	7·03	7·26	7·48	7·71	7·94	8·17	8·39	8·62	8·85	9·07	6
¼	7·45	7·70	7·95	8·20	8·45	8·70	8·95	9·19	9·44	9·69	9·94	¼
½	8·13	8·40	8·68	8·95	9·22	9·49	9·76	10·0	10·3	10·6	10·8	½
¾	8·85	9·14	9·44	9·73	10·0	10·3	10·6	10·8	11·2	11·5	11·8	¾
7	9·59	9·91	10·2	10·5	10·9	11·2	11·5	11·8	12·1	12·5	12·8	7
¼	10·4	10·7	11·1	11·4	11·7	12·1	12·4	12·8	13·1	13·5	13·8	¼
½	11·2	11·5	11·9	12·3	12·7	13·0	13·4	13·8	14·1	14·5	14·9	½
¾	12·0	12·4	12·8	13·2	13·6	14·0	14·4	14·8	15·2	15·6	16·0	¾
8	12·8	13·2	13·7	14·1	14·5	14·9	15·4	15·8	16·2	16·6	17·1	8
¼	13·7	14·2	14·6	15·1	15·5	16·0	16·4	16·9	17·4	17·8	18·3	¼
½	14·6	15·1	15·6	16·1	16·5	17·0	17·5	18·0	18·5	19·0	19·5	½
¾	15·5	16·0	16·6	17·1	17·6	18·1	18·6	19·1	19·7	20·2	20·7	¾
9	16·5	17·0	17·6	18·1	18·7	19·2	19·8	20·3	20·9	21·4	22·0	9
¼	17·4	18·0	18·6	19·2	19·8	20·4	20·9	21·5	22·1	22·7	23·3	¼
½	18·5	19·1	19·7	20·3	20·9	21·5	22·2	22·8	23·4	24·0	24·6	½
¾	19·5	20·1	20·8	21·4	22·1	22·7	23·4	24·0	24·7	25·3	26·0	¾
10	20·5	21·2	21·9	22·6	23·3	24·0	24·6	25·3	26·0	26·7	27·4	10
¼	21·7	22·4	23·1	23·8	24·5	25·3	26·0	26·7	27·4	28·1	28·9	¼
½	22·8	23·5	24·3	25·0	25·8	26·6	27·3	28·1	28·8	29·6	30·4	½
¾	23·9	24·7	25·5	26·3	27·1	27·9	28·7	29·5	30·3	31·1	31·9	¾

TABLE 27—*cont.* TARIFF TABLES FOR CONIFERS

B.H.Q.G. (inches)	Volumes in hoppus feet over bark — Tariff Number 30	31	32	33	34	35	36	37	38	39	40	B.H.Q.G. (inches)
11	25·1	25·9	26·7	27·6	28·4	29·2	30·1	30·9	31·7	32·6	33·4	11
¼	26·3	27·1	28·0	28·9	29·8	30·6	31·5	32·4	33·3	34·1	35·0	¼
½	27·5	28·4	29·3	30·2	31·1	32·0	33·0	33·9	34·8	35·7	36·6	½
¾	28·7	29·7	30·6	31·6	32·6	33·5	34·5	35·4	36·4	37·4	38·3	¾
12	30·0	31·0	32·0	33·0	34·0	35·0	36·0	37·0	38·0	39·0	40·0	12
¼	31·3	32·3	33·4	34·4	35·5	36·5	37·6	38·6	39·7	40·7	41·7	¼
½	32·6	33·7	34·8	35·9	37·0	38·1	39·2	40·2	41·3	42·4	43·5	½
¾	34·0	35·1	36·3	37·4	38·5	39·7	40·8	41·9	43·1	44·2	45·3	¾
13	35·4	36·6	37·7	38·9	40·1	41·3	42·5	43·6	44·8	46·0	47·2	13
¼	36·8	38·0	39·2	40·5	41·7	42·9	44·1	45·4	46·6	47·8	49·0	¼
½	38·2	39·5	40·8	42·0	43·3	44·6	45·9	47·1	48·4	49·7	51·0	½
¾	39·7	41·0	42·3	43·6	45·0	46·3	47·6	48·9	50·3	51·6	52·9	¾
14	41·2	42·5	43·9	45·3	46·7	48·0	49·4	50·8	52·1	53·5	54·9	14
¼	42·7	44·1	45·5	46·9	48·4	49·8	51·2	52·6	54·1	55·5	56·9	¼
½	44·2	45·7	47·2	48·6	50·1	51·6	53·1	54·5	56·0	57·5	59·0	½
¾	45·8	47·3	48·9	50·4	51·9	53·4	55·0	56·5	58·0	59.5	61·1	¾
15	47·4	49·0	50·5	52·1	53·7	55·3	56·9	58·4	60·0	61·6	63·2	15
¼	49·0	50·7	52·3	53·9	55·6	57·2	58·8	60·5	62·1	63·7	65·4	¼
½	50·7	52·3	54·0	55·7	57·4	59·1	60·8	62·5	64·2	65·9	67·6	½
¾	52·4	54·1	55·9	57·6	59·3	61·1	62·8	64·6	66·3	68·1	69·8	¾
16	54·1	55·9	57·7	59·5	61·3	63·1	64·9	66·7	68·5	70·3	72·1	16
¼	55·8	57·7	59·5	61·4	63·2	65·1	67·0	68·8	70·7	72.5	74·4	¼
½	57·6	59·5	61·4	63·3	65·2	67·1	69·1	71·0	72·9	74·8	76·7	½
¾	59·3	61·3	63·3	65·3	67·2	69·2	71·2	73·2	75·1	77·1	79·1	¾
17	61·1	63·2	65·2	67·3	69·3	71·3	73·4	75·4	77·5	79·5	81·5	17
¼	63·0	65·1	67·2	69·3	71·4	73·5	75·6	77·7	79·8	81.9	84·0	¼
½	64·9	67·0	69·2	71·3	73·5	75·7	77·8	80·0	82·2	84·3	86·5	½
¾	66·7	69·0	71·2	73·4	75·6	77·9	80·1	82·3	84·6	86·8	89·0	¾
18	68·7	70·9	73·2	75·5	77·8	80·1	82.4	84·7	87·0	89·3	91·6	18
¼												¼
½												½
¾												¾

TABLE 27—*cont.* TARIFF TABLES FOR CONIFERS

B.H.Q.G. (inches)	Volumes in hoppus feet over bark											B.H.Q.G. (inches)
	Tariff Number											
	40	41	42	43	44	45	46	47	48	49	50	
4	3·34	3·42	3·51	3·59	3·67	3·76	3·84	3·92	4·01	4·09	4·18	4
¼	3·92	4·02	4·11	4·21	4·31	4·41	4·50	4·60	4·70	4·80	4·90	¼
½	4·58	4·69	4·81	4·92	5·03	5·15	5·26	5·33	5·49	5·61	5·72	½
¾	5·24	5·37	5·50	5·63	5·76	5·89	6·02	6·15	6·28	6·42	6·55	¾
5	5·94	6·09	6·24	6·38	6·53	6·68	6·83	6·98	7·13	7·27	7·42	5
¼	6·64	6·81	6·97	7·14	7·30	7·47	7·63	7·80	7·97	8·13	8·30	¼
½	7·42	7·61	7·79	7·98	8·16	8·35	8·54	8·72	8·91	9·09	9·28	½
¾	8·25	8·45	8·66	8·87	9·07	9·28	9·48	9·69	9·90	10·1	10·3	¾
6	9·07	9·30	9·53	9·75	9·98	10·2	10·4	10·7	10·9	11·1	11·3	6
¼	9·94	10·2	10·4	10·7	10·9	11·2	11·4	11·7	11·9	12·2	12·4	¼
½	10·8	11·1	11·4	11·7	11·9	12·2	12·5	12·7	13·0	13·3	13·6	½
¾	11·8	12·1	12·4	12·7	13·0	13·3	13·6	13·9	14·2	14·4	14·7	¾
7	12·8	13·1	13·4	13·7	14·1	14·4	14·7	15·0	15·3	15·7	16·0	7
¼	13·8	14·2	14·5	14·9	15·2	15·5	15·9	16·2	16·6	16·9	17·3	¼
½	14·9	15·3	15·6	16·0	16·4	16·8	17·1	17·6	17·9	18·2	18·6	½
¾	16·0	16·4	16·8	17·2	17·6	18·0	18·4	18·8	19·2	19·5	19·9	¾
8	17·1	17·5	17·9	18·4	18·8	19·2	19·6	20·1	20·5	20·9	21·3	8
¼	18·3	18·7	19·2	19·6	20·1	20·6	21·0	21·5	21·9	22·4	22·8	¼
½	19·5	20·0	20·4	20·9	21·4	21·9	22·4	22·9	23·4	23·8	24·3	½
¾	20·7	21·2	21·7	22·3	22·8	23·3	23·8	24·3	24·8	25·4	25·9	¾
9	22·0	22·5	23·1	23·6	24·2	24·7	25·3	25·8	26·4	26·9	27·5	9
¼	23·2	23·8	24·4	25·0	25·6	26·2	26·7	27·3	27·9	28·5	29·1	¼
½	24·6	25·2	25·8	26·5	27·1	27·7	28·3	28·9	29·5	30·2	30·8	½
¾	26·0	26·6	27·3	27·9	28·6	29·2	29·9	30·5	31·2	31·8	32·5	¾
10	27·4	28·1	28·8	29·4	30·1	30·8	31·5	32·2	32·9	33·5	34·2	10
¼	28·9	29·6	30·3	31·0	31·8	32·5	33·2	33·9	34·6	35·4	36·1	¼
½	30·4	31·1	31·9	32·6	33·4	34·2	34·9	35·7	36·4	37·2	37·9	½
¾	31·9	32·7	33·5	34·3	35·1	35·9	36·7	37·5	38·3	39·0	39·8	¾
11	33·4	34·2	35·1	35·9	36·7	37·6	38·4	39·2	40·1	40·9	41·8	11
¼	35·0	35·9	36·8	37·6	38·5	39·4	40·3	41·1	42·0	42·9	43·8	¼
½	36·6	37·5	38·4	39·4	40·3	41·2	42·1	43·0	43·9	44·9	45·8	½
¾	38·3	39·3	40·2	41·2	42·1	43·1	44·1	45·0	46·0	46·9	47·9	¾

TABLE 27—*cont.* TARIFF TABLES FOR CONIFERS

B.H.Q.G. (inches)	Volumes in hoppus feet over bark											B.H.Q.G. (inches)
	Tariff Number											
	40	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	
12	40·0	41·0	42·0	43·0	44·0	45·0	46·0	47·0	48·0	49·0	50·0	12
¼	41·7	42·8	43·8	44·9	45·9	47·0	48·0	49·0	50·1	51·1	52·2	¼
½	43·5	44·6	45·7	46·8	47·9	49·0	50·0	51·1	52·2	53·3	54·4	½
¾	45·3	46·5	47·6	48·7	49·9	51·0	52·1	53·3	54·4	55·5	56·7	¾
13	47·2	48·4	49·5	50·7	51·9	53·1	54·3	55·4	56·6	57·8	59·0	13
¼	49·0	50·3	51·5	52·7	53·9	55·2	56·4	57·6	58·8	60·1	61·3	¼
½	51·0	52·2	53·5	54·8	56·1	57·4	58·6	59·9	61·2	62·4	63·7	½
¾	52·9	54·2	55·6	56·9	58·2	59·5	60·8	62·2	63·5	64·8	66·1	¾
14	54·9	56·3	57·6	59·0	60·4	61·8	63·1	64·5	65·9	67·2	68·6	14
¼	56·9	58·3	59·8	61·2	62·6	64·0	65·4	66·9	68·3	69·7	71·1	¼
½	59·0	60·4	61·9	63·4	64·9	66·4	67·8	69·3	70·8	72·2	73·7	½
¾	61·1	62·6	64·1	65·7	67·2	68·7	70·2	71·8	73·3	74·8	76·3	¾
15	63·2	64·8	66·4	68·0	69·5	71·1	72·7	74·3	75·9	77·4	79·0	15
¼	65·4	67·0	68·6	70·3	71·9	73·5	75·2	76·8	78·4	80·1	81·7	¼
½	67·6	69·2	70·9	72·6	74·3	76·0	77·7	79·4	81·1	82·7	84·4	½
¾	69·8	71·6	73·3	75·1	76·8	78·6	80·3	82·0	83·8	85·5	87·3	¾
16	72·1	73·9	75·7	77·5	79·3	81·1	82·9	84·7	86·5	88·3	90·1	16
¼	74·4	76·3	78·1	80·0	81·8	83·7	85·6	87·4	89·3	91·1	93·0	¼
½	76·7	78·7	80·6	82·5	84·0	86·4	88·3	90·2	92·1	94·0	95·9	½
¾	79·1	81·1	83·0	85·0	87·0	89·0	91·0	92·9	94·9	96·9	98·9	¾
17	81·5	83·6	85·6	87·6	89·7	91·7	93·8	95·8	97·8	99·9	102	17
¼	84·0	86·1	88·2	90·3	92·4	94·5	96·6	98·7	101	103	105	¼
½	86·5	88·6	90·2	93·0	95·1	97·3	99·4	102	104	106	108	½
¾	89·0	91·2	93·8	95·7	97·9	100	102	105	107	109	111	¾
18	91·6	93·8	96·1	98·4	101	103	105	108	110	112	114	18
¼												¼
½												½
¾												¾

TABLE 27—*cont.* TARIFF TABLES FOR CONIFERS

B.H.Q.G. (inches)	Volumes in hoppus feet over bark											B.H.Q.G. (inches)
	Tariff Number											
	50	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**	
6	11·3	11·6	11·8	12·0	12·2	12·5	12·7	12·9	13·2	13·4	13·6	6
¼	12·4	12·7	12·9	13·2	13·4	13·7	13·9	14·2	14·4	14·7	14·9	¼
½	13·6	13·8	14·1	14·4	14·6	14·9	15·2	15·5	15·7	16·0	16·3	½
¾	14·7	15·0	15·3	15·6	15·9	16·2	16·5	16·8	17·1	17·4	17·7	¾
7	16·0	16·3	16·6	16·9	17·3	17·6	17·9	18·2	18·5	18·9	19·2	7
¼	17·3	17·6	18·0	18·3	18·6	19·0	19·3	19·7	20·0	20·4	20·7	¼
½	18·6	19·0	19·4	19·7	20·1	20·5	20·8	21·2	21·6	22·0	22·3	½
¾	19·9	20·3	20·7	21·1	21·5	21·9	22·3	22·7	23·1	23·5	23·9	¾
8	21·3	21·8	22·2	22·6	23·0	23·5	23·9	24·3	24·8	25·2	25·6	8
¼	22·8	23·3	23·7	24·2	24·7	25·1	25·6	26·0	26·5	26·9	27·4	¼
½	24·3	24·8	25·3	25·8	26·3	26·8	27·2	27·7	28·2	28·7	29·2	½
¾	25·9	26·4	26·9	27·4	27·9	28·5	29·0	29·5	30·0	30·5	31·1	¾
9	27·5	28·0	28·6	29·1	29·7	30·2	30·8	31·3	31·9	32·4	33·0	9
¼	29·1	29·7	30·2	30·8	31·4	32·0	32·6	33·1	33·7	34·3	34·9	¼
½	30·8	31·4	32·0	32·6	33·2	33·9	34·5	35·1	35·7	36·3	36·9	½
¾	32·5	33·1	33·8	34·4	35·1	35·7	36·4	37·0	37·7	38·3	39·0	¾
10	34·2	34·9	35·6	36·3	37·0	37·7	38·3	39·0	39·7	40·4	41·1	10
¼	36·1	36·8	37·5	38·2	39·0	39·7	40·4	41·1	41·9	42·6	43·3	¼
½	37·9	38·7	39·5	40·2	41·0	41·7	42·5	43·2	44·0	44·8	45·5	½
¾	39·8	40·6	41·4	42·2	43·0	43·8	44·6	45·4	46·2	47·0	47·8	¾
11	41·8	42·6	43·4	44·3	45·1	45·9	46·8	47·6	48·4	49·3	50·1	11
¼	43·8	44·6	45·5	46·4	47·3	48·1	49·0	49·9	50·8	51·6	52·5	¼
½	45·8	46·7	47·6	48·5	49·1	50·4	51·3	52·2	53·1	54·0	54·9	½
¾	47·9	48·8	49·8	50·8	51·7	52·7	53·6	54·6	55·5	56·5	57·5	¾

TABLE 27—*cont.* TARIFF TABLES FOR CONIFERS

B.H.Q.G. (inches)	Volumes in hoppus feet over bark — Tariff Number 50	51	52	53	54	55	56	57	58	59	60	B.H.Q.G. (inches)
12	50·0	51·0	52·0	53·0	54·0	55·0	56·0	57·0	58·0	59·0	60·0	12
¼	52·2	53·2	54·3	55·3	56·3	57·4	58·4	59·5	60·5	61·6	62·6	¼
½	54·4	55·5	56·6	57·6	58·7	59·8	60·9	62·0	63·1	64·2	65·3	½
¾	56·7	57·8	58·9	60·0	61·2	62·3	63·4	64·6	65·7	66·8	68·0	¾
13	59·0	60·1	61·3	62·5	63·7	64·9	66·0	67·2	68·4	69·6	70·8	13
¼	61·3	62·5	63·7	65·0	66·2	67·4	68·6	69·9	71·1	72·3	73·6	¼
½	63·7	65·0	66·3	67·5	68·8	70·1	71·4	72·6	73·9	75·2	76·5	½
¾	66·1	67·5	68·8	70·1	71·4	72·8	74·1	75·4	76·7	78·0	79·4	¾
14	68·6	70·0	71·4	72·7	74·1	75·5	76·8	78·2	79·6	81·0	82·3	14
¼	71·1	72·6	74·0	75·4	76·8	78·3	79·7	81·1	82·5	83·9	85·4	¼
½	73·7	75·2	76·7	78·1	79·6	81·1	82·6	84·0	85·5	87·0	88·5	½
¾	76·3	77·9	79·4	80·9	82·4	84·0	85·5	87·0	88·6	90·1	91·6	¾
15	79·0	80·6	82·2	83·8	85·3	86·9	88·5	90·1	91·7	93·2	94·8	15
¼	81·7	83·3	85·0	86·6	88·2	89·9	91·5	93·1	94·8	96·4	98·0	¼
½	84·4	86·1	87·8	89·5	91·2	92·9	94·6	96·3	97·9	99·6	101	½
¾	87·3	89·0	90·8	92·5	94·2	96·0	97·7	99·5	101	103	105	¾
16	90·1	91·9	93·7	95·5	97·3	99·1	101	103	105	106	108	16
¼	93·0	94·8	96·7	98·6	100	102	104	106	108	110	112	¼
½	95·9	97·8	99·8	102	104	106	107	109	111	113	115	½
¾	98·9	101	103	105	107	109	111	113	115	117	119	¾
17	102	104	106	108	110	112	114	116	118	120	122	17
¼	105	107	109	111	113	115	118	120	122	124	126	¼
½	108	110	112	115	117	119	121	123	125	128	130	½
¾	111	113	116	118	120	122	125	127	129	131	133	¾
18	114	117	119	121	124	126	128	130	133	135	137	18
¼												¼
½												½
¾												¾

Appendix IV

THE PRECISION OF THE VOLUME ESTIMATE

THE error of a volume estimate made by the tariff table method has three components. These will be considered in turn and the calculation of each will be illustrated by using the figures from the worked example in Appendix III.

(1) The mean tariff number T_m, calculated from the sample trees, is rounded up or down to the nearest whole number in order to fit the tariff tables. In the worked example, $T_m = \frac{698}{22} = 31{\cdot}7$, the tariff used T_u was number 32, and the error in the tariff number was therefore : $T_u - T_m = 32 - 31{\cdot}7 = 0{\cdot}3$.

The corresponding proportionate error p in the volume estimate Y' is equal to the proportionate error in the tariff number :

$$p = \frac{T_u - T_m}{T_u}$$

For the example :

$$p = \frac{32 - 31{\cdot}7}{32} = 0{\cdot}009$$

The actual error k, due to this cause, can be obtained by multiplying by p the volume estimate Y'.

Thus, in the example :

$$k = p\,(\Sigma Y')$$
$$= 0{\cdot}009 \times 14021$$

but usually it is more convenient to work with p, as this facilitates combination with errors from other sources.

As a result of this error caused by rounding up and down to the nearest whole tariff number, the volume estimate $(\Sigma Y')$ is not in the centre of the fiducial limits of the estimates. That centre is the volume (ΣY) that would have been obtained by using the actual mean tariff number. (ΣY) is given by :

$$(\Sigma Y) = (\Sigma Y') - k$$

This correction can, if desired, be applied as a routine to the crude volume estimate $(\Sigma Y')$. However, as can be seen from the example above, the effect of ignoring this correction is unlikely to be appreciable, except possibly when T_m is very small. Its routine use in the field would therefore seem to be an unnecessary complication, particularly as the errors due to its omission, being sometimes positive and sometimes negative, will tend to cancel out over a series of determinations.

(2) The mean tariff number T_m, being estimated from a sample instead of being determined from all trees, is subject to a sampling error. The standard error of T_m will be referred to as s_t and the corresponding standard error in the volume estimate as s_{yt}, these standard errors being conveniently expressed as decimal fractions of T_m and $(\Sigma Y')$ respectively.

The error s_t may be calculated, from the squared deviations of the tariff numbers of the sample trees, from T_m. Alternatively, the range r_t of the tariff numbers of the sample trees may be used to give a rough estimate of s_t. The estimate is obtained by means of a table, reproduced below, as Table 28, giving the theoretical ratio of range to standard deviation in samples of various sizes (Snedecor, 1946 ; Jeffers, 1952). This simpler, but less precise, method of estimating s_t will be used here.

TABLE 28

RANGE/STANDARD DEVIATION RATIOS

n	Mean value of r/σ	n	Mean value of r/σ
2	1·13	20	3·73
3	1·69	30	4·09
4	2·06	50	4·50
5	2·33	75	4·81
6	2·53	100	5·02
7	2·70	150	5·3
8	2·85	200	5·5
9	2·97	300	5·8
10	3·08	500	6·1
15	3·47	700	6·3

In the example, the tariff numbers for individual trees range from 24 to 38, thus :

$$r_t = 38 - 24 = 14.$$

The number of felled sample trees (n_t) is 22. For : $n = 22$, the ratio r/σ, according to the table, is approximately 3·75, the nearest tabulated value being 3·73 for : n = 20.

Thus :

$$\frac{r_t}{s} = 3{\cdot}75$$

$$s = \frac{r_t}{3{\cdot}75} = \frac{14}{3{\cdot}75}$$

where s is the standard deviation of tariff numbers for individual trees. Then, since T_m is the mean of n_t tariff numbers, the standard deviation of T_m is :

$$s_t = \frac{s}{\sqrt{n_t}} = \frac{14}{3{\cdot}75\sqrt{22}}$$

and, expressing this as a decimal fraction of T_m (= 31·7 in the example) :

$$s_t = \frac{14}{3{\cdot}75 \times \sqrt{22} \times 31{\cdot}7} \times T_m$$
$$= 0{\cdot}025 \times T_m.$$

Then since, as in (1) above, a proportionate deviation in tariff number produces an equal proportionate deviation in volume :

$$s_{yt} = 0{\cdot}025 \times (\Sigma Y')$$
$$= 0{\cdot}025 \times 14021$$

(3) The mean girth g_m, and hence also the mean volume $\bar{y}'$ and total volume ($\Sigma y'$), being estimated by girthing a sample instead of all trees, are subject to sampling errors. The standard error of g_m will be referred to as s_m, the corresponding component of the standard error of $\bar{y}'$ as s_g, and that of ($\Sigma Y'$) as s_{yg}, these standard errors being conveniently expressed as decimal fractions of g_m, and $\bar{y}'$ and ($\Sigma Y'$) respectively.

It is possible to determine s_g from the volumes (Y') given in the appropriate tariff T_u for each girth class, without at first calculating s_m; s_g may be calculated from the squared deviations of the volumes (Y') for each girth class from the mean volume $\bar{y}'$, but for most practical purposes a rough estimate of s_g by means of the range/standard deviation table will suffice, and this method will be adopted here.

In the example, the girths range from 9¼ inches to 3 inches, and the volumes for these girths, according to the appropriate tariff (T_u = 32), are 18·6 and 1·09 hoppus feet respectively. The volume range r_g is given by :

$$r_g = 18{\cdot}6 - 1{\cdot}1 = 17{\cdot}5$$

The number of trees girthed n_g is 218. For n = 218, the ratio r/σ, according to the table, is approximately 5·5, the nearest tabulated value being 5·5 for n = 200. Thus :

$$\frac{r_g}{s} = 5{\cdot}5$$

$$s = \frac{r_g}{5{\cdot}5} = \frac{17{\cdot}5}{5{\cdot}5}$$

where s is the standard deviation of individual tree volumes from the mean volume $\bar{y}'$. Then, since $\bar{y}'$ is the mean volume of n_g trees, its standard deviation s_g is given by the equation :

$$s_g = \frac{s}{\sqrt{n_g}} = \frac{17{\cdot}5}{5{\cdot}5 \times \sqrt{218}}$$

and, expressing this as a decimal fraction of $\bar{y}'$ (= 6·43 hoppus feet in the example) :

$$s_g = \frac{17{\cdot}5}{5{\cdot}5 \times 218 \times 6{\cdot}43} \times \bar{y}'$$
$$= 0{\cdot}033 \times \bar{y}'$$

Then, since a proportionate deviation in mean volume produces an equal proportionate deviation in total volume :

$$s_{yg} = 0{\cdot}033 \times (\Sigma Y')$$
$$= 0{\cdot}033 \times 14021$$

The Combination of Errors from Different Sources

The standard error of the volume estimate will be referred to as s_y, and like its components s_{yt} and s_{yg}, is conveniently stated as a decimal fraction of the estimate ($\Sigma Y'$). As the tariff number and the distribution of volumes among the girth classes are estimated from entirely different samples, the corresponding sampling errors will be uncorrelated, so that :

$$s_y = \sqrt{s_{yt}^2 + s_{yg}^2}$$

In the example :

$$s_y = \sqrt{0.025^2 + 0{\cdot}033^2} \times 14021$$
$$= 0{\cdot}0414 \times 14021$$

Estimates, at the 5 per cent probability level, of the fiducial limits F_1 and F_2 of the volume estimate are given by the equations :

$$F_1 = Y' - k + 2\,s_y$$
$$\text{and : } F_2 = Y' - k - 2\,s_y$$

In the example :

$$F_1 = 14021\ (1 - 0{\cdot}009 + 2 \times 0{\cdot}0414) = 14021 \times 1{\cdot}0738 = 15060$$

$$F_2 = 14021\ (1 - 0{\cdot}009 - 2 \times 0{+}0414) = 14021 \times {\cdot}9082 = 12730$$

There is thus a probability of about 20 to 1 against the actual volume being either greater than

15,060 hoppus feet (7·4 per cent more than the estimated 14,021 hoppus feet) or smaller than 12,730 hoppus feet (9·2 per cent less than the estimate).

The use of a systematic, rather than a random, method for selecting the two samples of trees (for felling and girthing) has been recommended in Appendix III, on the grounds that systematic sampling is easier to carry out in practice and will usually, although not always, result in a better representation of the population and hence of a more precise estimate. The use of the formula :

$$s^2/\sqrt{n}$$

will therefore generally result in an overestimation of the true sampling standard errors $\sigma^2_{\bar{y}t}$ and $\sigma^2_{\bar{y}g}$. Furthermore, the use of a systematic sample, simply because it gives a better coverage of the population, is likely to result in wider ranges of girths and tariff numbers than would be expected from a random sample. Since the values in the range/standard deviation table are based on random samples, the use of this table with ranges from systematic samples will provide a further source of overestimation of standard errors. Thus the methods of this appendix are appropriate for random samples, but for systematic samples they should be regarded as giving only an upper limit to the standard error, and upper and lower limits respectively to F_1 and F_2. For a more detailed discussion of the precision of systematic sampling methods see Finney (1948).

Appendix V

PARTICULARS OF THE PLOTS EXAMINED IN PART II OF THE INVESTIGATION

Locality	Plot Number	Thinning Grade	Number of Remeasurements	Top height in feet at 1st Measurement	Top height in feet at Last Measurement	Quality Class
			SCOTS PINE			
Bagshot, Windsor, Surrey	E.35	A	5	35½	50	IV
	E.36	B	5	35½	53½	IV
	E.37	L.C.	5	38½	55½	III
	E.38	D	5	38	58½	III
	E.39	C	5	40	62	III
Healey, Northumberland	E.74	B	5	49	57½	IV
	E.75	C	5	50½	61½	III
Dilston, Northumberland	E.76	D	3	52½	65½	III
	E.77	B	3	52½	60½	III
Dilston, Northumberland	E.80	A	5	40½	58	IV
	E.81	B	5	42½	56½	IV
New Forest, Hants	E.103	C	5	71½	83½	II
	E.104	B	5	82	95	I
Brandon Park, Suffolk	E.130	L.C.	4	32	41½	III
	E.131	A	4	30	42	III
Glendye, Kincardine	S.15	B	6	37½	61½	III
	S.16	C	6	38½	63	II
Balmoral, Aberdeen	S.29	B	6	41	66½	II
	S.30	D	6	43	62½	III
Evanton, Ross-shire	S.33	B	6	42	69½	I
	S.34	D	6	41	65½	II
Seafield, Moray	S.64	B	5	38½	55	III
	S.65	C/D	5	39	55½	III
Broomhill, Inverness-shire	S.68	D	5	43½	56	III
	S.69	B	5	43	57	III
	S.70	D	5	43	56	III
	S.71	D	5	45½	57½	III
			CORSICAN PINE			
Minehead, Somerset	E.46	B	5	35	60½	II
	E.47	C	6	32½	63	III
Highclere, Hants	E.57	L.C.	4	41½	66½	I
	E.58	B	4	42	68½	I
Highclere, Hants	E.59*	B	4	38½	65½	I
	E.60*	D	4	37	66	I
Delamere, Cheshire	E.66	B	6	33	73	II
	E.67	C	6	32½	69	II
Sherwood Forest, Notts	E.118	B	5	39	56½	II
	E.126	D	5	39	56½	II

*Felled plots.

APPENDIX V—*cont.*

Locality	Plot Number	Thinning Grade	Number of Remeasurements	Top height in feet at 1st Measurement	Top height in feet at Last Measurement	Quality Class
			EUROPEAN LARCH			
Cressage, Shropshire	E.15	B	5	43½	61½	III
	E.16	C	3	47½	55	III
	E.17	D	5	49½	67	II
Haldon, Devonshire	E.32	D	4	49½	72½	I
	E.33	D	6	52	77	II
	E.34	B	6	52½	77½	II
Highclere, Hants	E.63	B	3	33½	49½	II
	E.64	D	3	33½	49½	I
Highmeadow, Forest of Dean	E.105 (1 & 2)	B	4	38	58	II
	E.125 (1 & 2)	D	4	40	53½	II
Tintern, Monmouth	E & W.1	D	7	72½	93	I
	E & W.2	C/D	6	73½	86½	II
	E & W.3	C	7	52½	70½	III
Haystoun, Peebles	S.1	B	5	42½	56½	III
	S.2	D	5	44½	58	III
Haystoun, Peebles	S.3	B	5	32	45	IV
	S.4	D	5	32½	43½	IV
Fyvie, Aberdeenshire	S.20	B	6	33½	62	II
	S.21	D	6	34	60½	III
	S.22	L.C.	6	34½	62	II
Shambellie, Kirkcudbright	S.24	D	5	56½	78½	II
	S.25	B	5	57	79½	II
Murthly, Perthshire	S.44	L.C.	6	40	67½	II
	S.45	B	6	40	67½	II
	S.46	D	6	39½	67½	II
Seafield, Moray	S.66	D	5	45	63½	III
	S.67	B	5	43	66	III
Shambellie, Kirkcudbright	S.74	C	5	31½	58½	II
	S.75	D	5	32½	60	II
Drummond Hill, Perthshire	S.103, A & B	C	3	36	48½	II
	S.104, A & B	D	3	36	48	II
			JAPANESE LARCH			
Stourhead, Wilts	E.54	D	6	38	79	II
	E.55	B	6	38	74½	II
	E.56	C	6	38	75½	II
Hafod Fawr, Merioneth	E & W.112	C	3	29½	37	V
	E & W.113	C	3	31½	40½	V
	E & W.114	C	3	45	49½	III
	E & W.115	C	5	41½	64	II
Kirkennan Hill, Kirkcudbright	S.27	D	6	34	70½	II
	S.28	C	6	34½	72	II
Dunach, Argyllshire	S.54	B	4	29½	56½	II
	S.55	D	4	29	56	II

APPENDIX V—*cont.*

Locality	Plot Number	Thinning Grade	Number of Remeasurements	Top height in feet at 1st Measurement	Top height in feet at Last Measurement	Quality Class
			JAPANESE LARCH—*cont.*			
Ardgowan, Renfrewshire	S.58	B	4	40½	67	II
	S.59	D	4	40½	64½	II
Benmore, Argyll	S.99	D	4	55	68½	II
Bowmont, Kelso, Roxburgh	S.109	L.C.	4	59	69	III
	S.110	D	4	59	70	III
Inverliever, Argyll	S.120	D	3	41½	51½	I
Knapdale, Argyll	S.121	C/D	3	27	38	I
	S.122	C/D	3	26½	39½	I
			NORWAY SPRUCE			
Highclere, Hants	E.61	D	4	44	64	I
	E.62	B	4	44½	63½	I
Hexham, Northumberland	E.78	B	3	53	64	III
	E.79	C	3	53	64½	III
Tintern, Monmouthshire	E & W. 99	D	4	44	73½	I
	E & W.100	C	4	45½	73	I
	E & W.101	B	4	45	73½	I
Dumfries	S.5	D	6	38½	70	II
	S.6	B	6	38	66	II
Grandtully, Perthshire	S.48	D	6	40½	75	I
	S.49	B	6	40½	70½	II
Grandtully, Perthshire	S.50	D	5	32½	67	I
	S.51	B	5	32½	61½	II
*Bowmont, Kelso, Roxburgh	S.85	B	5	25	47	III
	S.86	C	5	24	46	III
	S.87	D	5	24½	48	III
	S.88	L.C.	5	25	47	III
			SITKA SPRUCE			
Minehead, Somerset	E.41	C/D	9	37	100	I
Fenwick, Northumberland	E.69	C	3	32	55	IV
	E.70	B	3	27	57	III
Dumfries	S.9	C	6	44½	93	II
Drumlanrig, Dumfries	S.73	B/C	5	50	87½	II
Benmore, Argyll	S.84	C/D	4	68	92	IV
Drummond Hill, Perthshire	S.105	B	3	41	62	II
	S.106	L.C.	3	38	57	III
Corrour, Inverness-shire	S.108	C/D	4	38	62½	IV

*Each plot contains 4 Sub plots.

APPENDIX V—*cont.*

Locality	Plot Number	Thinning Grade	Number of Remeasurements	Top height in feet at 1st Measurement	Top height in feet at Last Measurement	Quality Class
			SITKA SPRUCE—*cont.*			
Inverliever, Argyll	S.116	C	3	36½	54	II
	S.117	L.C.	3	34½	51½	II
	S.118	C	3	37½	55	I
	S.119	L.C.	3	38½	54	II
			DOUGLAS FIR			
Dunster, Somerset	E.18	C/D	5	87½	107	II
Tortworth, Glos.	E.19	B	6	109½	130	I
	E.20	B	6	81½	104½	III
Dunster, Somerset	E.44	B	5	62½	96	III
	E.45	D	5	58	91	III
Stourhead, Wilts	E.52	B	4	51½	90	II
	E.53	D	4	52½	86½	II
Fenwick, Northumberland	E.71	H.C.	5	47	73½	IV
	E.72	D	5	47	77½	III
Lake Vrynwy, Montgomery	E & W.27*	D	4	29	59	III
	E & W.28	L.C.	4	29½	60	III
Kildrummy, Aberdeen	S.32*	C	5	47	76	III
Murthly, Perthshire	S.41	C	6	31	67½	IV
Dunach, Argyll	S.52	C	3	36	68	II
Culloden, Inverness-shire	S.79	D	5	44½	83½	III
	S.80	B	5	43	77	III

*Felled plots.

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