Forestry Commission Bulletin

44

Operational Research and the Managerial Economics of Forestry

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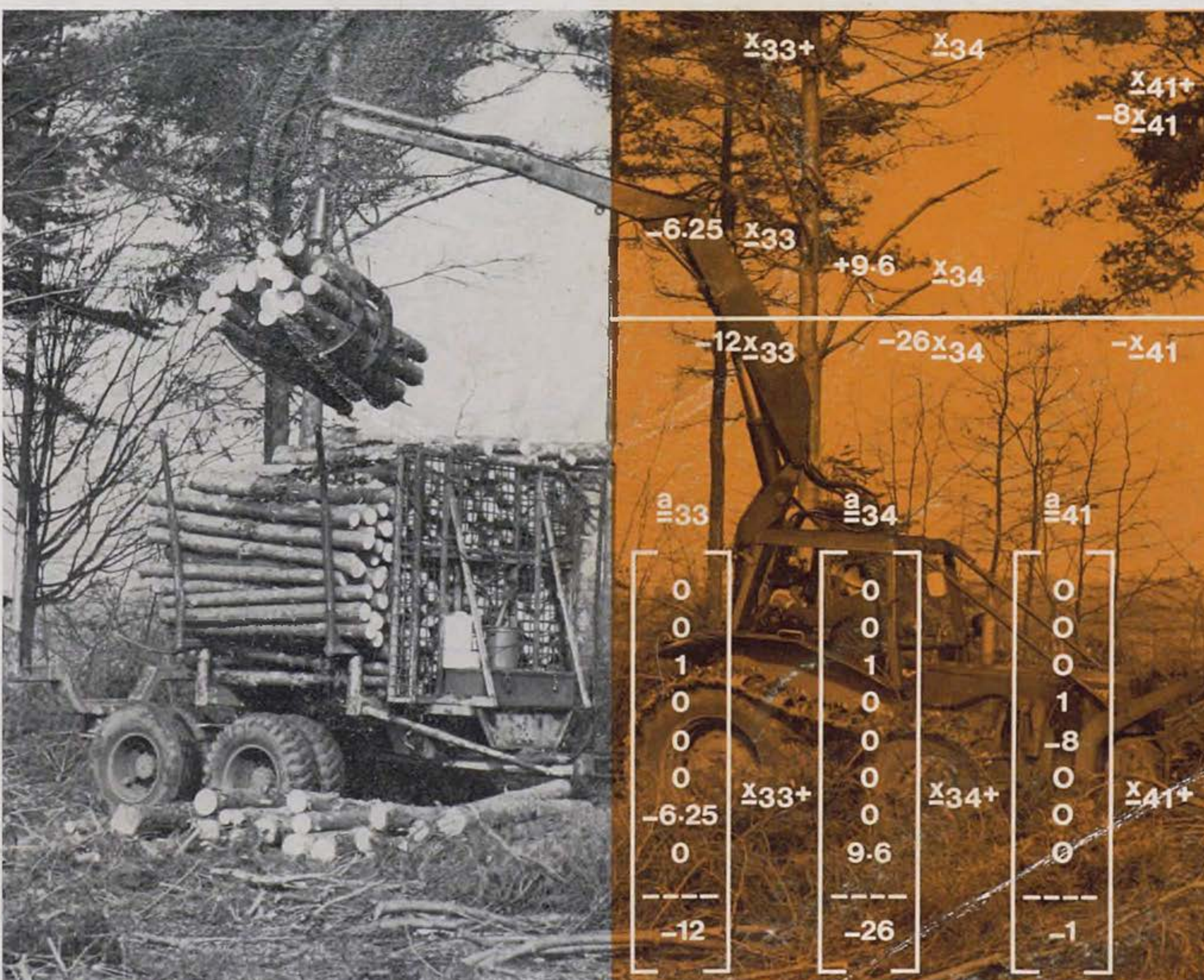

FORESTRY COMMISSION BULLETIN
No. 44

Operational Research and the Managerial Economics of Forestry

PROCEEDINGS OF A MEETING OF A WORKING GROUP OF THE INTERNATIONAL UNION OF FOREST RESEARCH ORGANISATIONS HELD AT THE FORESTRY COMMISSION RESEARCH STATION, ALICE HOLT LODGE IN SEPTEMBER, 1970

Edited by
P. A. WARDLE, B.Sc.
Forestry Commission

LONDON: HER MAJESTY'S STATIONERY OFFICE
1971

FOREWORD

At the Fourteenth Congress of the International Union of Forest Research Organisations held in Munich in September, 1967 a Working Group was set up in the Economics Section, Section 31, to consider the contribution of operational research to studies in the field of the managerial economics of forestry.

In September, 1970, members of this working group met at the Research Station of the Forestry Commission, Alice Holt Lodge, Farnham, Surrey, England. This Bulletin contains the papers presented at that meeting and a record of the discussion.

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EDITOR'S PREFACE

When the Working Group on Operational Research and Managerial Economics was formed in Section 31 of the International Union of Forest Research Organisations, the terms of reference selected were "to study the contribution of operational research to the solution of the central problems of managerial economics of forestry". We felt that we should be most effective if we gathered as active members of the group people who were actually working on operational research problems in the field of managerial economics, and collected together their experience in tackling problems, rather than attempting to list the appropriateness of some catalogue of operational research techniques to the problems of forestry. We hoped, by adopting the former method, to find at the same time examples of the use of several well-known operational research techniques. Having assembled examples, we hoped to bring the authors together and draw some general conclusions about the contribution of operational research from their discussion.

The opportunity for this discussion was provided by the meeting of the Working Group at Alice Holt Lodge, 14–19 September, 1970, to which 15 papers were presented and when 15 members of the Working Group from 9 countries were assembled. The small number allowed lively and informal discussion and I think we all counted it a very happy occasion. Our only disappointment was the absence of three members who had taken the trouble to send excellent papers. They were: Dr. Vilmos Farkas of Hungary, Mr. Milan Novotny and Professor Juri Ruprich of Czechoslovakia. We missed their experience in discussion and in reaching our conclusions.

The papers are published in the order in which they were discussed. Each paper is followed by a note of discussion where that discussion added to what is contained in the paper. Any classification of the papers will inadequately indicate the particular contribution of each author. However, when originally placing them in order in the programme, I had in mind that the papers of Höfle, Paillé, Kostov and Risvand were concerned with some aspect of harvesting or timber production management. Farkas' paper, while using the problem of choice of species as his example, is a very clear presentation of the logical steps in the solution of the linear programming problem. Von Malmborg and Sayers were concerned with modelling the private forestry enterprise. Ruprich, Duerr and Wardle dealt with aspects of the systems approach to forest management. Bittig and Novotny were both concerned with the solution of short-term management problems, while Morgan and Bjora, Vornstad and Jackson described models of large enterprises and the forestry sector of the economy.

The presentations did, in fact, exemplify the use of a variety of operational research techniques. Linear programming is most frequently referred to and is central to work presented by Farkas, Höfle, Jackson, Kostov, von Malmborg and Vornstad, and a component in the case of Duerr, Novotny, Paillé, Risvand and Ruprich's work. Duerr's work also includes reference to the use of Leontief input-output analysis. Simulation is central to the work of Morgan and Bjora, Paillé and Sayers and a component for Duerr's. Dynamic programming is Risvand's main concern and is mentioned by Ruprich. Bittig and Novotny deal with network analysis, while Duerr, Ruprich and Wardle are principally concerned with systems analysis.

The papers received by the editor were as the authors had prepared them for presentation in discussion. In every case the author had kindly prepared an English text. Because of our desire to publish quickly I have not been able to go back to the author in every case where I felt a revision of the text desirable, and I hope in the case where a change has been made that the original intention of the author has been correctly presented.

I would like on behalf of the Working Group to thank Mr. George Holmes, Director Research, and the Forestry Commission for their hospitality at Alice Holt and for agreeing to publish the proceedings of the Working Group's meeting.

I am particularly grateful to Dr. Paillé and to Dr. Glück for preparing the French and German language translations of the summaries, to Mr. Jim Dickinson for preparing diagrams and to Miss Monica Hopkin and her colleagues for typing and retyping.

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SUMMARIES OF PAPERS

1. OPTIMISATION OF THE HARVEST OF SMALL-SIZE WOOD THROUGH LINEAR PROGRAMMING

By H. H. HÖFLE

West Germany

If we deal with parts of the forest firm, we have to start with a systems analysis of the firm in order to avoid suboptimisation incompatible with the firm's main objective. In doing so, we find that, though maximum utility is the main objective, profit maximisation is still a valid objective for parts of the firm. This, for instance, is true if we optimise the harvest of small size wood.

The firm owns various stands with certain stand characteristics. The allowable cut of these stands is limited by upper and lower limits for silvicultural reasons. The firm also has a certain labour force and some machines at its disposal. It is able to use various logging methods for producing various products which will yield different profits. Finally, the market requires certain amounts of some products whereas the maximum amount of others is limited. Now the product mix must be found that maximises profit within the limitations mentioned.

This problem in the allocation of resources complies with the linear programming model: profit maximisation provides the objective function, the alternative activities consist of the possible combinations of stand, product, and logging method, while the limitations of capacities provide the constraints.

A hypothetical model, the data of which draw heavily from reality, has demonstrated that this problem may, in fact, be solved by linear programming. The optimal solution, a sensitivity analysis, and the implication of both for the firm are briefly discussed. In conclusion, there are some hints to improving the model and to further applying linear programming to forestry problems.

2. USES OF SIMULATION IN FORECASTING STAND GROWTH AND MORTALITY

By G. PAILLÉ and J. H. G. SMITH

Canada

A review of some recent advances of simulation in forestry is made. Stand modelling is put into perspective, and some of the basic principles for reproducing tree and stand growth are outlined. Mortality tables for individual trees are presented as a reliable method for generating competition mortality in a stochastic manner. A new stand model, based on such tables, is introduced; apart from predicting growth and yield of natural stands of Douglas fir, it gives detailed information on amount, timing and distribution of mortality. It is suggested that efforts should be directed at improving such models so that they soon become practical tools in operational forest management.

3. OPTIMISATION OF THE SPECIES COMPOSITION OF A FOREST AREA

By P. KOSTOV

Bulgaria

On the basis of data of the forest arrangement of the forest enterprise Nove Selo possibilities are considered for determining the optimum tree species composition by site types in a given forest area. To this end a mathematical model is suggested and the prerequisites are pointed out for its application in planning the forest production. The mathematical expression of the model is as follows:

$$F = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} c_{ij} \rightarrow \max \qquad \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

$$\sum_{j=1}^{n} x_{ij} = a_i \ (i = 1,2,\ldots,m)$$

$$\sum_{i=1}^{m} x_{ij} = b_j \ (j = 1,2,\ldots,n) \quad x_{ij} \geqslant 0$$

where x_{ij} is the area of site type i planted to species j

a_i is the area of site type i

b_j is the area of species j required

c_{ij} is the maximum mean volume increment per unit area on site type i of species j.

4. An Algorithm of the Simplex Method Applied to the Solution of the General Linear Programming Problem in a Forestry Example

By V. FARKAS

Hungary

The mathematical model of a general linear programming problem is usually converted for solving by simplex method into a system of equations in such a way that vector **b** composed of the constant terms of constraints doesn't contain, from the initial simplex tableau on, any negative co-ordinate related to any basis.

The present paper deals with a simplex algorithm, which lets each constant term belonging to constraints of type "$\geqslant$" appear with negative sign in column **b** of the first simplex tableau. The temporary toleration of these negativities which are subsequently to be driven out, permits one to avoid either (1) the additional columns that would otherwise have to be opened up for coefficients of surplus variables, or (2) the additional operations that would otherwise have to be performed in every case when an element for pivot in a row that contains negative co-ordinate in column **b** is selected.

The algorithm is shown in application to a forestry problem.

5. Economic Planning of the Farm Forest Operating Unit

By G. von MALMBORG

Sweden

In the paper a planning model for long-term planning of primarily private woodlots, separately or in combination with other activities, is presented briefly. For a more complete description reference is made to previous publications. The paper therefore is directed primarily to a discussion of the applicability of the model for practical planning of actual farms or farm woodlots. As a background farm forestry structure and the intended use of the model is described. In the latter connection the need for strategic and tactical plans as a framework for operational logging plans is pointed out.

The model uses linear programming technique and special programs have been developed for calculating input data for the LP-matrix as well as for converting the output data into a readable form.

The model is general, that is to say it can be used on different farms without changing its general form. It is also total in that all activities, including investment and financing are included. In the same vein impact of taxation is taken account of through special coefficients included in the LP-matrix.

6. Dynamic Programming for Determining Optimum Cutting Policies for a Forest Enterprise

By J. RISVAND

Norway

The long perspective of production, in addition to the more short-term problems of co-ordination arising because of the large areas involved, presents special problems of planning in forestry. Most of the forest planning models constructed, do not take into consideration the location of the individual stands, and the co-ordination problems have, therefore, been avoided.

The present paper shows how a more realistic model can be constructed by means of dynamic programming. To take the location of the stands into account, the forest area has been divided into management blocks. The management block, in addition to the transport distance, is characterised by the location of the stand.

The optimum has been found in two steps. First, it is determined how the stands within a management block should be treated, when different requirements are made on the cutting quantity in the block. Secondly the results of these calculations, together with some other information, are used in determining distribution of the total cutting quantity on the management blocks.

It is also shown how the problem can be formulated for solution by linear programming. The location of the stands however has not been considered in this model.

7. A Simulation Model for Comparing Plans of Management on Private Forestry Estates in Scotland

By A. R. SAYERS

Great Britain

This paper describes a computer model of private forestry enterprises in Scotland. Simulation is used in preference to mathematical programming both for its flexibility and the ease with which the mechanism of the model and the output information can be understood by the forest owner and manager.

The individual crops which comprise the forest growing stock are described and the Forestry Commission's Yield Tables are used as the model for their growth. The management plan which it is desired to study is expressed as a list of forest operations to be executed each year on the individual crops. The output of the model includes a detailed forest operations account, a capital account with valuation of the growing stock and an analysis of timber yields, thus helping the forest owner and manager in choosing the plan most suited to his policy.

The model has been used successfully on three large private forestry estates in Scotland, but further improvement of the growth model is desirable.

8. Methods of Operational Research and Planning Selected Operations in Forestry

By J. RUPRICH

Czechoslovakia

A general scheme of a forest enterprise is described in the form of matrices. Then the methods of operational analysis applied at the level of forestry organisation in the CSSR are described. The approach to solutions involves the following steps (i) the optimisation of individual activities in isolation, (ii) taking account of the mutual relations of the individual activities, (iii) modifying the detailed scheme for the individual activities, (iv) development of a plan for organisation for 3–10 years ahead and a detailed plan for operations for 1 year ahead, and (v) evaluation at the end of each year of both the model and its application in this last period and to prepare a plan for the rest of the long term planning cycle. The methods used in analysis of a number of activities are described.

9. "Guides to Land Management"; An Integrated Operations-Research Project with a Systems Viewpoint

By W. A. DUERR

United States of America

Guides to Land Management is a research and development project of the Bureau of Land Management in the United States Department of the Interior. Its basic purpose is to produce aids to the making of wise land management and planning decisions.

In support of the basic purpose, two phases of the Project are devoted to building and implementing (1) large-system and (2) small-system models of the Bureau's activity. All models are intended to predict the consequences of alternative courses of action and thus give the decision maker a better basis for choice. Large-system models are of three kinds: (a) economy models, of the inter-industry type; (b) Bureau models, encompassing Bureau activities within an administrative unit; and (c) tract models, providing on-the-ground detail for specific forests or other landholdings.

Secondary purposes of the Guides to Land Management Project which derive from its basic purpose are (3) employee training and (4) research problem analysis.

Experience with the Project suggests the need for close collaboration between the researchers and the users of their results. It points to certain guides in the use of computers. And it emphasises the value of Bureau models as decision aids.

10. Operational Research and the Design of a Management Control System for a Forestry Enterprise

By P. A. WARDLE

Great Britain

The approach to the design of a revised system of financial control in the Forestry Commission has been first to establish the functions that have to be performed by management in order that the enterprise will operate effectively, secondly to identify these functions with particular managers or groups of managers within the organisation and thirdly to set up the procedures necessary to support these managers in the functions allocated to them.

The main functions identified are (i) strategic planning and decision making relating to the type and level of activity of the enterprise and its objectives, (ii) tactical planning and decision making relating to such things as the staff, installation, machinery and services necessary to carry out the operations and the specification of those operations and (iii) operational planning relating to the deployment of particular resources to specific jobs.

Features of the system are the prime importance attached to strategic and tactical planning involving the appraisal of the contribution of possible courses of action to pursuit of the objectives, the concept of operational plans being constructed within the framework of tactical plans and the provision for reporting back to show the correspondence between actual performance and planned performance and between local and central plans. The main function of reporting back is to ensure that adjustment and revision of plans is carried out on a basis of up-to-date information on the state of the enterprise.

11. Experience Gained with Network Technique in Avalanche Control

By B. BITTIG and F. PFISTER

Switzerland

Research about applying Network Technique in forest engineering has shown that the critical path method (CPM) is the best form.

Due to different reasons, the volume of projects in avalanche control has grown heavier lately. For smaller projects most difficulties could be eliminated by clever improvisations. This was not possible for big projects. For reasons of economics it was necessary to search for new methods for planning and management. As we have explained in an example of the year 1966, the CMP is an excellent way to improve project planning and surveillance.

In the future it will be necessary to find an integral system for project research. If more interested parties will study these problems it should be possible to find a satisfying system of integral planning.

12. Application of Mathematical Methods in Operational Planning of Logging Operations

By M. NOVOTNY

Czechoslovakia

In testing the suitability of linear programming application in the short-time management of timber transport we have acquired experience that one of the factors hampering the success is a want of reliable information on probable time order of logging operations in the short term (week, month, quarter of year).

In this paper the method of critical path is described, simplified and supplemented with a decision model. In general, this procedure consists in the use of the combination of network diagram with Gantt progress chart and with linear programming methods.

13. A Simulation Model for Enterprise Planning

By J. MORGAN
Great Britain
and E. BJORA
Norway

The paper describes a model of the forestry activities of the Forestry Commission which is intended to be an aid to the corporate planning process.

The main decision variables considered are cutting and planting policies. The model allows the exploration of the effects of different policies on a set of variables which include net discounted revenue, net income, timber production and employment. The relationships considered within the model are described.

The use of the model is illustrated through examples of the projection of industrial man power requirements for different cutting policies and different productivity assumptions. Examples are also given of the effect of different cutting policies on the present value of the growing stock.

14. The Linear Programming Planning System of the Swedish Forest Service

By B. F. FORNSTAD
Sweden

The linear programming technique is often thought of as a tool of operations research intended to help the analyst to find optimal solutions to allocation problems. As such it is successful insofar as it leads to optimal solutions with less computational effort than other techniques of operations research.

The central theme of this paper is that the optimal solution aspect of linear programming tends to lose importance at the stage of implementation and that other considerations such as sensitivity analysis, presentability, etc. tend to come into the foreground. It is maintained by the author that the dual prices associated within a linear programming solution can be of great help at this stage.

15. A Linear Programming Model of the United Kingdom Forest Products Sector

By B. G. JACKSON
Great Britain

The object of the study is to develop an optimum strategy for the U.K. forest products sector, in terms of maximum net discounted social benefit. State and private forests are taken together. One section of the model covers wood-producing, with British forests grouped into 4 "species", 8 age-classes and various management systems. These activities supply wood to the other section, which covers primary processing into 10 possible products, e.g. sawnwood, newsprint. The time horizon is 60 years, divided into 7 periods. Demand forecasts for products are made on the basis of past trends and estimates of future technical changes.

RESUMÉ DES COMMUNICATIONS

1. Optimisation de la Recolte du Bois de Petite Dimension par Programmation Lineaire

Par H. H. HÖFLE

Allemagne de l'Ouest

En considérant les parties de la firme forestière, nous devons débuter par une analyse de systèmes de façon à éviter la sous-optimisation incompatible avec l'objectif principal de la firme. Ce faisant, nous trouvons que, même si l'utilité maximum est l'objectif principal, la maximisation du profit reste un objectif valide pour les parties de la firme. Ceci, par exemple, est vrai si nous optimisons la récolte du bois de faible dimension.

La firme possède une variété de peuplements ayant certaines caractéristiques. La possibilité de ces peuplements est limitée par des limites inférieures et supérieures, pour des raisons sylvicoles. La firme dispose aussi de main d'oeuvre et de machinerie. Elle peut utiliser diverses techniques d'exploitation pour produire divers produits qui vont rapporter des profits différents. Finalement, le marché requière certaines quantités de certains produits, alors que la quantité maximum d'autres produits est limitée. En considérant ces limitations, la combinaison de produits qui maximise le profit doit être déterminée.

Ce problème d'allocation des ressources est résolu à l'aide d'un modèle de programmation linéaire: la maximisation du profit fournit la fonction objective, les alternatives sont déterminées par les combinaisons possibles de peuplements, produits et méthodes d'exploitation; les contraintes proviennent des limitations de capacité.

Un modèle hypothétique, basé en bonne partie sur des données réelles, a démontré que ce problème peut en fait être résolu par programmation linéaire. La solution optimale, une analyse de sensibilité et l'implication des deux pour la firme sont discutées brièvement. En conclusion, quelques suggestions sont faites pour améliorer le modèle et pour promouvoir les applications de la programmation linéaire aux problèmes de la foresterie.

2. Usages de la Simulation pour Predire L'accroissement des Peuplements Forestiers et la Mortalité

Par G. PAILLE et J. H. G. SMITH

Canada

Les développements récents de la simulation en foresterie sont passés en revue. Les principes à la base de la reproduction de la croissance des arbres et des forêts employés dans les modèles de peuplements sont mis en perspective et des tables de mortalité, applicables aux tiges individuelles, sont présentées comme une nouvelle méthode probabiliste pour générer la mortalité due à la compétition. Un modèle de peuplement semi-stochastique est décrit, au moyen duquel il est possible de prédire l'accroissement et le rendement des forêts naturelles de sapin de Douglas et d'obtenir des informations détaillées concernant la quantité de mortalité et sa distribution dans le temps et dans l'espace. Il est suggéré que des efforts soient faits pour améliorer ces modèles de façon à ce qu'ils deviennent utiles non seulement en recherche et en enseignement mais aussi dans la pratique courante de l'aménagement forestier.

3. Optimisation de la Composition des Espèces sur une Superficie Forestiere

Par P. KOSTOV

Bulgarie

Les possibilités de détermination de la composition optimale des espéces par type de station sur une superficie forestière donnée sont considérées, basées sur l'arrangement des forêts de l'entreprise forestière

Nove Selo. A cette fin, un modèle mathématique est suggéré et les préalables à son application dans la planification de la production forestière sont soulignés. L'expression mathématique du modèle est la suivante:

$$F = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij}\, c_{ij} \longrightarrow \max \qquad \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

$$\sum_{j=1}^{n} x_{ij} = a_i \; (i = 1,2,....,m)$$

$$\sum_{i=1}^{m} x_{ij} = b_j \; (j = 1,2,....,n) \qquad x_{ij} \geqslant 0$$

où x_{ij} est la superficie de type de station i reboisée avec l'essence j

a_i est la superficie du type de station i

b_j est la superficie de l'essence j requise

c_{ij} est l'accroissement maximum moyen en volume de l'essence j par unité de superficie du type de station i.

4. Algorithme de la Methode du Simplex Appliqué a la Solution du Probleme General de Programmation Lineaire dans un Exemple de Foresterie

Par V. FARKAS

Hongrie

Le modèle mathématique d'un problème général de programmation linéaire est d'ordinaire converti, dans le but d'en arriver à une solution, par la méthode du simplex, en un système d'équations tel que le vecteur **b**, composante des termes constants des contraintes, ne contient pas de coordonnées négatives reliées à aucune base à partir du tableau simplex initial.

Le présent travail traite d'un algorithme simplex qui permet à chaque terme constant, appartenant aux contraintes de type "$\geqslant$" d'apparaître avec un signe négatif dans la colonne **b** du premier tableau simplex. Le fait de tolérer temporairement ces valeurs négatives qui doivent être éliminées subséquemment permet d'éviter ou bien (1) les colonnes additionnelles qui devraient autrement être formées pour les coefficients des variables en surplus, ou (2) les opérations additionnelles qui devraient autrement être exécutées dans chaque cas quand un élément est choisi comme pivot dans une rangée qui contient une coordonnée négative dans la colonne **b**.

L'algorithme est appliqué à un problème forestier.

5. Planification Economique de la Ferme Forestiere

Par G. von MALMBORG

Suède

Dans ce travail, un modèle de planification est présenté brièvement, visant principalement la planification à long-terme de boisés privés, indépendammment ou combinée à d'autres activités. Référence est faite à des publications antérieures pour une description plus détaillée. Ainsi, l'étude est orientée principalement vers la discussion de l'applicabilité du modèle à la planification pratique de fermes réelles ou de boisés de ferme. Comme information supplémentaire, la structure des fermes forestières et l'usage correct du modèle sont décrits. En relation avec le modèle, le besoin de plans stratégiques et tactiques comme cadres aux plans opérationnels d'exploitation est montré.

Le modèle utilise la technique de programmation linéaire; des programmes spéciaux ont été développés pour calculer les données d'entrée de la matrice-PL aussi bien que pour convertir les données de sortie sous forme lisible.

Le modèle est général, c'est-à-dire qu'il peut être utilisé pour des fermes diverses sans qu'on doive changer sa forme générale. Il est aussi total en ce que toutes les activités sont inclues, y compris l'investissement et le financement. Dans la même veine, l'influence de la taxation est considérée par l'addition de coefficients spéciaux à la matrice-PL.

6. Programmation Dynamique pour la Determination de Politiques Optimales de Coupe dans L'entreprise Forestiere

Par J. RISVAND

Norvège

La perspective de production à long-terme, en surplus des problèmes de coordination à plus court-terme qui se posent à cause des vastes superficies impliquées, présente des problèmes spéciaux de planification en foresterie. La plupart des modèles de planification forestière construits ne tiennent pas compte de la position des peuplements individuels et, ainsi, les problèmes de coordination sont évités.

Le présent travail montre comment on peut construire un modèle plus réaliste par programmation dynamique. En vue de tenir compte de la position des peuplements, la superficie forestière a été subdivisée en blocs d'aménagement. Le bloc est caractérisé par la position du peuplement et la distance de charroyage.

L'optimum est trouvé en deux étapes. D'abord, on détermine comment les peuplements devraient être traités à l'intérieur d'un bloc d'aménagement quand différents impératifs sont posés concernant le volume de la récolte dans le bloc. Ensuite, les résultats de ces calculs ainsi que d'autres informations sont utilisés pour déterminer la distribution du volume total de la récolte dans les blocs d'aménagement.

La solution de ce problème par programmation linéaire est aussi illustrée. Toutefois, dans ce modèle, la localisation des peuplements n'est pas considérée.

7. Un Modele de Simulation pour Comparer les plans D'amenagement de Forets Privees en Ecosse

Par A. R. SAYERS

Grande Bretagne

Ce travail décrit un modèle d'ordinateur électronique pour les entreprises forestières privées d'Ecosse. La simulation est utilisée de préférence à la programmation mathématique à cause de sa flexibilité et de la facilité avec laquelle le mécanisme du modèle et les résultats peuvent être compris du propriétaire et de l'aménagiste forestier.

Les peuplements individuels qui forment le stock forestier en croissance sont décrits et les tables de production du Service Forestier (*Forestry Commission*) sont utilisées comme modèles de croissance. Le plan d'aménagement à l'étude est décomposé en une liste d'opérations forestières à exécuter chaque année dans les peuplements individuels. Les résultats comprennent le détail des opérations forestières, une évaluation monétaire du stock en croissance et une analyse des rendements en matière ligneuse qui aident le propriétaire et l'aménagiste forestier à choisir le plan le plus approprié pour atteindre leurs objectifs.

Le modèle a été employé avec succès dans trois grandes forêts privées en Ecosse, mais d'autres améliorations restent à faire concernant la croissance.

8. Methodes de Recherche Operationnelle et de Planification D'operations Choisies en Foresterie

Par J. RUPRICH

Tchécoslovaquie

Le plan général d'une entreprise forestière est décrit sous forme de matrices. Les méthodes d'analyse opérationnelle appliquées en RSST sont ensuite décrites. L'approche vers des solutions implique les étapes suivantes: (i) l'optimisation des activités individuelles isolées, (ii) l'établissement des relations communes aux activités individuelles, (iii) la modification du plan détaillé des activités individuelles, (iv) le développement d'un plan d'organisation pour 3 à 10 années à venir et d'un plan détaillé d'opération pour une année, et (v) l'évaluation à la fin de chaque année du modèle et de son application durant la dernière période et la préparation d'un plan pour le reste du cycle de planification à long-terme. Les méthodes utilisées dans l'analyse d'un certain nombre d'activités sont décrites.

9. "Guides pour l'Amenagement du Territoire"; Un Projet Integré de Recherche Operationnelle avec un Point-de-vue Systematique

Par W. A. DUERR

Etats-Unis

Guides pour l'Aménagement du Territoire est un projet de recherche et de développement du Bureau de l'Aménagement du Territoire du Ministère de l'Intérieur des Etats-Unis. Son but premier est de fournir des aides à la prise de décisions sages en aménagement du territoire et en planification.

Pour réaliser l'objectif de base, deux phases du Projet sont consacrées à construire et à mettre en oeuvre (1) des modèles à grande échelle et (2) à petite échelle de l'activité du Bureau. Tous les modèles visent à prédire les conséquences d'alternatives dans les modes d'action de façon à fournir au preneur de décisions une meilleure base pour choisir. Il y a trois sortes de modèles à grande échelle: (a) les modèles de l'économie, du type interindustrie; (b) les modèles "Bureau", incluant les activités du Bureau à l'intérieur d'une unité administrative; et (c) les modèles de superficie (*tract models*) qui fournissent des détails de terrain pour des forêts en particulier ou pour d'autres tenures.

Les objectifs secondaires du Projet qui découlent de l'objectif de base sont (3) la formation de main d'oeuvre et (4) l'analyse de problèmes de recherche.

L'expérience acquise avec ce Projet suggère la nécessité d'une collaboration étroite entre chercheurs et utilisateurs des résultats. Elle fournit certains guides à l'utilisation des ordinateurs et elle démontre la valeur des modèles de type "Bureau" comme aides à la prise de décisions.

10. Recherche Operationnelle et Construction d'un Systeme de Controle de l'Amenagement pour une Entreprise Forestiere

Par P. A. WARDLE

Grande Bretagne

L'approche à la construction d'un système revisé de contrôle financier pour le Service Forestier a consisté d'abord à établir les tâches qui doivent être exécutées par les dirigeants (*managers*) de façon que l'entreprise fonctionne efficacement, ensuite à identifier ces fonctions à des dirigeants en particulier ou à des groupes de dirigeants au sein de l'organisation et enfin à prendre les mesures nécessaires pour maintenir ces dirigeants dans leurs fonctions.

Les principales fonctions identifiées sont (i) la planification stratégique et prise de décision reliées au type et au niveau d'activité de l'entreprise et à ses objectifs, (ii) la planification tactique et prise de décision reliées à des choses telles que le personnel, l'installation, la machinerie et les services nécessaires à la poursuite des opérations et à la préparation des spécifications pour ces opérations et (iii) la planification opérationnelle reliée au déploiement des ressources particulières pour des fins spécifiques.

Les caractéristiques de ce système sont l'importance primordiale accordée à la planification stratégique et tactique impliquant une évaluation de la contribution des modes d'action possibles dans la poursuite des objectifs, le concept de plans opérationnels construits dans le cadre de plans tactiques et la possibilité de communication à rebours pour montrer la correspondance entre l'accomplissement réel et planifié et entre les plans locaux et centraux. L'utilité principale de la communication à rebours est de s'assurer que l'ajustement et la revision des plans sont faites sur la base d'information récente sur l'état de l'entreprise.

11. Experience Acquise avec la Technique de Reseau pour le Controle des Avalanches

Par B. BITTIG et F. PFISTER

Suisse

La recherche sur l'application de la technique de réseau (*network technique*) en génie forestier a montré que la méthode du chemin critique (*critical path*) est la meilleure.

Récemment, pour des raisons diverses, le nombre de projets de contrôle des avalanches s'est augmenté sensiblement. Dans la plupart des projets de faible envergure, des improvisations intelligentes ont pu éliminer la plupart des difficultés. Ceci ne fut pas possible dans le cas de grands projets. Pour des raisons économiques, il est devenu nécessaire de chercher de nouvelles méthodes de planification et d'aménagement. Tel qu'expliqué dans un exemple de 1966, la méthode du chemin critique est excellente pour améliorer la planification et la surveillance d'un projet. Dans l'avenir, il sera nécessaire de trouver un système intégral pour la recherche sur les projets. Si un plus grand nombre d'intéressés étudiaient ces problèmes, il deviendrait possible de découvrir un système satisfaisant de planification intégrale.

12. Application de Methodes Mathematiques en Planification Operationnelle des Operations d'exploitation Forestiere

Par M. NOVOTNY

Tchécoslovaquie

En vérifiant l'applicabilité de la programmation linéaire à l'aménagement en courte période du transport du bois, l'expérience nous a montré qu'un des facteurs empêchant la réussite est le besoin d'information valable sur la distribution probable dans le temps des opérations d'exploitation à court-terme (semaine, mois, trimestre).

Dans ce travail, la méthode du chemin critique (*critical path*) est décrite, simplifiée et complétée d'un modèle de décision. En général, cette procédure consiste à utiliser une combinaison d'un diagramme de réseau (*network diagram*), d'un graphique de Gantt (*Gantt progress chart*) et de méthodes de programmation linéaire.

13. Un Modele de Simulation pour la Planification de l'entreprise

Par J. MORGAN

Grande Bretagne

et

E. BJORA

Norvège

Ce travail décrit un modèle des activités forestières du Service Forestier visant à faciliter le procédé de planification intégré.

Les principales variables de décision considérées sont les politiques de coupe et de reboisement. Le modèle permet l'exploration des effets de diverses politiques sur une série de variables, notamment le revenu net escompté, le revenu net, la production ligneuse et l'emploi. Les relations considérées dans le modèle sont décrites.

L'utilisation du modèle est illustrée par des exemples de projection de la demande en main d'oeuvre industrielle pour différents programmes de coupe et différentes hypothèses de productivité. En plus, des exemples des effets des diverses politiques de coupe sur la valeur présente du stock en croissance sont donnés.

14. Le Systeme de Planification par Programmation Lineaire du Service Forestier Suedois

Par B. F. FORNSTAD

Suède

La technique de programmation linéaire est souvent envisagée comme un outil de recherche opérationnelle destiné à aider l'analyste à trouver des solutions optimales aux problèmes d'allocation. Comme telle, elle est fructueuse en autant qu'elle conduit aux solutions optimales en requérant moins de calculs que les autres techniques de recherche opérationnelle.

Le thème central de ce travail est que l'aspect solution optimale de la programmation linéaire tend à perdre de l'importance au stage de la mise en application et que d'autres considérations, telles que l'analyse de sensibilité, de présentabilité, etc. prennent beaucoup d'importance. L'auteur soutient que la dualité des prix associée avec la solution per programmation linéaire peut être d'un grand secours à ce stage.

15. Un Modele de Programmation Lineaire du Secteur des Produits Forestiers du Royaume-uni

Par B. G. JACKSON

Grande Bretagne

Le but de cette étude est de développer une stratégie optimale pour le secteur des produits forestiers du RU en termes de profit social net escompté. Les forêts publiques et privées sont considérées ensembles. Une partie du modèle considère la production de bois, les forêts du RU étant regroupées en 4 "espèces", 8 classes d'âge et divers systèmes d'aménagement. Ces activités fournissent le bois à l'autre partie, qui couvre l'usinage primaire en 10 produits possibles, e.g. bois d'oeuvre, papier journal. L'horizon est de 60 ans, divisé en 7 périodes. Les prévisions de la demande pour les produits sont faites sur la base des tendances passées et d'estimés de changements techniques futurs.

ZUSAMMENFASSUNG DER STUDIEN

1. Optimierung der Schwachholzsortierung mit Hilfe der Linearen Programmierung

von H. H. HÖFLE

Bundesrepublik Deutschland

Wenn man sich mit Teilen des Forstbetriebs beschäftigt, muß man mit einer Systemanalyse des Betriebs beginnen, um eine Suboptimierung zu vermeiden, die mit den wichtigsten Betriebszielen nicht vereinbar ist. Bei dieser Vorgangsweise findet man, daß die Gewinnmaximierung noch immer ein gültiges Ziel für Teilbereiche des Betriebes ist, wenngleich das Hauptziel die Nutzenmaximierung bleibt. Das gilt z.B. auch, wenn man die Ernte von Schwachholz optimiert.

Der Forstbetrieb verfügt über verschiedene Bestände mit bestimmten Bestandesmerkmalen. Der Hiebssatz dieser Bestände ist durch obere und untere Beschränkungen waldbaulicher Natur festgesetzt. Dem Betrieb steht auch eine bestimmte Zahl an Arbeitskräften und Maschinen zur Verfügung. Er kann zur Erzeugung verschiedener Produkte verschiedene Erntemethoden anwenden, die verschiedene Gewinne zur Folge haben. Schließlich verlangt der Markt nach bestimmten Mindestmengen einzelner Produkte; hingegen ist bei anderen Produkten die Maximalmenge beschränkt. Nun soll das "product-mix" gefunden werden, das den Gewinn innerhalb der erwähnten Beschränkungen maximiert.

Dieses Zuteilungsproblem wird mit hilfe der linearen Programmierung gelöst: Die Gewinnmaximierung erfolgt in der Zielfunktion—die verschiedenen Aktivitäten bestehen aus den möglichen Kombinationen von Bestand, Produkt und Ernteverfahren—während die Kapazitätsbeschränkungen die Beschränkungsgleichungen besorgen.

Ein hypothetisches Modell—die Daten hierfür sind weitgehend von der Wirklichkeit abgeleitet—hat gezeigt, daß dieses Problem tatsächlich mit hilfe der linearen Programmierung gelöst werden kann. Die optimale Lösung, eine Sensitivitätsanalyse und die Anwendung von beiden auf den Forstbetrieb werden kurz behandelt. Als Schlußfolgerung werden einige Hinweise zur Verbesserung des Modells und zur weiteren Anwendung der linearen Programmierung auf forstliche Probleme gegeben.

2. Verwendung der Simulation zur Vorhersage des Bestandeswachstums und der Mortalität

von G. PAILLÉ und J. H. G. SMITH

Canada

Es wird von einigen Fortschritten der Simulation in der Forstwirtschaft der jüngsten Zeit berichtet; es werden Bestandesmodelle betrachtet und grundlegende Überlegungen über Wachstum von Bäumen und Beständen zur Erstellung von Ertragstafeln angestellt. Als verläßliche Methode, den Überlebenskampf auf stochastische Art zu erzeugen, werden Sterblichkeitstafeln für einzelne Bäume dargestellt. Es wird ein neues Bestandesmodell, das auf solchen Tafeln beruht, eingeführt. Neben der Vorhersage von Wachstum und Ertrag natürlicher Bestände der Douglasie gibt es eine detaillierte Information über Ausmaß, Verbreitung und Steuerung der Sterblichkeit. Es wird empfohlen, Anstrengungen zur Verbesserung solcher Modelle zu unternehmen, damit sie bald praktische Werkzeuge der forstlichen Wirtschaftsführung werden.

3. Optimierung der Holzartenzusammensetzung eines Waldgebietes

von P. KOSTOV

Bulgarien

Auf der Basis von Forsteinrichtungsdaten der Forstunternehmung "Nove Selo" werden Möglichkeiten betrachtet, die optimale Holzartenzusammensetzung nach Standortstypen in einem gegebenen Waldgebiet zu bestimmen. Hierfür wird ein mathematisches Modell empfohlen; es werden die Voraussetzungen für seine Anwendung bei der Planung der forstlichen Produktion aufgezeigt. Die mathematische Formulierung des Modells ist wie folgt:

$$F = \sum_{i=1}^{m} \quad \sum_{j=1}^{n} x_{ij}\, c_{ij} \rightarrow \max \qquad \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

$$\sum_{j=1}^{n} x_{ij} = a_i \;(i = 1,2,\ldots,m)$$

$$\sum_{i=1}^{m} x_{ij} = b_j \;(j = 1,2,\ldots,n) \quad x_{ij} \geqslant 0$$

x_{ij} ist die Fläche der Standortstype i, bepflanzt mit der Holzart j

a_i ist die Fläche der Standortstype i

b_j ist die Fläche der erforderlichen Holzart j

c_{ij} ist der maximale mittlere Volumszuwachs pro Flächeneinheit der Holzart j auf Standortstype i.

4. Ein Algorithmus der Simplex-Methode zur Lösung des Allgemeinen Linearen Programmierungsproblems: Angewandt an einem Beispiel aus der Forstwirtschaft

von V. FARKAS

Ungarn

Normalerweise wird das mathematische Modell eines allgemeinen linearen Programmierungsproblems zur Lösung mit Hilfe der Simplexmethode derart in ein System von Gleichungen verwandelt, daß der Begrenzungsvektor **b**, der aus den Konstanten der Beschränkungen besteht, vom ersten Simplextableau an keine negative Koordinate in Bezug zu irgendeiner Basis enthält. Das vorliegende Papier handelt von einem Simplex-Algorithmus, der jede Konstante, die zu Beschränkungen des Typs "$\geqslant$" gehört, mit negativen Vorzeichen in der **b**-Spalte des ersten Simplex-Tableaus erscheinen läßt. Die vorläufige Tolerierung dieser Negativitäten, die später beseitigt werden, erlaubt, entweder

(1) die zusätzlichen Spalten, die anderenfalls für die Koeffizienten der Überschußvariablen eröffnet werden müßten, oder

(2) die zusätzlichen Operationen zu vermeiden, die anderenfalls dann ausgeführt werden müßten, wenn ein Pivotelement in einer Zeile gewählt wird, die eine negative Koordinate in der **b**-Spalte enthält.

Der Algorithmus wird in Anwendung auf ein forstliches Problem dargestellt.

5. Wirtschaftsplanung im Bauernwald

von G. von MALMBORG

Schweden

Es wird in diesem Papier ein Planungsmodell für die langfristige Planung von hauptsächlich privaten Wäldern, getrennt oder in Kombination mit anderen Aktivitäten, kurz dargelegt. Eine ausführlichere Beschreibung findet sich in früheren Publikationen. Das Papier ist daher hauptsächlich auf eine Diskussion über die Anwendbarkeit des Modells für die praktische Planung wirklicher bäuerlicher Betriebe oder Bauernwälder ausgerichtet. Als Hintergrund ist die Struktur des Bauernwaldes und die angestrebte Verwendung des Modells beschrieben. Im Anschluß wird der Bedarf nach strategischen und taktischen Plänen als Rahmen für operative Erntepläne betont.

Das Modell benützt die lineare Programmierung; spezielle Programme wurden für die Errechnung der Input-Daten für die LP-Matrix und für die Output-Daten entwickelt, um sie in eine lesbare Form zu bringen.

Das Modell ist "allgemein", d.h. es kann für verschiedene Bauernhöfe ohne Veränderung seiner allgemeinen Form verwendet werden. Es ist auch ein "Gesamtmodell", weil alle Aktivitäten, die mit Investitionen und Finanzierung verbunden sind, darin enthalten sind. Außerdem ist der Einfluß der Steuern durch Einbeziehung spezieller Koeffizienten in die LP-Matrix berücksichtigt worden.

6. Dynamische Programmierung zur Bestimmung der Optimalen Einschlagspolitik

von J. RISVAND

Norwegen

Der lange Zeitraum der forstlichen Produktion sowie die mehr kurzfristigen Probleme der Koordination als Folge der Bewirtschaftung großer Flächen stellen besondere Probleme der Planung in der Forstwirtschaft dar. Die meisten forstlichen Planungsmodelle berücksichtigen den Standort der einzelnen Bestände nicht und sind daher dem Koordinationsproblem aus dem Weg gegangen.

Die vorliegende Studie zeigt, wie ein realistisches Modell mit Hilfe der dynamischen Programmierung konstruiert werden kann. Um den Standort der Bestände zu berücksichtigen, wurde die Forstfläche in "Management-Blöcke" unterteilt. Der Management-Block ist zusätzlich zur Transportentfernung durch den Standort des Bestandes charakterisiert.

Das Optimum wurde in 2 Schritten gefunden. Im ersten Schritt wird bestimmt, wie die Bestände innerhalb eines Management-Block's behandelt werden sollen, wenn verschiedene Anforderungen bezüglich Einschlagsmenge an den Block gestellt werden. Im 2. Schritt werden die Ergebnisse dieser Berechnungen zusammen mit einigen anderen Informationen verwendet, um die Verteilung der gesamten Einschlagsmengen auf die Management-Blöcke zu bestimmen.

Es wurde auch gezeigt, wie das Problem zur Lösung mit Hilfe der linearen Programmierung formuliert werden kann. Der Standort der Bestände wurde jedoch in diesem Modell nicht berücksichtigt.

7. Ein Simulationsmodell zum Vergleich von Management Privater Forstbetriebe in Schottland

von A. R. SAYERS

Grossbritanien

Dieses Papier beschreibt ein Computer Modell privater Forstbetriebe in Schottland. Der Simulation wird gegenüber mathematischen Programmierungsmethoden wegen ihrer Anpassungsfähigkeit und wegen der Leichtigkeit, mit welcher der Mechanismus des Modells und die Ergenbnisse von Forsteigentümern und Wirtschaftsführern verstanden werden können, Vorzug gegeben.

Es werden die einzelnen Bestände, die den stehenden Holzvorrat darstellen, beschrieben; die Ertragstafeln der Forestry Commission dienen als Modell für ihren Zuwachs. Der Management-Plan, dessen Auswirkungen untersucht werden soll, besteht aus einer Liste von forstlichen Maßnahmen, die jedes Jahr in den einzelnen Beständen durchgeführt werden sollen. Der Output des Modells enthält eine detaillierte Aufstellung der forstlichen Operationen und einen finanziellen Bericht, der eine Bewertung des stehenden Holzvorrats und eine Analyse der Holzerträge umfaßt; das alles soll dem Forsteigentümer und Manager bei der Auswahl des für seine Politik am besten geeigneten Planes helfen.

Das Modell wurde bei 3 großen privaten Forstbetrieben in Schottland erfolgreich angewandt; eine künftige Verbesserung der Ertragstafeln wäre jedoch wünschenswert.

8. Methoden der Operations Research und Planung. Ausgewählte Operationen in der Forstwirtschaft

von J. RUPRICH

Czechoslovakien

Es wird ein allgemeines Schema eines Forstbetriebes in Matrixform beschrieben. Dann werden Operations Research-Methoden, die in der tschechoslowakischen Forstwirtschaft angewandt werden, beschrieben. Die Lösungsmethode besteht aus folgenden 4 Schritten: (i) Optimierung der einzelnen Aktivitäten, jede für sich allein (ii) Berücksichtigung der Interdependenz der einzelnen Aktivitäten (iii) Modifikation des detaillierten Schemas für die einzelnen Aktivitäten (iv) Entwicklung eines Organisationsplanes für 3–10 Jahre im voraus und eines detaillierten Operationsplanes für 1 Jahr im voraus und (v) Bewertung am Ende jedes Jahres des Modells und seiner Anwendung in dieser letzten Periode und Vorbereitung eines Planes für den Rest des langfristigen Planungszyklus. Es werden die Methoden, die zur Analyse einer Zahl von Aktivitäten benützt werden, beschrieben.

9. Richtlinien zum "Land Management"; Ein integriertes Operations Research Projekt

von W. A. DUERR

Vereinigten Staaten von Amerika

Richtlinien zum" Land Management" ist ein Projekt des "Bureau of Land Management" im Innenministerium der Vereinigten Staaten. Sein grundlegendes Ziel besteht darin, Entscheidungshilfen für eine weitblickende Bodennutzungspolitik zu erarbeiten.

In Verfolgung dieses Ziels ergaben sich bei der Durchführung zwei Phasen des Projekts: (1) große Modelle und (2) kleine Modelle der Tätigkeiten des Büros. Alle Modelle dienen der Vorhersage der Auswirkungen verschiedener Maßnahmen und geben daher dem Entscheidungsträger eine bessere Basis für seine Wahl. Es gibt 3 Arten von kleinen Modellen: (a) wirtschaftliche Modelle (b) Büro-Modelle, die die Tätigkeiten des Büros innerhalb einer Verwaltungseinheit umschließen und (c) Trakt-Modelle, die grundlegende Details für bestimmte Wälder oder andere Bodennutzungsarten liefern.

Sekundäre Ziele der Richtlinien des "Land-Mangement"-Projekts, die sich von den Basiszielen ableiten, sind (3) Weiterbildung der Angestellten und (4) Problemanalyse.

Die Erfahrung aus dem Projekt läßt enge Zusammenarbeit zwischen Forschern und den Benützern ihrer Ergebnisse geraten sein; sie verweist auf bestimmte Richtlinien beim Gebrauch von Computern und sie unterstreicht den Wert der Büro-Modelle als Entscheidungshilfen.

10. Operational Research und die Planung eines Management-kontroll-systems für ein Forstunternehmen

von P. A. WARDLE

Grossbritanien

Die Vorgangsweise bei der Planung eines revidierten finanziellen Kontroll systems in der Forestry Commission war erstens die Festlegung der Funktionen, die vom Management ausgeübt werden müssen, damit die Unternehmung wirksam arbeiten kann, zweitens die Identifikation dieser Funktionen mit einzelnen Managern oder Gruppen von Managern und drittens die Festsetzung der notwendigen Vorgangsweisen, um diese Manager in den ihnen zugeteilten Funktionen zu unterstützen.

Die identifizierten Hauptfunktionen sind (i) strategische Planung und Entscheidungsbildung in bezug zu Typ und Art der Aktivität des Unternehmens und seiner Ziele (ii) taktische Planung und Entscheidungsbildung in bezug zu Fragen wie Personal, Baulichkeiten, Maschinen und Leistungen, die zur Durchführung und zur Bestimmung dieser Maßnahmen erforderlich sind und (iii) operative Planung in bezug zum Einsatz einzelner Mittel für bestimmte Arbeiten.

Hauptmerkmale des Systems sind die vorrangige Bedeutung der strategischen und taktischen Planung, die die Bewertung des Beitrags verschiedener Alternativen der Zielverfolgung involviert, das Konzept operativer Pläne, die innerhalb des Rahmens taktischer Pläne erstellt werden, und die Rückkopplungseinrichtungen, die die Übereinstimmung zwischen tatsächlicher und geplanter Durchführung und zwischen lokalen und zentralen Plänen zeigen sollen. Die Hauptfunktion der Rückkopplung besteht in der Sicherstellung, daß Erstellung und Revision der Pläne auf der Basis zutreffender Informationen über den Zustand des Unternehmens durchgeführt werden.

11. Erfahrung über die Anwendung der Netzplantechnik bei der Lawinenkontrolle

von B. BITTIG und F. PFISTER

Schweiz

Untersuchungen über die Anwendung von Netzplantechnik im forstlichen Ingenieurwesen haben gezeigt, daß die "Critical Path Method" (CPM) am besten geeignet ist.

Aus verschiedenen Gründen ist das Ausmaß an Projekten über Lawinenkontrolle in den letzten Jahren stärker angewachsen. Bei kleineren Projekten konnten die meisten Schwierigkeiten durch kluge Improvisationen beseitigt werden; bei großen Projekten war dies allerdings nicht möglich. Aus wirtschaftlichen Gründen war es notwendig, nach neuen Methoden der Planung und Durchführung zu suchen. Wie an einem Beispiel des Jahres 1966 dargestellt wird, ist die CPM ein ausgezeichneter Weg, Projektplanung und-überwachung zu verbessern.

In Zukunft wird es notwendig sein, ein integriertes System für die Projektforschung zu finden. Wenn mehr interessierte Stellen diese Probleme studierten, sollte es möglich sein, ein befriedigendes System der Integralplanung zu finden.

12. Anwendung Mathematischer Methoden bei der Operativen Planung von Rückoperationen

von M. NOVOTNY

Czechoslovakien

Bei der Prüfung, ob die lineare Programmierung für die kurzfristige Planung des Holztransports geeignet ist, haben wir festgestellt, daß bei der kurzfristigen Planung (Woche, Monat, Vierteljahr) einer der ausschlaggebenden Faktoren für den Erfolg der Wunsch nach verläßlicher Information über die wahrscheinliche Zeitangabe der Erntemaßnahmend ist.

Es wird in dieser Studie die "Methode des kritischen Weges" (CPM) beschrieben, vereinfacht und ergänzt durch ein Entscheidungsmodell. Grundsätzlich besteht dieses Modell in der Verwendung der Kombination der Netzplantechnik mit linearen Programmierungsmethoden.

13. Ein Simulationsmodell zur Unternehmensplanung

von J. MORGAN

Grossbritanien

und E. BJORA

Norwegen

Das Papier beschreibt ein Modell der forstlichen Aktivität in der Forestry Commission, das eine Hilfe für den gesamten Planungsprozeß darstellen soll.

Die betrachteten wichtigsten Entscheidungsvariablen sind Einschlag und Neuaufforstung. Das Modell erlaubt die Erklärung der Auswirkungen verschiedener Maßnahmen auf einen Satz von Variablen, die den auf den Jetztzeitpunkt diskontierten Gewinn, Nettoeinkommen, Holzproduktion und Beschäftigtenzahl enthalten. Es werden die im Modell unterstellten Beziehungen beschrieben.

Die Anwendung des Modells ist durch Beispiele der Vorhersage der Nachfrage nach Arbeitskräften bei verschiedener Einschlagspolitik und verschiedenen Annahmen der Arbeitsproduktivitätssteigerung erläutert. Es werden auch Beispiele über die Auswirkungen verschiedener Einschlagspolitik auf den gegenwärtigen Wert des stehenden Holzvorrats gegeben.

14. Das Linear Programming-planungssystem der Schwedischen Staatsforste

von B. FORNSTAD

Schweden

Die lineare Programmierung wird oft als ein Werkzeug der Operations Research bezeichnet, das dem Analytiker hilft, optimale Lösungen bei Zuteilungsproblem zu finden. Sie ist erfolgreich angewandt worden und führt auch zu optimalen Lösungen mit weniger Rechenaufwand als mit anderen Rechentechniken.

Das zentrale Thema dieses Papiers ist, daß der Aspekt der optimalen Lösung der linearen Programmierung gegenwärtig an Bedeutung verliert und daß andere Überlegungen wie Sensitivitätsanalyse, Präsentabilität usw. in den Vordergrund rücken. Der Autor ist der Meinung, daß die dualen Preise, die mit einer Linear Programming-Lösung verbunden sind, von großer Hilfe sein können.

15. Ein Lineares Programmierungsmodell des Forstprodukten-sektors des United Kingdom

von B. G. JACKSON

Grossbritanien

Ziel der Studie ist die Entwicklung einer optimalen Strategie für den Forstprodukten-Sektor des United Kingdom—ausgedrückt durch den maximalen volkswirtschaftlichen Nutzen—. Staatliche und private Wälder werden zusammen betrachtet. Ein Teil des Modells behandelt die Holzproduktion, indem die Wälder des United Kingdom in 4 "Holzarten", 8 Alterklassen und verschiedene Bewirtschaftungssysteme gegliedert werden. Diese Aktivitäten bieten Holz dem anderen Sektor an, der in erster Linie die holzverarbeitende Industrie mit 10 Produkten wie z.B. Schnittholz, Zeitungspapier umfaßt. Der Zeitraum ist 60 Jahre und ist in 7 Perioden geteilt. Die Nachfrageprognosen für die Produkte sind auf der Basis vergangener Trends und Schätzungen der künftigen technischen Veränderungen gemacht.

Paper 1

OPTIMISATION OF THE HARVEST OF SMALL-SIZE WOOD THROUGH LINEAR PROGRAMMING

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Introduction

There have not been many applications of operational research methods to forestry in Germany yet (Höfle, 1967; Schopfer 1970; Sperber, 1969). Those models existing are simplified and comprise only parts of the forest operations. We are, however, presently refining these models and building more realistic ones. The following model belongs to the first category.

The problem

While there are few alternative methods of harvesting and converting large timber available, the forest firm faces a more complicated decision-making problem when it looks at the operation of harvesting small sized wood. (In this paper small sized wood is defined as timber that is cut in stands which include no lumber, or only insignificant amounts of lumber.)

Let me outline this decision-making problem by the questions which the managers have to answer: What is the main objective of the firm? What is the objective of this small size wood logging operation if it is to be in line with the main objective? Should they produce pulpwood, poles for special uses or posts of various dimensions? Should they debark the products, should they sell them by weight or volume, or should they forget about cutting altogether? If they decide to produce any of these products, how much of each should they produce? In which stand should they produce them and when? What are the optimal economic units for production? What logging techniques should they use?

To answer these questions, we have to analyse the system of the forest firm.

A brief systems analysis of the forest firm and restatement of the problem

The forest firm does not pursue the objective of profit maximisation exclusively since besides producing timber it also provides erosion control, water regulation, recreational facilities and the like. We might rather say that it aims to maximise utility. I do not, however, want to press this point here.

If the firm is to realise this objective, it must apply various planning activities. The "organic production plan" determines where which trees in what species composition are to be planted. It determines the rotation period, the thinning regime and the annual allowable cut. As a matter of time I may omit further details.

The "mechanical production plan" decides upon how trees selected for cutting have to be harvested and converted. It must consider the market conditions and ensure that trees are converted only into products that can be sold. These considerations in turn, may influence the allowable cut determined by the organic production plan.

Logging must take place in units whose size and location makes the operation economic. These things, however, are not only determined by short-run considerations of the mechanical production but also by long-run considerations of the organic production. Finally, the mechanical production has to follow an annual plan that outlines the sequence of all the operations involved including, for instance, planting and road construction.

This outline already discloses that there are interactions and even conflicts between the firm's various operations. This demonstrates the necessity for co-ordination by planning. In the past it was necessary to make several plans and to integrate them in successive steps. Now, by means of systems analysis and operations research we are able to integrate all activities into one plan. This has yet only been realised for parts of our German forest firms. We now put such a model into the above framework of the firm.

The organic production plan determines the annual allowable cut of small size wood. In defining the allowable cut it takes into account the various aspects of utility maximisation and allows some variation by setting upper and lower limits that must be satisfied for silvicultural reasons. It may also restrict the logging methods to be used following the same arguments.

The firm knows which products it is able to sell at what prices. It also knows maximum and minimum requirements for products which have to be met because of purchase agreements.

Further, the factors of production available for the small size wood operation are known from the annual plan of the sequence of production. This plan and the location and size of areas of operation is already determined at a higher level of planning. The factors of production restrict the logging methods that may be used up to the road-side landing.

Finally, since the aspect of maximising utility is taken into account in planning the organic production, the small size wood operation may be planned with the objective of profit maximisation within the constraints laid down. Thus, while the objective of maximising utility is valid for the firm as a whole, profit maximisation may still be applied to parts of the firm.

The problem of logging small size wood may be restated as follows: the forest firm wants to allocate its allowable cut of small size wood and its other factors of production to several products and to various methods of production in such a way that it maximises its profit without violating its capacity and market constraints.

Formulation of the problem by linear programming

The problem corresponds to the structure of the basic linear programming model. The goal of profit maximisation supplies the objective function, the capacities of the factors of production provide the constraints, and the alternatives lie with the production of various products in several stands. Each combination of stand category, product, and logging technique determines one process with a given combination of factors of production. It can be demonstrated that these processes fulfil the assumptions of linear programming such as proportionality, independence, and additivity.

The model was tested by a quantitative example the data of which originate from time studies, mensurational investigations, and other properties rather common to our forest firms. The structure of the illustrative firm and—at the same time—of the model may be seen from Table 1.

TABLE 1

THE HYPOTHETICAL FOREST FIRM, A SURVEY OF PARAMETERS AND VARIABLES

Character of the item considered	Parameter or variable	Representation of parameter or the variable in the hypothetical firm
Fixed parameters	area	4 000 ha
	species of trees	100% spruce
	yield class (average) (according to WIEDEMANN, yield table for spruce 1936)	II·4
	rotation period	100 years
	cutting cycle	5 years (yields an area to be cut each year of 800 ha)
	age distribution	normal in each unit
	road system	50 m per ha roads of truck standard, each compartment with a complete system of skidding lines
	spatial distribution units	5, each of 800 ha one part of the forests with two units in plain terrain
		one part with two units: 1 200 ha of which on medium slopes 400 ha of which on steep slopes
		one part with one unit: 400 ha of which on medium slopes 400 ha of which on steep slopes
	compartments	each of the size 400 × 200 m (rectangle) with a system of skidding lines and space for inventories
		compartments with small-sized wood (referring to trees to be cut) organised in three diameter classes: 6·25— 9·74 cm, mean diameter 8·0 cm 9·75—13·24 cm, mean diameter 11·5 cm 13·25—16·75 cm, mean diameter 15·0 cm

TABLE 1 *Continued*

<table>
<tr><th>Character of the item considered</th><th>Parameter or variable</th><th>Representation of parameter or the variable in the hypothetical firm</th></tr>
<tr><td></td><td></td><td>three classes of terrain and three diameter classes yield 9 stand categories
yield classes: plain terrain II·0
medium slopes II·5
steep slopes III·0
form factor (concerning the shape of the trees): = 0·550
stands to be cut determined through age, cutting cycle and diameter</td></tr>
<tr><td>Restrictions</td><td>allowable cut of small-sized wood per year</td><td>allowable cut determined by means of the yield table mentioned
organised according to stand categories
values of the yield table
+25%=minimum constraint
−25%=maximum constraint</td></tr>
<tr><td></td><td>factors of production</td><td><table>
<tr><th></th><th>capacity per year in hours</th></tr>
<tr><td>8 forest workers</td><td>13 600</td></tr>
<tr><td>power saws</td><td>no constraint</td></tr>
<tr><td>skidding system (tractor with crane and trailer)</td><td>1 600</td></tr>
<tr><td>1 debarking machine</td><td>500</td></tr>
<tr><td>1 skidding device "Flott"</td><td>1 800</td></tr>
</table></td></tr>
<tr><td></td><td>market restrictions</td><td>minimum constraints for posts and pulpwood, maximum constraints for poles</td></tr>
<tr><td>Variables</td><td>combinations of products</td><td>pulpwood, posts, poles } called after its main component
timber conversion factors connect the combination with the allowable cut
the combinations "Thinnings" just cut the trees</td></tr>
<tr><td></td><td>methods of production</td><td>only one method per combination of products and stand category:
<table>
<tr><th></th><th>plain terrain</th><th>medium slopes</th><th>steep slopes</th></tr>
<tr><td></td><td colspan="3">mechanical debarking</td></tr>
<tr><td>pulpwood:</td><td colspan="2">sorted skidding</td><td>tree-length skidding</td></tr>
<tr><td>posts:</td><td colspan="2">sorted skidding</td><td>tree-length skidding</td></tr>
<tr><td>poles:</td><td colspan="3">tree-lengths skidding</td></tr>
</table>
input coefficients of the factor of production deduced from experimental data</td></tr>
<tr><td>Objective</td><td>objective of the conversion of the small-sized wood: maximisation of the profit contribution (gross return)</td><td>prices of combinations of products= weighted average of the prices of the components of the combinations

costs=products of the inputs of the factors times the prices of the factors

differences of prices and costs=profit contributions (gross return)</td></tr>
</table>

The firm covers 4,000 ha of spruce stands distributed on plain terrain and on medium and steep slopes. The stands with small size wood fall into three diameter classes, so that we find nine stand categories altogether. The allowable cut of each is given by Table 2. A careful analysis proved that in the first approach it would be wise to concentrate on two stand properties, diameter and slope, because the influence of other factors had not been previously clearly established.

Concerning the other factors of production, the firm is able to use up to eight forest workers, a forwarder, a sort of cableway for steep slopes, and a debarking machine. Table 3 lists the available capacities of each.

TABLE 2

ANNUAL ALLOWABLE CUT OF SMALL SIZED WOOD

m^3

Diameter	Distribution of the cut on stand categories								
	Terrain								
	plain			medium slope			steep slope		
	min.	normal	max.	min.	normal	max.	min.	normal	max.
8·0	192	256	320	111	148	185	69	92	115
11·5	552	736	920	969	1 292	1 615	471	628	785
15·0	924	1 232	1 540	579	772	965	417	556	695
Sum	1 668	2 224	2 780	1 659	2 212	2 765	957	1 276	1 595

Total allowable cut:	minimum	4 284
	normal	5 712
	maximum	7 140

TABLE 3

CAPACITY OF FACTORS OF PRODUCTION

Factor	Capacity of the factor per year in hours		
	Initial formulation of the model	Variation I	Variation II
Normal labour (forest workers)	13 600	7 200	7 200
Driver of the skidding-system	1 600	450	450
Foreman of the debarking machine	500	300	400
Power saws	no restrictions		
Skidding system	1 600	450	450
Skidding device "Flott"	1 800	1 800	1 800
Debarking machine	500	300	400

Three different products—pulpwood, posts and poles—can be produced in each stand by various logging methods. The market restrictions for these products are given in Table 5.

The logging methods, which I will not comment on further, and their coefficients of production are given in Table 4. The alternatives labelled "thinnings" only include cutting trees as a silvicultural treatment. The "timber conversion factor" indicates how much standing timber is necessary to produce one cubic metre solid volume of products.

Finally, Table 6 shows the prices of products, costs of producing them, and their profit contributions. We refer to them as profit contributions because only variable costs are taken into account. You may notice that the profit contributions of some products and of all so-called thinnings are negative. These alternatives are, however, not excluded because of the lower limits of the annual allowable cut.

TABLE 4

INPUTS OF FACTORS OF PRODUCTION FOR EACH COMBINATION OF PRODUCTS

1/100 hours per m^3

Combination of products	Inputs per stand category and combination of products								
	Diameter								
	8·0			11·5			15·0		
	Terrain			Terrain			Terrain		
	plain	medium slope	steep slope	plain	medium slope	steep slope	plain	medium slope	steep slope
Pulpwood									
normal labour	3·61	3·67	6·50	1·56	2·22	2·72	0·84	0·90	1·40
driver of skidding system	0·13	0·15	—	0·10	0·11	—	0·07	0·08	—
foreman of debarking machine	0·14	0·14	0·14	0·07	0·07	0·07	0·05	0·05	0·05
power saws	1·52	1·68	1·86	0·68	0·75	0·83	0·32	0·37	0·64
skidding system	0·13	0·15	—	0·10	0·11	—	0·07	0·08	—
skidding device "Flott"	—	—	2·23	—	—	0·86	—	—	0·38
debarking machine	0·14	0·14	0·14	0·07	0·07	0·07	0·05	0·05	0·05
timber conversion factor		1·49			1·23			1·20	
Posts									
normal labour	2·85	3·17	5·51	1·41	1·50	2·55	0·79	0·85	1·31
driver of skidding system	0·19	0·21	—	0·14	0·16	—	0·08	0·08	—
foreman of debarking machine	0·17	0·17	0·17	0·08	0·08	0·08	0·06	0·06	0·06
power saws	1·15	1·28	1·57	0·56	0·61	0·83	0·28	0·31	0·39
skidding system	0·19	0·21	—	0·14	0·16	—	0·08	0·09	—
skidding device "Flott"	—	—	1·74	—	—	0·76	—	—	0·30
debarking machine	0·17	0·17	0·17	0·08	0·08	0·08	0·06	0·06	0·06
timber conversion factor		0·79			0·93			1·12	
Poles									
normal labour	2·59	2·86	4·72	1·31	1·39	2·26	0·80	0·86	1·28
driver of skidding system	0·20	0·22	—	0·15	0·16	—	0·09	0·10	—
foreman of debarking machine	0·18	0·18	0·18	0·08	0·08	0·08	0·04	0·04	0·04
power saws	0·12	0·14	1·27	0·50	0·56	0·68	0·28	0·31	0·38
skidding system	0·20	0·22	—	0·15	0·16	—	0·09	0·10	—
skidding device "Flott"	—	—	1·51	—	—	0·69	—	—	0·31
debarking machine	0·18	0·18	0·18	0·08	0·08	0·08	0·04	0·04	0·04
timber conversion factor		0·79			0·93			1·12	
Thinnings									
normal labour	0·43	0·45	0·49	0·21	0·22	0·24	0·12	0·13	0·14
power saws	0·35	0·36	0·40	0·16	0·17	0·19	0·10	0·10	0·11
timber conversion factor		1·00			1·00			1·00	

TABLE 5

MARKET RESTRICTIONS ON THE PRODUCT COMBINATIONS

m^3

Diameter of the stand categories	Terrain (slope)	Market restrictions	
		Maximum amount of poles	Minimum amount of posts and pulpwood
8·0	plain	100	40
	medium	100	30
	steep	50	30
11·5	plain	400	300
	medium	700	500
	steep	300	200
15·0	plain	500	400
	medium	300	300
	steep	200	200

TABLE 6

PRICES, COSTS AND PROFIT CONTRIBUTION OF EACH COMBINATION OF PRODUCTS

DM per m^3

Combination of products, prices, costs, and profit contributions	Prices, costs, and profit contribution per stand category and combination of products								
	Diameter								
	8·0			11·5			15·0		
	Terrain			Terrain			Terrain		
	plain	medium slope	steep slope	plain	medium slope	steep slope	plain	medium slope	steep slope
Pulpwood									
prices	50·37	50·37	50·37	53·33	53·33	53·33	58·55	58·55	58·55
costs(1)	38·11	39·19	60·73	17·49	23·26	25·78	9·94	10·69	13·93
profit contribution(2)	12·26	11·18	−10·36	35·84	30·07	27·55	48·61	47·86	44·62
profit contribution(3)	3·95	2·69	−24·27	24·92	24·92	21·69	46·51	45·60	41·56
Posts									
prices	47·45	47·45	47·45	51·69	51·69	51·69	57·12	57·12	57·12
costs(1)	33·07	36·25	52·78	17·02	18·19	24·61	9·90	10·62	13·06
profit contributions(2)	14·38	11·20	−5·33	34·67	33·50	27·08	47·22	46·50	44·06
profit contributions(3)	7·38	3·48	−17·28	31·08	29·66	21·55	45·16	44·28	41·16
Poles									
prices	45·25	45·25	45·25	49·85	49·85	49·85	57·12	57·12	57·12
costs(1)	31·14	33·90	46·00	16·27	17·20	21·96	9·64	10·36	12·28
profit contributions(2)	14·51	11·75	−0·35	33·58	32·65	27·89	47·88	46·76	44·84
profit contributions(3)	7·99	4·61	−10·69	30·17	29·04	22·95	45·43	44·55	42·06
Thinnings									
profit contributions	−4·10	−4·28	−4·67	−1·98	−2·09	−2·09	−1·15	−1·23	−1·33

(1) without fixed components of machine costs.
(2) fixed components of machine costs are not covered.
(3) fixed components of machine costs are covered.

Solution and results

The mathematical formulation of the problem is shown in Table 7. The problem was solved on a computer by means of the modified simplex algorithm.

The solution specifies the optimum quantities and combinations of products and methods of production. It shows which factors of production are scarce and evaluates them by implicit prices. For products not included in the optimum solution it shows the loss which we incur if we want to produce them. The sensitivity analysis determines whether the optimum solution remains the same if we change costs, prices, coefficients of production, and capacities of factors of production. It also enables us to assess the chances of new products or methods of production.

TABLE 7

SIMPLIFIED MATRIX FORMULATION OF THE PROBLEM

	Poles produced in stand category 1	Posts produced in stand category 2	Pulpwood produced in stand category 3	Thinning in stand category 4	...	Sign	Capacities of resources or market restrictions
Variable	x_1	x_2	x_3	x_4	...		
Profit contribution	p_1	p_2	p_3	p_4	...		
Stand categories 1	a_{11}					$\geq$	$b_1 \leq b_2$
Stand categories 2		a_{22}				$\geq$	$b_3 \leq b_4$
Stand categories 3			a_{33}			$\geq$	$b_5 \leq b_6$
Stand categories 4				1		$\geq$	$b_7 \leq b_8$
. . .							
Other resources							
labour	a_{51}	a_{52}	a_{53}	a_{54}		$\leq$	b_9
forwarder	a_{61}	a_{62}	a_{63}			$\leq$	b_{10}
. . .							
Market restrictions							
Max. for poles	1					$\leq$	b_{11}
Min. for posts		1				$\geq$	b_{12}
Min. for pulpwood			1			$\geq$	b_{13}
. . .							

Some solutions are given in Table 8. I do not, however, want to discuss these solutions in detail. I will rather indicate information which we get by performing a limited sensitivity analysis.

We find that poles are preferred to pulpwood because of their small timber conversion factors, and pulpwood, in turn, is preferred to posts. But this changes if the debarking machine and the forwarder are scarce because these resources are better used by other products. We may mention in passing that the advantage of poles is also eliminated if we substitute the objective of profit maximisation for that of cost minimisation. The thinnings are only performed to achieve the minimum cut required if the resources saved can compensate the loss on the thinnings through their use in the production of more profitable products.

The market restrictions decrease the profit because they limit the production of optimal products and force that of less valuable ones. What changes in the optimum solution variations in the profit contributions initiate can only be determined by carefully investigating the entire set of data.

If there are enough resources the maximum amounts of the stands are cut. If, however, resources are scarce, there is, generally speaking, a decline of unused cut from the stand class with the highest to that with the lowest diameter.

In the first place the debarking machine and the forwarder are scarce. Their implicit prices indicate how scarce these resources are. They change with every variation of the profit contribution or the capacity of other scarce resources so that they are only valid for the situation investigated.

TABLE 8

SOME SOLUTIONS OF THE PROBLEM

Combination of products			Quantities of the optimal combination of products m³ — Solutions			
Diameter	Product	Terrain	1	2	3	4
8·0	pulpwood	medium	74·89			
	poles	plain	405·06			
		medium	92·93			
	thinnings	plain		192·00	192·00	192·00
		medium		111·00	111·00	111·00
		steep	69·00	69·00	69·00	69·00
11·5	pulpwood	plain		747·97	747·97	747·97
		steep		335·89	335·89	
	posts	medium		1 033·08	1 033·08	
	poles	plain	989·25			
		medium	1 737·56			1 381·48
		steep	844·09	339·97	339·97	597·59
15·0	pulpwood	plain				1 283·33
		medium				804·17
	poles	plain	1 375·00	1 375·00	1 375·00	
		medium	861·61	861·61	861·61	
		steep	620·54	620·54	620·54	620·54

Factors of production		1 Unused capacities	1 Implicit prices	2 Unused capacities	2 Implicit prices	3 Unused capacities	3 Implicit prices	4 Unused capacities	4 Implicit prices
normal labour		3 723·82			4·24		4·17		12·01
foreman of debarking machine			55·05		228·92		221·67	60·12	
debarking machine			55·05		228·92		221·67	60·12	
driver of skidding system		851·16			55·19		44·42		84·63
skidding system		851·16			55·19		44·42		84·63
"Flott"		1 025·21		1 084·18		1 084·18		1 195·59	
stands 8·0	plain terrain	upper limit	+5·82	lower	−5·92	lower	−5·89	lower	−9·27
	medium slope	upper	2·33	lower	−6·19	lower	−6·15	lower	−9·69
	steep slope	lower	−4·67	lower	−6·75	lower	−6·71	lower	−10·56
stands 11·5	plain terrain	upper	31·37	upper	5·25	upper	6·28	upper	5·67
	medium slope	upper	30·37	323·65		323·65		330·25	
	steep slope	upper	25·25	55·68		55·68		229·24	
stands 15·0	plain terrain	upper	40·43	upper	26·76	upper	26·75	upper	26·19
	medium slope	upper	39·78	upper	25·39	upper	25·39	upper	24·16
	steep slope	upper	38·07	upper	27·02	upper	27·04	upper	25·98
Total profit contribution		254 342·04		211 965·07		204 476·48		207 922·42	

Remarks:

Solution 1: original capacities of factors, thinnings only for steep slope of diameter class 8·0 in the model, only variable components of machine costs included, restrictions of stand categories minimum—maximum allowable cut.

Solution 2: variation I for capacities of factors, thinnings for all stand categories included, remainder like solution 1.

Solution 3: fixed components of machine costs included, too; remainder like solution 3.

Solution 4: like solution 3 but variation II for capacities of factors.

Consequences and conclusions

Since the model does not employ the parameters of a particular firm, it cannot be verified directly. We can, however, infer that the results comply with reality.

Only firms whose small size wood operation has the same structure and that are willing to accept the simplifications made are able to apply the model. Then, the managers still have to decide upon the economic units, the sequence of production, and upon how to convert the individual stands.

Using sensitivity analysis the firm is able to experiment with new logging methods or new products through the model instead of by trial and error. It also can determine optimal reactions to a changing environment and to specific demands of products.

Of course, the model still has to be improved. Possible improvements include, for instance, the collection of better data, the introduction of more products and logging methods, and a better stratification of stands. Furthermore, the model should encompass more activities of the forest firm, e.g. the logging of large timber and the decisions of the organic production. Finally, we should come to a complete model of the forest firm.

But even if, or just because, we have not reached that stage yet, this model may in spite of its simplifications prove to be a valuable beginning since it shows the structure and requirements of linear programming, its advantages, and its limitations.

Thus, I may conclude with what I consider the main advantages of applying linear programming to forest problems. The formulation of the model requires a complete and thorough analysis of the problem and of the process of decision-making. It also requires exact data. The solution simultaneously combines and co-ordinates all alternatives and provides unbiased results to managers. It gives valuable information about deviating from the optimum and about the optimal reactions to changing parameters.

But these advantages do not permit the use of linear programming in every case. Rather, the problem must have the structure of linear programming, all information about the situation must be quantified, and the model has to be solvable mathematically.

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DISCUSSION OF HÖFLE'S PAPER

Duerr. A general problem arising from this paper is that of defining the boundaries of study area—there is no optimisation in the general sense. In this case you are going to expand the model and take some of the givens and incorporate them as variables. *Höfle.* One is faced with the problems of sub-optimisation. There are two ways in which I would like to expand the model, they are (i) in the area of mechanical production to introduce the possibility of varying the capacity of the factors of production and the type of equipment; (ii) instead of having to define the allowable cut, to introduce choice of species and the allowable cut as well as harvesting technique and assortments as variables within the model itself. I see no way of doing the second at the moment. *Glück.* In respect of assortments there are many possibilities for any one tree and if you want to reach the true optimum you have to define all the available possibilities. *Höfle.* It should be recognised that this is a first shot model and, in fact, products were product combinations. In order to develop here more elaborate data has to be assembled. *Von Malmborg.* I understand you would like to expand the model to cover more levels. *Höfle.* Yes, the tradition is to plan at different levels and integrate plans in a separate step. For example, given the "organic production" we plan the production process, finance and markets. What we should do is to include all these things as variables and solve not just for parts but for the whole enterprise simultaneously. *Von Malmborg.* You want to integrate long and short-term planning. *Höfle.* Yes, but I don't think it is practicable. *Duerr*. The paper and the discussion have raised the problems the researcher must face about the degree of abstraction in respect not only of physical variables but the value system including the immeasurables—how does one take account of these values. The question of the appropriate model has also been raised. What are the advantages of linear programming as contrasted with simulation, and of course one can simulate using a linear programme model.

Paper 2

USES OF SIMULATION IN FORECASTING STAND GROWTH AND MORTALITY

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Introduction

One hundred years ago, most of the empirical knowledge about North American forests was gathered by "Grand Walkers" who developed eye-estimation procedures and rules of thumb. Fifty years ago, foresters started making descriptive studies of North American lands and forests to begin the task of managing some of the most valuable natural resources on the continent. Five years ago, forest scientists turned to high-speed digital electronic computers for summarising background information already collected, and for diagnosing what would likely happen in the near and distant future to the forest industry, to forest stands, and even to individual trees.

Accumulated industrial and economic information led to the creation of BUSINESS MODELS for simulating the behavior of forest firms, and forecasting their role in large sectors of the economy (Manetsch, 1964; Hool, 1965; Clutter and Bamping, 1966; Schweitzer, 1968; and Ware, 1968).

Operational and mensurational information permitted the development of FOREST MANAGEMENT MODELS aimed at simulating the long-term effects of certain managerial decisions. Thus, a number of simulators were devised to analyse forest sampling (Payandeh, 1968; Arvanitis and O'Regan, 1969), reforestation (Grevatt and Wardle, 1967), harvests and designs of logging equipment (Newnham, 1965, 1966b, 1967, 1968a; Newnham and Sjunnesson, 1969), pest and fire control (Watt, 1963; Kourtz, 1966), forest fuels (Smith, 1968), and the complete management of small forest properties for timber production (Gould and O'Regan, 1965; Myers, 1968; Bare 1970).

However, during the past five-year period, the largest scientific effort in forest simulation has been concentrated in the building of MATHEMATICAL STAND MODELS for forecasting stand growth from individual tree analyses. Hereafter, we intend to review briefly the advances in that field, to suggest a new approach to such studies, and to point out where further ameliorations are possible and desirable.

Previous studies in stand modelling

Stand growth simulation can be performed by using a deterministic or a stochastic approach. In the first case, a given set of conditions will yield only one definite answer; in the second case, the relations will be stated in terms of probability distributions, and the answers will vary according to random elements. Simulation models developed by Newnham (1964) and Dress (1968) are examples of these two types.

Both hypothetical and empirical data can serve in establishing relationships, in building a simulator, and in testing the outputs. Simulations carried out by Smith *et al.*, (1965) were performed on such hypothetical data. Most models, however, are based, at least partly, on hard data; e.g. Newnham and Smith (1964), and Mitchell (1969).

Early stand models represented only a small number of trees located in a square lattice. Recent ones can accommodate more realistic populations even if they are still limited to even-aged forests containing only one (Bella, 1969; Lin, 1969) or two tree species (Pennycuick *et al.*, 1968).

Many of the aforementioned stand models were created to evaluate the relative merits of several possible combinations of cultural treatments. In some (Lee, 1967, Tsolakides, 1968), the authors went as far as breaking down the products into logs of different sizes and quality.

Simulation of growth and death

In order to model the growth of a forest stand, one has to write a computer program which (1) generates or determines the spatial arrangement of each individual tree, and allocates or defines its dimensions; (2) evaluates the degree of competition bearing on it, and simulates how its growth is affected by this stress and by other impeding agents; and (3) removes it after death or cut, by following a dynamic schedule.

Some programs are available to auto-generate upon request an almost unlimited number of artificial forest stands of various spatial arrangements and frequency distributions of tree diameters and heights (Newnham, 1968; Newnham and Maloley, 1970). Otherwise, the consideration of actual tree patterns and sizes does not present any particular problem, except for their location on the ground, which is also computerised (Brace, 1970).

On the other hand, the evaluation of both the competition supported by trees and its influence on tree growth is theoretically a much more challenging aspect of stand modelling. Usually, it consists in developing some mathematical function that explains the largest proportion possible of the variation in tree size increment, based on stand and site parameters, and a competition index. Stand age and site quality are often used because they serve in classifying even-aged forests composed of one species. Stand density can be considered as an index of the average competition intensity, or point density can be evaluated to appreciate the degree of competition around each tree (Lemmon and Schumacher, 1962). Alternately, the proportion of the zone of influence of each subject tree occupied by competitors can be calculated, and its reduction in growth related to number, size and distance of competitors. Maximum zone of influence and rate of increment are normally measured on "open-grown" individuals, from the size of stem at breast height and from the width of their crowns. Stem or crown growth reductions are empirically related to the amount of zone or crown overlap, evaluated directly or by some angle summation procedure. Applications of these methods can be found in Newnham (1966a); Opie (1968), and Gerrard (1969).

Most of the growth-competition regressions thus developed are efficient tools to forecast tree development under various management regimes. However, they are inadequate for predicting mortality, due to the variability in resistance of trees submitted to limiting degrees of competition. In practice, thresholds are set somewhat arbitrarily, beyond which trees are deemed dead. For instance, Newnham (1964) considered as dead, any trees that increased their diameter by no more than a given percentage (between 0·1 and 5 per cent) in 5-year periods; Lin (1969) "killed" trees which had a zero growing space index (i.e. trees that subtended a cumulative view angle from their stem to that of competitors larger than 5·25 degrees in four quadrants), and were growing at less than 0·1 inch per two-year period for 6 years. These procedures for predicting mortality were obviously not refined enough by comparison to the growth characteristics of the model in which they were included.

A new mortality generator

We attempted to improve the technique of mortality allocation by building mortality tables for second-growth pure and mixed stands of Douglas fir (Paillé, 1970). These tables were based on a large number of trees (13 thousand) observed during periods of from 10 to 30 years in permanent sample plots; they give the probability that trees die from competition within a decade, according to their relative size and position within the forest canopy. Figure 1 shows decadal probabilities that individual trees die between the age of 20 and 90, depending on the ratio of tree diameter at breast height (dbh) to average stand diameter (DBH), and on the ratio of total tree height (h) to stand top height (TOPH); Figure 2 relates the probabilities of mortality to individual tree crown class (4 = suppressed; 3 = intermediate; 2 = codominant; 1 = dominant). Probability distributions built on relative increment in diameter and height were also calculated, but are not shown here. These probabilities were computed from unmanaged stands, representing a wide variety of conditions encountered throughout the Coastal Douglas fir Region of the Pacific Northwest. The assumption underlying this approach was that trees having the same relative characteristics as their predecessors by comparison to the forest in which they grow should behave the same way.

The probability distributions permit generation of competition mortality in both a deterministic and a stochastic manner. For instance, a manager who wants to keep only those trees having less than 60 per cent chances to die between the age of 40 and 50 will harvest every tree having a diameter smaller than half of the average, being shorter than half of the stand top height (from Figure 1), or being classified as suppressed (from Figure 2). Alternatively, if he wants to adopt a statistical approach, he will kill randomly 6 trees out of 10 having those characteristics.

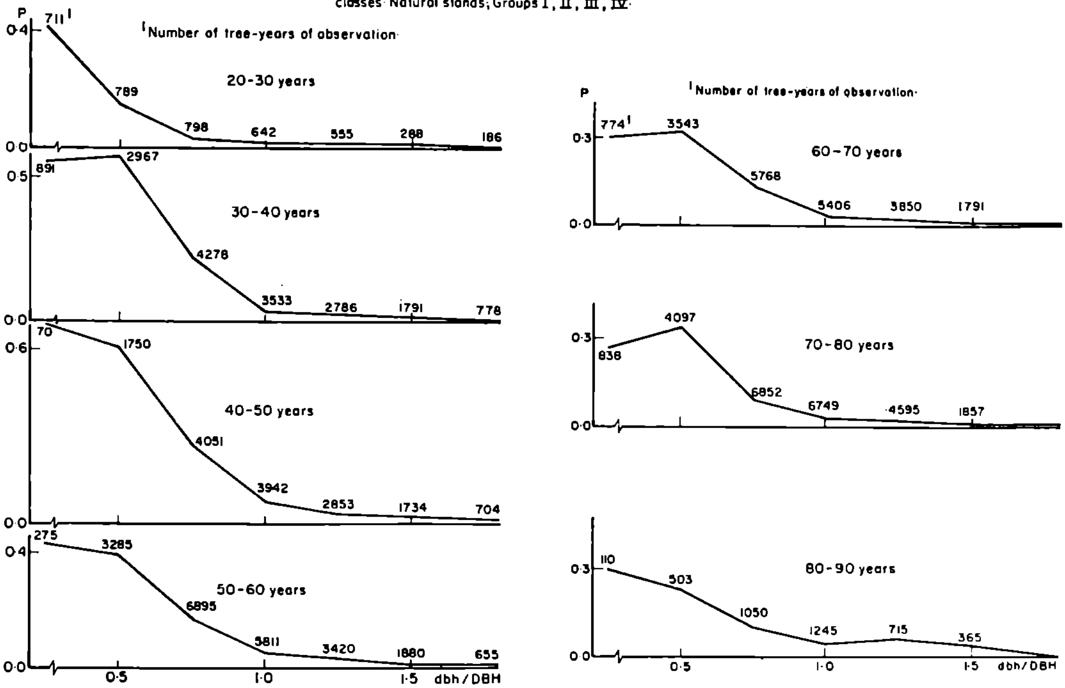
Moreover, they allow for irregular and catastrophic types of mortality (caused by fire, insects, diseases, wind, and the like) to be taken into account as well. When the probability of such events is calculated for a given region, it can be directly added to those shown here over the full range of sizes, because it usually affects the entire stand.

Uses of mortality tables

(1) For improving actual stand models

When introduced into most actual stand models, mortality tables could efficiently replace the kind of arbitrary threshold previously described. Trees could be directed through these tables and killed systematically if a basic risk level is determined by the forest manager, or stochastically when full use of the curves is made.

Better answers would be obtained by building a specific set of tables for each climatic region, each topographic class, each forest type, association, or species, each quality of site, and the like. The ones shown in Figures 1 and 2 represent a wide range of conditions, for one species; as such, they offer an average degree of reliability. Moreover, since they are based on data collected in unmanaged stands, Periodic probability of individual tree mortality by relative size classes· Natural stands; Groups I, II, III, IV·



Periodic probability of individual tree mortality by relative height classes· Natural stands; Groups I, II, III, IV·

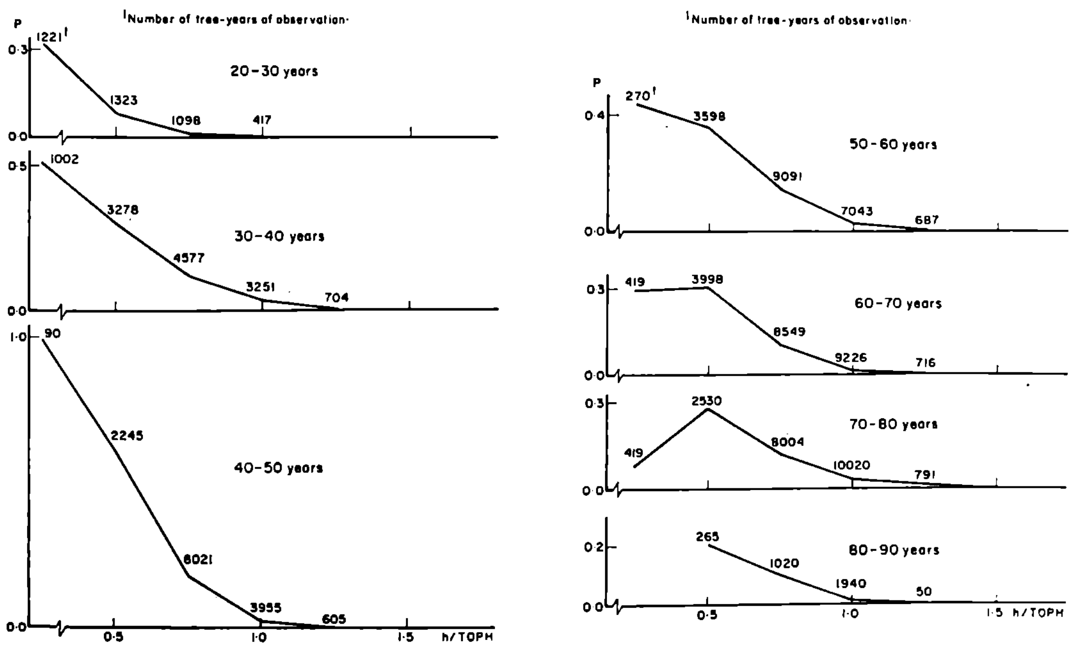


Figure 1. Periodic probability of individual tree mortality by relative diameter and height classes.

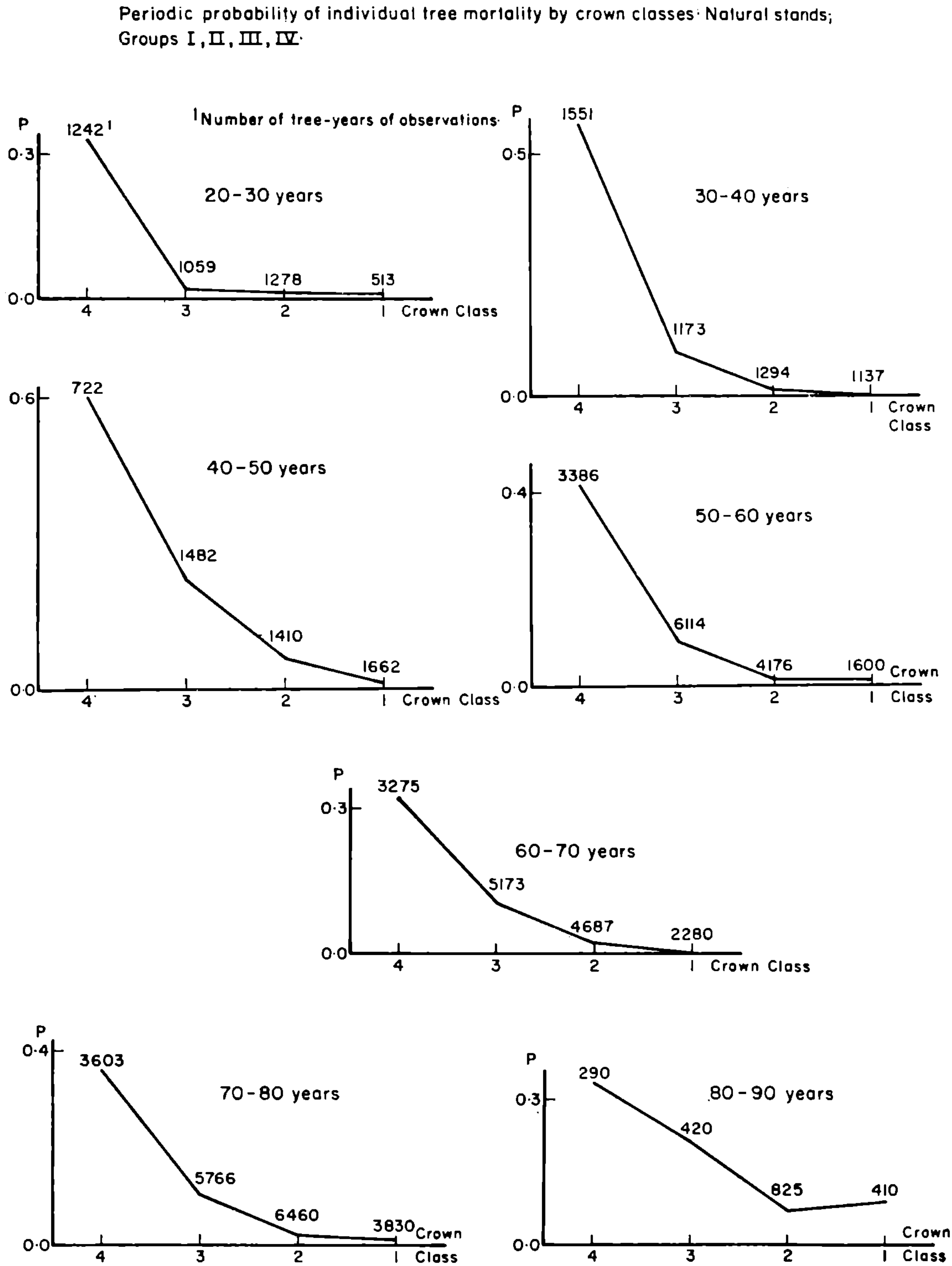


Figure 2. Periodic probability of individual tree mortality by crown classes.

they should be most useful to forecast the development of extensively managed natural stands, where cultural treatments are contemplated. Once stands are frequently thinned, however, the amount of expected mortality becomes minimal, and any mortality generator is more or less useless, unless salvage of only a few trees at a time is economically feasible.

(2) For devising marking rules

One of the main purposes of silvicultural operations, at least in North America, is to salvage natural mortality by effecting one or two partial cuts within a rotation period. For that purpose, trees to be taken out are often being designated on the basis of their vigour, quality, and location, and according to some criteria of merchantability. However, the actual chances that trees have to live or die are seldom considered, the operation relying mainly on educated guesses. Mortality tables could be used to facilitate decisions made "at the stump", and to quantify the degree of confidence that can be put in such operations aimed at the recuperation of anticipated losses.

(3) For developing a semi-stochastic stand model

A new stand model has just been developed (Paillé, 1970) to predict, for a restricted number of 10-year periods, gross and net growth and yield of second-growth Douglas fir stands. The input consists of an actual or hypothetical plot tally of individual tree diameters (up to 1,000), and a linear regression of tree height on diameter. The output is composed of histograms of dead tree diameters, maps of their most likely spatial arrangement, and growth, yield and mortality information for a specified number of prediction periods (up to 7). Inside the model, trees are grown with a regression of annual diameter increment on stand characteristics; their height is predicted periodically from their diameter, and their volume is computed in cubic feet from diameter and height. Trees are "killed" by going through the mortality generator described above. Their spatial arrangement is plotted directly if it is read as an input; otherwise, it is determined according to a complicated mathematical process, devised from analyses of empirical data on spatial patterns of dead trees.

This model gives much information on amount, timing, and distribution of mortality, for it is meant to be used by forest managers who must take decisions about the urgency of thinning or salvage operations, and calculate in advance expected financial returns.

Characteristics of the ideal model

Much remains to be done before stand models become fully operational. The consideration of advantages and drawbacks in several existing models has led to some suggestions concerning the ideal model. Eventually, it should offer the following features:

(1) Be fully stochastic and accept hypothetical as well as actual data.

(2) Provide possibilities for handling several species simultaneously, or in sequence.

(3) Integrate mensurational data, and relevant non-mensurational data about soil characteristics, microclimate, microsite, physiography, and other such factors affecting tree and stand development.

(4) Evaluate competition and growth in a multidimensional universe. Ultimately, up to six dimensions should form the picture, three underground and three above. This would facilitate the study of complete biomass production.

(5) Allow external interference in the simulation process, produce many outputs in graphical and tabular forms, display them visually, and contain a self-testing mechanism.

Conclusion

This review of what has been accomplished in the past 5-year period, in a relatively small sector of forestry research, is indicative of the modern scientific pace. Let us hope that what is today one of the best tools of investigation and education in forest land management planning will soon become a more widely used operational technique in practical decision-making.

Simulation has much to offer in forestry because it may destroy the old myth that the next generation of foresters has to cope with the mistakes of the present one. Now, one can forecast mistakes before they are made, and hopefully correct them in no time.

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DISCUSSION OF PAILLÉ'S PAPER

Paillé. This type of simulation I see as providing information for higher level management models. The particular case presented is not appropriate for intensively managed forests. *Glück.* One can certainly see the relevance of such models to managed forests, particularly a truncated form in which the mortality component is replaced by a thinning one. *Vornstad.* Has one been able to assess the degree of error associated with the model and what sort of consequences it might have using it for the higher level management planning? *Paillé.* It has not been possible to check it in detail, but it has been possible to verify against stand inventory data. *Howell.* To what extent is this a stochastic model? *Paillé.* It is semi-stochastic, the mortality part is stochastic, the rest is based on deterministic equations.

Paper 3

OPTIMISATION OF THE SPECIES COMPOSITION OF A FOREST AREA

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Introduction

The development of mathematical methods and of electronic computer technique are closely related. As new models are worked out in various fields of science and industry, their application to forestry has to be tested and their adoption for wider application should follow checks on their validity. The papers so far published in Bulgaria in this field (Vasilev, 1968; Garelkov, 1966; Kostov, 1968; Cimev, 1968) and the trial described below follow this pattern.

This paper is concerned with the optimisation of the species composition of a given forest area. The question of the optimum mixture of species within a plantation on a given site is not considered. A model solving such problems is put forward by Nestorov, (1967) and has been adopted to the conditions in Bulgarian forests by Garelkov, (1966). The model described in this paper is a linear programming model which seeks the optimum growing stock according to species, taking account of the sites available and the requirements of the various species on the one hand and the needs of the national economy for wood of different species on the other.

On a given site many species have equal or almost equal productive potential. If the sole aim were to obtain the highest possible production without taking into consideration the need for wood of various species, the problem would be solved easily by choosing the most productive species for each site type. It is well known however that because of their varying technical properties the national economy requires supplies of wood of a number of different tree species.

Where a forest area includes many site types on each of which a number of tree species may be grown and each species having different productive potential, the selection of species which will meet the wood requirements by species and at the same time yield maximum production from the whole area is complicated, and cannot be achieved by traditional computing methods. The selection of the combination of species which will secure the highest production calls for the utilisation of linear programming.

The problem, to obtain the distribution of species to site types which secures maximum production has the following mathematical expression:

$$F = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij}\, c_{ij} \rightarrow \max \qquad \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

$$\sum_{j=1}^{n} x_{ij} = a_i \; (i = 1,2,....,m) \qquad a_i = a_{io} - a_{ip}$$

$$\sum_{i=1}^{m} x_{ij} = b_j \; (j = 1,2,....,n) \qquad x_{ij} \geqslant 0$$

The x_{ij} stand for the area on site type i of species j obtained when the problem is solved. The available area of site type i is a_i and is obtained from the total area of the site type a_{io} by deduction of the area having a meliorative role a_{ip}. The deduction of this latter area which is not significant (2–3 per cent) is carried out before solving the problem. Data for the area according to site type is obtained from forest site surveys. The area of each species b_j is determined on the basis of the long-term requirement for wood of that species. It is specified for the region being studied and takes account of the site types in the region. Mean annual increment at maturity c_{ij} is a measure of the crop's contribution to production. Mean annual increment is obtained from experimental tables or by observation as for example in the experiments of Krastov, (1967). In the case that the existing crop is to be preserved for reasons such as production of special wood or seed sources, the increment for alternative crops on these sites is expressed as zero indicating incompatibility. Similarly tree species which would involve great expense and labour on particular sites are regarded as incompatible.

The mathematical method surveyed above was applied in determining the optimal composition of the forests in the forest enterprise Novo Selo (Kostov and Stoimenov, 1968).

In deciding the tree species capable of growing on each site type and their productivity on the site, use is made of the work of Garelkov, (1967). Three basic variants were prepared with respect to the correlation of the consumption according to tree species, which are distinguished mainly for the participation of beech and of other broad-leaved and coniferous tree species. It cannot be asserted that this is the best correlation but it is quite suitable for testing the method surveyed. The three variants were processed by an electronic computer at the Electronic Computer Centre of the Ministry of Transport. (In an earlier publication (Kostov and Stoimenov, 1968) the problem was solved by means of ordinary calculating machines after the method of Lurie.) The results obtained are shown on tables 9 and 10. All variants give higher increment than that obtained by preserving the present species composition.

The real increment is 24520 cubic metres. It should be mentioned that when the proportion of coniferous species is preserved unchanged the difference in the increment is small. Each increase in the proportion of coniferous species leads to a rise in increment. Solving the problem without restriction on the composition by species, the area is distributed exclusively between the coniferous tree species as follows: Scots pine 59·5%, Silver fir 21·9%, Black pine 12·0%, Douglas fir 5·6%, spruce 0·1%. With this species composition the mean annual increment reaches 9·35 m^3 per hectare. As is well known, however, from the silvicultural point of view appropriate place should be secured for broad-leaved species. It may be mentioned also that the increase in the proportion of beech among the broad-leaved species leads to an increase in increment.

Whatever variant is chosen as optimal, it remains optimal only under constant natural and economic conditions. Since the conditions change in the course of time, for example as the composition of the forest is revised the solution determined earlier should be revised and improved accordingly.

The difference between the second and the third variant is not big and in this case the proportion of beech is the decisive factor. If it is expected that the consumption of beech wood will decrease in the future, no doubt the third variant will be adopted as a basis for planning the species composition of the region investigated.

TABLE 9

OPTIMUM SPECIES COMPOSITION WITH VARYING CONSUMPTION

Tree Species	Present composition of the forest area	Future composition of the forest					
		I variant		II variant		III variant	
		Area in %	Mean annual increment m^3	Area in %	Mean annual increment m^3	Area in %	Mean annual increment m^3
Douglas fir	0·2	3·0	3010	3·0	3143	5·0	5020
Spruce	1·8	10·0	9722	5·0	5264	10·0	9722
Silver fir	0·1	1·0	1040	1·0	1060	1·0	1040
Black pine	5·6	6·0	4033	6·0	4033	6·0	4033
Scots pine	12·3	20·0	15064	34·0	25427	35·5	26557
Beech	55·9	45·0	27450	40·0	26400	32·0	20636
Birch	—	2·0	1273	1·5	942	1·0	617
Acacia	0·6	0·5	290	0·5	290	0·5	290
Hornbeam	2·2	1·0	680	0·5	340	0·5	340
Sessile oak	2·0	3·0	1170	2·0	784	2·0	784
Lime	—	2·0	864	2·0	864	2·0	864
Adriatic oak	7·9	2·0	1073	1·0	540	1·25	675
Quercus conferta	7·7	2·0	811	1·0	430	1·0	430
European aspen	0·1	1·0	500	1·0	500	1·0	500
Eastern hornbeam	3·2	0·5	40	0·5	40	0·25	20
Others	0·4	1·0	400	1·5	400	1·0	400
Total	100	100	67420	100	70458	100	71928

Table 10 shows the distribution of the tree species according to site type in the third variant. It appears that the alteration of the tree composition compared with the second variant the most important changes from the second variant are on comparatively poor site types (Numbers 1, 2, 5, 7, 9, 10, 11, 27, 25, 29, 30, 31, 32). In these cases the plantations should be subjected to reconstruction. The composition is practically unchanged on moderately rich types (Numbers 12 and 14) typical for certain native tree

TABLE 10

OPTIMUM SPECIES COMPOSITION FOR EACH SITE TYPE UNDER THE THIRD VARIANT

No.	Site type	Type of forest	Area ha	Future tree species—area in ha
		Lower beech forest belt		
1	A_1	Dry Eastern hornbeam forest on stony soils	137·7	Black pine—137·7
2	$A_{1.2}$	Crops	16·5	Black pine—16·5
3	AB_1	Dry hornbeam forest	261·6	Black pine 236·6, hornbeam—25·0
4	$AB_{1.2}$	Crops	79·5	Black pine—79·5
5	B_1	Dry oak coppice *Quercus conferta* with *Q. sessiliflora*	651·2	Black pine—131·7, Scots pine—318·5 *Quercus sessiliflora*—201·0
6	$B_{1.2}$	Crops	59·9	Scots pine—9·9, Acacia—50·0
7	BC_1	Dry beech-hornbeam forest	370·5	Scots pine—370·5
8	BC_2	Fresh beech forest with wood rush (*Luzula maxima*)	1118·8	Scots pine—44·5, beech—1034·3, others—40·0
9	$BC_{1.2}$	Dry to fresh beech forest and beech-hornbeam forest	708·5	Scots pine—658·5, hornbeam—50·0
10	C_1	Dry oak coppice (*Q. conferta* with *Q. cerris*)	530·9	Scots pine—104·9, lime—201·0, *Q. cerris*—125·0, *Q. conferta*— *Q. conferta*—100·0
11	$C_{1.2}$	Fresh oak coppice (*Q. sessiliflora*) forest	483·6	Scots pine—483·6
12	C_2	Fresh beech-hornbeam coppice	252·1	Beech—242·1, other species—20.0
13	CD_2+ CD_3	Crops	48·8	Scots pine—48·8
14	$CD_{2.3}$	Fresh beech forest with *Mercurialis perennis*	651·0	Douglas fir—449·2, beech—181·8, other species 20·0
15	$CD_{2.3}$	Fresh to moist beech-hornbeam forest	163·2	Beech—163·2
		Middle beech forest belt		
16	A_1	Crops	28·8	Birch—28·8
17	AB_1	Crops	26·3	Birch—26·3
18	$AB_{1.2}$	Crops	83·4	Scots pine—83·4
19	$B_{1.2}$	Crops	27·1	Scots pine—27·1
20	$B_{2.3}$	Crops	10·5	Scots pine—10·5
21	BC_1+ BC_2	Crops	19·8	Scots pine—19·8
22	BC_2	Fresh beech forest with wood rush (*Luzula maxima*)	597·9	Silver fir—100·0, beech—497·9
23	C_2+ $C_{1.2}$	Fresh beech forest with mountain fescue	198·4	Spruce—198·4
24	CD_2	Fresh beech forest with *Asperula odorata*	1401·6	Spruce 200·2, beech—1093·7, aspen—877, other tree species—20·0
25	D_2	Crops	12·3	Aspen—12·3
26	$D_{2.3}$	Fresh beech forest with litter	508·2	Spruce—498·2, other tree species—10·0
		Lower beech forest belt		
27	A_1	Dry beech forest with lichen and moss	513·5	Scots pine—513·5
28	AB_1	Dry hornbeam coppice	241·7	Scots pine—241·5
29	B_1	Dry oak coppice (*Q. conferta*)	288·0	Scots pine—288·0
30	$B_{1.2}$	Dry to fresh beech forest on stony soil	344·7	Scots pine—299·8, birch—44·9
31	$B_{2.3}$	Crops	12·9	Spruce—12·9
32	C_1	Dry oak coppice (*Q. conferta* with *Q. cerris*)	35·0	Scots pine—35·0
33	$C_{2.3}$	Fresh to moist beech forest	146·1	Douglas fir—52·8, spruce—93·3

species (beech, oak). The forest composition should nevertheless be slightly changed on very rich sites on account of the potential participation of rapid growing coniferous species. (Numbers 15, 22, 23, 25 and 26). On certain site types, mainly on poorer ones (Numbers 2, 4, 16, 17, 18, 19, 20, 21, 31) crops are already established. Regardless of that, however, the optimal tree species for such sites is pointed out. The composition of the site $C_{2,3}$ in the upper beech belt is radically changed. In the place of "the fresh to moist beech forest with litter" the establishment of Douglas fir and spruce should be envisaged.

In concluding this paper the following inferences may be drawn. The model suggested makes use of linear programming to determine the optimal species composition of a given forest area. It is appropriate to solve this problem for forest or economic regions as well as for the whole territory of the country. The first prerequisite for the application of this method with electronic computers is the establishment of the composition of the forest by site types—this in general is already done during the forest survey revisions. The second prerequisite for the application of the method is the establishment of tree species productivity by site types expressed in terms of the mean annual increment. The third prerequisite is an estimate of wood consumption by species (or groups of species). That prognosis is worked out for a period not shorter than 20 years for the whole country and for the forest or economic region concerned.

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DISCUSSION OF KOSTOV'S PAPER

Glück. The fundamental problem is that the long-term determination of species composition cannot be made on the basis of short-term requirements of industry. *Kostov*. One has to decide what the requirements of industry are going to be, how else does one bring the requirements of industry and the productive potential of the forest sites into relationship. *Duerr*. Industrial requirements will change, what is optimal today may not be optimal for future market conditions. *Glück*. This does not seem to me to be a linear programming problem. *Jackson*. Why was cost minimisation rather than volume maximisation not the criterion function. *Kostov*. This would be a possibility, volume was used, the use of costs would require a different approach. *Duerr*. Possibly Dr. Kostov is making two assumptions: first, supply will create its own demand, and second, technology will be developed to cope with whatever situation is found in the forest at the time. *Wardle*. I think more is being read into Dr. Kostov's presentation than he intends. Dr. Glück says, for example, one should not confuse short- and long-term situations. All that Dr. Kostov is saying is that we must take account of the market in deciding the species composition of the forest. I think he is talking about the long-term market, certainly there is nothing in the paper to suggest that he does not have the long-term market in mind. It is accepted that the model imperfectly represents the complexities of the real situation. I understand that it is seen as providing a very general approach to the selection of species or most appropriate to the choice over a region rather than for a specific forest. *Höfle*. The large area does not get round the implication that technology will not change and supply create its own demand. We have this problem of integrating long- and short-term models. *Wardle*. I do not think the model implies acceptance of a constant market or technology. Indeed, the analysis presented shows the author has explored various possible market assumptions. *Höfle*. In this way the model becomes a sort of simulation to find species according to different possible demands. *Bittig*. This model can only apply when the market is fixed and could not apply in a western free market economy. *Kostov.* You have to make some estimate of what the market will require 30 or 40 years hence in order to decide on afforestation now and afforestation and reafforestation are major problems with us. So we are using this method to help to decide species composition in our forward plans. These are not fixed but can be revised. *Duerr.* In closing the discussion I should like to thank Dr. Kostov for his patience in dealing with us in various languages.

Paper 4

AN ALGORITHM OF THE SIMPLEX METHOD APPLIED TO THE SOLUTION OF THE GENERAL LINEAR PROGRAMMING PROBLEM IN A FORESTRY EXAMPLE

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Linear programming is used to find optimum solution to problems that can be formulated by means of a mathematical model in which all constraints stating the conditions and the objective function to be optimised are linear.

The constraints are to be formulated with linear inequalities and/or equalities. Inequalities occur more often. It will generally suffice to use two of their possible types in practice. With one of the types restrictions imposed by the availability of resources are usually stated: the sum of the amounts required by several activities *must not exceed* (may not be greater than) the individual resource supplies. With the other type of inequalities there can be considered, e.g. constraints as follows: *at least* a certain amount of some resource is required by the processes of technology, *at least* a certain quantity of any product is to be sold or within the firm internally allocated, etc.

In any linear inequality expressing a constraint of type "must not exceed" the algebraic sum of the left-hand side has to be less than or equal to ($\leqslant$) the amount of the right-hand side, for constraints of type "at least", however, the left-hand side has to be greater than or equal to ($\geqslant$) the right-hand one. If formulating the constraints requires the use of each of these types of linear inequalities and moreover the use of any linear equality, we are dealing with a general linear programming problem.

The mathematical modelling of an optimisation problem arising in any sphere of the economy can effect a case of the general linear programming problem. The well known simplex method can provide solution even to such complicated problems, therefore it has a great importance among the methods for solving linear programming models.

Here a particular algorithm of the simplex method will be presented, applied to the solution of the general linear programming problem in a forestry example. The data of the example are mostly taken from the article of Dissescu, (1966), but the algorithm for solving the problem is quite different from that applied in it.

The known and unknown numbers arranged for view in Table 11 suggest the meaning exposed below.

Concerning a highland forest the following are known:

- —the area in hectares of each of the sites Nos. 1, 2, 3, 4 of differing productive capacity,
- —the average annual increment in cubic metres and the gross or net revenue per hectare expected persistently on each site from the species spruce, fir, larch and beech, that are considered as possible ingredients of the final composition of stands,
- —volumes (in m^3) in demand for a sustained annual yield of the mentioned species, where the data are to be interpreted as follows: the volumes to be harvested per year considering the species spruce, fir, larch are *at least* 10,500, 6,000, 2,000 m^3 respectively, the beech, however, *equal* to 8,000 m^3.

The problem to be solved is: which of the species, on what kind of sites, on how large area should be produced, if we are to attain from the production on the total area the maximum annual revenue subject to the conditions stated above.

If the unknown areas (hectare-values) are denoted by $x_{11}, x_{12}, \ldots, x_{44}$—where the first subscript always refers to the site and the second to the species through the numbering given in Table 11—we have to determine the values of altogether 16 unknowns (variables). It is for this purpose that the *mathematical model* reflecting the constraints and the objective of the problem is formulated. (See Table 11.)

We state in mathematical terms that the summed area of the four species on any site cannot be greater than the whole area available:

$$x_{11}+x_{12}+x_{13}+x_{14} \leqslant 500 \ldots\ldots (1)$$

$$x_{21}+x_{22}+x_{23}+x_{24} \leqslant 1{,}100 \ldots\ldots (2)$$

$$x_{31}+x_{32}+x_{33}+x_{34} \leqslant 700 \ldots\ldots (3)$$

$$x_{41}+x_{42}+x_{43}+x_{44} \leqslant 300 \ldots\ldots (4)$$

We prescribe that the sum of the annual yield per species expected on the four sites from the spruce, fir, larch is to be *at least*, from the beech, however, to be *just equal* to the sustained yield in demand per year (thence add columnwise products obtained by multiplying the known quantities of average annual

TABLE 11

DATA OF THE PROGRAMMING PROBLEM

Site No.	Site reference items associated with the species	Known and unknown quantities in relation of the sites to the species: spruce 1	fir 2	larch 3	beech 4	Total area available per site (hectare)
1	Superficial extent (hectare) Average annual increment (m^3 per hectare) Revenue (thousand money units per hectare)	x_{11} 15 16	x_{12} 12 9	x_{13} 8 13	x_{14} 6·4 8	500
2	Superficial extent (hectare) Average annual increment (m^3 per hectare) Revenue (thousand money units per hectare)	x_{21} 12 11	x_{22} 15 13	x_{23} 6·25 12	x_{24} 8 15	1,100
3	Superficial extent (hectare) Average annual increment (m^3 per hectare) Revenue (thousand money units per hectare)	x_{31} 10 5	x_{32} 10 9	x_{33} 6·25 12	x_{34} 9·6 26	700
4	Superficial extent (hectare) Average annual increment (m^3 per hectare) Revenue (thousand money units per hectare)	x_{41} 8 1	x_{42} 8 5	x_{43} 5 7	x_{44} 6·4 8	300
	Sustained yield expected per year (m^3)	10,500	6,000	2,000	8,000	

yield by the unknown areas belonging to those, and relate the sum to the demand):

$$15x_{11}+12x_{21}+10x_{31}+8x_{41} \geqslant 10{,}500 \ldots (5)$$

$$12x_{12}+15x_{22}+10x_{32}+8x_{42} \geqslant 6{,}000 \ldots (6)$$

$$8x_{13}+6{\cdot}25x_{23}+6{\cdot}25x_{33}+5x_{43} \geqslant 2{,}000 \ldots (7)$$

$$6{\cdot}4x_{14}+8x_{24}+9{\cdot}6x_{34}+6{\cdot}4x_{44} = 8{,}000 \ldots (8)$$

Any feasible solution to the system of linear relationships (1) to (8) cannot have negative value for any of the 16 variables (since none of the species is able to occupy an area less than zero), i.e. each feasible solution has to satisfy the non-negativity restrictions:

$$x_{11}, x_{12}, \ldots, x_{44} \geqslant 0.$$

Out of the feasible solutions to the system of relationships (1) to (8), if any, that one is to be found whose hectare-values multiplied by the associated amounts of the annual revenue per hectare (i.e. the total revenue per year) gives the greatest money-value. Since the hectare-values to be determined are represented by the unknowns $x_{11}, x_{12}, \ldots, x_{44}$, the maximisation of the sum z of the total revenue per year can be laid down in form of the *objective function:*

$$16x_{11}+9x_{12}+13x_{13}+8x_{14}+11x_{21}+13x_{22}+$$
$$12x_{23}+15x_{24}+5x_{31}+9x_{32}+12x_{33}+26x_{34}+$$
$$x_{41}+5x_{42}+7x_{43}+8x_{44} = z \rightarrow \max.$$

The mathematical model consisting of the formulated constraints and objective function will be solved by the *simplex method.* Application of the simplex method can be carried out by means of various kinds of algorithms, which have some *common* parts as follows:

(1) Rearrange the terms involved in each constraint of the model so, that the constant term stands alone on the right-hand side;

(2) Convert the system of constraints by introducing additional variables, into a system of equalities (which takes place in course of a routine programming only in the mathematical background), and recognise the restriction, that only such solutions of the system of equalities, which contain no variable with negative value can be obtained;

(3) Find the coefficients of each variable in all equations and in the objective function and put them in association with the variable in the order of the constraints in a column (columnvector), consider the right-hand side values, too, as elements (components) of a columnvector;

(4) Find the co-ordinates (scalar factors) of the established vectors in relation to an initial basis, which usually consists of an appropriate set of unit vectors;

(5) Transform the initial basis by iterations with the aim of generating a new basis, which can allocate to the primal variables values that satisfy the constraints and optimise the objective function.

Hereinafter a particular algorithm of the simplex method is presented, applied to the model formulated above, with due emphasis on its *special* traits.

The inequalities representing in the model the type "$\geqslant$" are to be multiplied by (-1) *and so turned into those of type "$\leqslant$".*

The system of constraints, consiting from now on only of two types "$\leqslant$" and "$=$" of relation, is to be converted into a system of equations. The conversion takes place by adding one of the additional variables $u_1, u_2, \ldots, u_8$ in order of their subscripts to the left-hand side of the partly rephased constraints (1) through (8) respectively. We require the additional variable in each constraint to represent always the quantity that makes the sum of the left-hand side equal to the amount of the right-hand side. For this reason the additional variable u_8 belonging to the equality constraint (8) must receive the value zero. *The equation of the objective function is to be multiplied by* (-1), for we are going to have the variable z with positive sign on the left hand-side and hence values of correct sign for it in the simplex tableaux.

The converted model obtained as described is shown in the upper part of Table 12. The terms on the left-hand side of every equation are spaced out for making it easier to compose column vectors of the coefficients of every single variable, i.e. considering the primal variables $x_{11}, x_{12}, \ldots, x_{44}$ and dual variables $u_1, u_2, \ldots, u_8$ and variable z, respectively. The coefficient-columns associated with the *primal* variables (shown in the lower part of Table 12) have been designated according to the subscripts of those by the symbols $\mathbf{a}_{11}, \mathbf{a}_{12}, \ldots, \mathbf{a}_{44}$. (Every symbol with double-underscored or bold faced type letter, for example $\mathbf{a}_{11}$, refers in this paper to a particular *set* column—of *explicit* numbers, but every symbol with single-underscored or italicised one, for example x_{11}, u_4 or z, refers to a *scalar variable* which may represent any *one* of the real numbers). The coefficient-vectors associated with the *dual* variables and with z have been denoted by the symbols $\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_9$, which represent, taken one by one a unitvector and collectively a system of them. This system of unitvectors is a purposive consequence of the manner (multiplications by minus 1 and introduction of additional variables afterwards) in which we have derived the converted model from the initial one.

The column-vector composed of the right-hand side elements of the converted model and designated by $\mathbf{b}$ appears in the lower part of Table 12 as a sum of products obtained by multiplying every column of coefficients by the variable associated with it, viz. $\mathbf{b}$ is written as linear combination of the coefficient-vectors by the variables associated with them.

Considering this linear combination form of the model, we have for the 25 variables involved to find a solution set consisting of 25 non-negative values that contains zero for u_8 and the greatest value possible for z and satisfies the linear combination equation. That is to say if we substitute the solution values in the linear combination and perform the indicated multiplications and addition, the left-hand side becomes a column-vector equal to the right-hand side column-vector $\mathbf{b}$. Within the set of the values for all the variables the subset of values for the 16 primal variables $(x_{11}, x_{12}, \ldots, x_{44})$ will be that one, which exists as optimal solution of the model formulated at the outset and thus of the programming problem itself.

In order to arrive at the optimal solution we firstly compose an initial basis consisting of as many coefficient-vectors as there are rows in the model and find the co-ordinates of $\mathbf{b}$ and those of the other (non-basic) vectors in relation to the basis. Then we transform the initial basis by means of iterative interchanges between a vector inside and one outside the basis until a new basis has been developed, in relation to which all the co-ordinates of $\mathbf{b}$ are non-negative and in addition to this $u_8 = 0$ and the value of z is the greatest one among those obtainable with any of the feasible solutions for it.

It is convenient to compose the initial basis of the unitvectors $\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_8, \mathbf{e}_9$ (which are coefficient-vectors of the variables $u_1, u_2, \ldots, u_8$, z respectively), since *co-ordinates* of the primal coefficient-vectors and those of $\mathbf{b}$ in relation to the basis of unitvectors are equivalent to their *components* shown in columns $\mathbf{a}_{11}, \mathbf{a}_{12}, \ldots, \mathbf{a}_{44}$ and $\mathbf{b}$ in the lower part of Table 12.

According to this the initial simplex tableau (Tableau I of Table 13) lists in a column the first basis involving the unitvectors $\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_8, \mathbf{e}_9$ and in subsequent columns the components of every primal coefficient-vector and those of $\mathbf{b}$, transcribed all of them without any modification from Table 12. Transcribing the coefficient-columns of the additional variables $u_1, \ldots, u_8$ could be omitted, since those are still considered as unitvectors constituting the basis in Tableau I. Therefore this tableau involves every coefficient-vector, which may be constituent part of the basis that will generate the optimal solution.

Hence *if we convert in the described manner the model of a general linear programming problem, in the simplex tableaux we shall never have to use for the coefficients of the variables more columns than the number of the primal ones.* In case we should have converted the inequalities of type "$\geqslant$" into equalities by *subtracting* a non-negative additional variable from their respective original left-hand side, the extent of the simplex tableaux would have increased in number of columns for coefficients of the additional (surplus) variables.

The general linear programming problem can also be solved without either multiplying the

TABLE 12

CONVERTING THE MODEL INTO A SYSTEM OF EQUATIONS (ABOVE), AND INTO A LINEAR COMBINATION (BELOW)

$$
\begin{array}{ccccccccccc}
x_{11}+ & x_{12}+ & x_{13}+ & x_{14} & & & & & & & \\
 & & & & x_{21}+ & x_{22}+ & x_{23}+ & x_{24} & & & \\
 & & & & & & & & x_{31}+ & x_{32}+ & x_{33}+ \\
-15x_{11} & & & & -12x_{21} & & & & -10x_{31} & & \\
 & -12x_{12} & & & & -15x_{22} & & & & -10x_{32} & \\
 & & -8x_{13} & & & & -6{\cdot}25x_{23} & & & & -6{\cdot}25x_{33} \\
 & & & 6{\cdot}4x_{14} & & & & +8x_{24} & & & \\
\hline
-16x_{11} & -9x_{12} & -13x_{13} & -8x_{14} & -11x_{21} & -13x_{22} & -12x_{23} & -15x_{24} & -5x_{31} & -9x_{32} & -12x_{33}
\end{array}
$$

$$
\overset{\mathbf{a}_{11}}{\left[\begin{array}{c}1\\0\\0\\0\\-15\\0\\0\\0\\\hline -16\end{array}\right]}x_{11}+
\overset{\mathbf{a}_{12}}{\left[\begin{array}{c}1\\0\\0\\0\\0\\-12\\0\\0\\\hline -9\end{array}\right]}x_{12}+
\overset{\mathbf{a}_{13}}{\left[\begin{array}{c}1\\0\\0\\0\\0\\0\\-8\\0\\\hline -13\end{array}\right]}x_{13}+
\overset{\mathbf{a}_{14}}{\left[\begin{array}{c}1\\0\\0\\0\\0\\0\\0\\6{\cdot}4\\\hline -8\end{array}\right]}x_{14}+
\overset{\mathbf{a}_{21}}{\left[\begin{array}{c}0\\1\\0\\0\\-12\\0\\0\\0\\\hline -11\end{array}\right]}x_{21}+
\overset{\mathbf{a}_{22}}{\left[\begin{array}{c}0\\1\\0\\0\\0\\-15\\0\\0\\\hline -13\end{array}\right]}x_{22}+
\overset{\mathbf{a}_{23}}{\left[\begin{array}{c}0\\1\\0\\0\\0\\0\\-6{\cdot}25\\0\\\hline -12\end{array}\right]}x_{23}+
\overset{\mathbf{a}_{24}}{\left[\begin{array}{c}0\\1\\0\\0\\0\\0\\0\\8\\\hline -15\end{array}\right]}x_{24}+
\overset{\mathbf{a}_{31}}{\left[\begin{array}{c}0\\0\\1\\0\\-10\\0\\0\\0\\\hline -5\end{array}\right]}x_{31}+
\overset{\mathbf{a}_{32}}{\left[\begin{array}{c}0\\0\\1\\0\\0\\-10\\0\\0\\\hline -9\end{array}\right]}x_{32}+
\overset{\mathbf{a}_{33}}{\left[\begin{array}{c}0\\0\\1\\0\\0\\0\\-6{\cdot}25\\0\\\hline -12\end{array}\right]}x_{33}+
$$

Table 12—continued

Converting the model into a system of equations (above), and into a linear combination (below)

$$
\begin{array}{lllllllllllllll}
 & & & & & +u_1 & & & & & & & & = & 500 \\
 & & & & & & +u_2 & & & & & & & = & 1{,}100 \\
x_{34} & & & & & & & +u_3 & & & & & & = & 700 \\
 & x_{41}+ & x_{42}+ & x_{43}+ & x_{44} & & & & +u_4 & & & & & = & 300 \\
 & -8x_{41} & & & & & & & & +u_5 & & & & = & -10{,}500 \\
 & & -8x_{42} & & & & & & & & +u_6 & & & = & -6{,}000 \\
 & & & -5x_{43} & & & & & & & & +u_7 & & = & -2{,}000 \\
+9{\cdot}6x_{34} & & & & +6{\cdot}4x_{44} & & & & & & & & +u_8 & = & 8{,}000 \\
\hline
-26x_{34} & -x_{41} & -5x_{42} & -7x_{43} & -8x_{44} & & & & & & & & +z & = & 0
\end{array}
$$

$$
\overset{\mathbf{a}_{34}}{\begin{bmatrix} 0\\0\\1\\0\\0\\0\\0\\9{\cdot}6\\ \hline -26 \end{bmatrix}} x_{34}+
\overset{\mathbf{a}_{41}}{\begin{bmatrix} 0\\0\\0\\1\\-8\\0\\0\\0\\ \hline -1 \end{bmatrix}} x_{41}+
\overset{\mathbf{a}_{42}}{\begin{bmatrix} 0\\0\\0\\1\\0\\-8\\0\\0\\ \hline -5 \end{bmatrix}} x_{42}+
\overset{\mathbf{a}_{43}}{\begin{bmatrix} 0\\0\\0\\1\\0\\0\\-5\\0\\ \hline -7 \end{bmatrix}} x_{43}+
\overset{\mathbf{a}_{44}}{\begin{bmatrix} 0\\0\\0\\1\\0\\0\\0\\6{\cdot}4\\ \hline -8 \end{bmatrix}} x_{44}+
\overset{\mathbf{e}_{1}}{\begin{bmatrix} 1\\0\\0\\0\\0\\0\\0\\0\\ \hline 0 \end{bmatrix}} u_1+
\overset{\mathbf{e}_{2}}{\begin{bmatrix} 0\\1\\0\\0\\0\\0\\0\\0\\ \hline 0 \end{bmatrix}} u_2+
\overset{\mathbf{e}_{3}}{\begin{bmatrix} 0\\0\\1\\0\\0\\0\\0\\0\\ \hline 0 \end{bmatrix}} u_3+
\overset{\mathbf{e}_{4}}{\begin{bmatrix} 0\\0\\0\\1\\0\\0\\0\\0\\ \hline 0 \end{bmatrix}} u_4+
\overset{\mathbf{e}_{5}}{\begin{bmatrix} 0\\0\\0\\0\\1\\0\\0\\0\\ \hline 0 \end{bmatrix}} u_5+
\overset{\mathbf{e}_{6}}{\begin{bmatrix} 0\\0\\0\\0\\0\\1\\0\\0\\ \hline 0 \end{bmatrix}} u_6+
\overset{\mathbf{e}_{7}}{\begin{bmatrix} 0\\0\\0\\0\\0\\0\\1\\0\\ \hline 0 \end{bmatrix}} u_7+
\overset{\mathbf{e}_{8}}{\begin{bmatrix} 0\\0\\0\\0\\0\\0\\0\\1\\ \hline 0 \end{bmatrix}} u_8+
\overset{\mathbf{e}_{9}}{\begin{bmatrix} 0\\0\\0\\0\\0\\0\\0\\0\\ \hline 1 \end{bmatrix}} z =
\overset{\mathbf{b}}{\begin{bmatrix} 500\\1{,}100\\700\\300\\-10{,}500\\-6{,}000\\-2{,}000\\8{,}000\\ \hline 0 \end{bmatrix}}
$$

TABLE 13: PART A

APPLYING THE SIMPLEX ALGORITHM TO THE SOLUTION OF THE LINEAR PROGRAMMING PROBLEM: TABLEAUX I, II, III, IV

Tab-leau	Basis	In each tableau: co-ordinates of each heading vector in relation to the current basis																
		a_{11}	a_{12}	a_{13}	a_{14}	a_{21}	a_{22}	a_{23}	a_{24}	a_{31}	a_{32}	a_{33}	a_{34}	a_{41}	a_{42}	a_{43}	a_{44}	b
I	e_1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	$500 = u_1$
	e_2	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	$1,100 = u_2$
	e_3	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	$700 = u_3$
	e_4	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	$300 = u_4$
	e_5	−15	0	0	0	−12	0	0	0	−10	0	0	0	−8	0	0	0	$-10,500 = u_5$
	e_6	0	−12	0	0	0	−15	0	0	0	−10	0	0	0	−8	0	0	$-6,000 = u_6$
	e_7	0	0	−8	0	0	0	−6·25	0	0	0	−6·25	0	0	0	−5	0	$-2,000 = u_7$
	e_8	0	0	0	6·4	0	0	0	$\boxed{8}$	0	0	0	9·6	0	0	0	6·4	$8,000 = u_8$
	e_9	−16	−9	−13	−8	−11	−13	−12	−15	−5	−9	−12	−26	−1	−5	−7	−8	$0 = z$
		a_{11}	a_{12}	a_{13}	a_{14}	a_{21}	a_{22}	a_{23}	e_8	a_{31}	a_{32}	a_{33}	a_{34}	a_{41}	a_{42}	a_{43}	a_{44}	b
II	e_1	$\boxed{1}$	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	$500 = u_1$
	e_2	0	0	0	−0·8	1	1	1	−0·125	0	0	0	−1·2	0	0	0	−0·8	$100 = u_2$
	e_3	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	$700 = u_3$
	e_4	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	$300 = u_4$
	e_5	−15	0	0	0	−12	0	0	0	−10	0	0	0	−8	0	0	0	$-10,500 = u_5$
	e_6	0	−12	0	0	0	−15	0	0	0	−10	0	0	0	−8	0	0	$-6,000 = u_6$
	e_7	0	0	−8	0	0	0	−6·25	0	0	0	−6·25	0	0	0	−5	0	$-2,000 = u_7$
	a_{24}	0	0	0	0·8	0	0	0	0·125	0	0	0	1·2	0	0	0	0·8	$1,000 = x_{24}$
	e_9	−16	−9	−13	4	−11	−13	−12	1·875	−5	−9	−12	−8	−1	−5	−7	4	$15,000 = z$

Tableau	Basis	In each tableau: co-ordinates of each heading vector in relation to the current basis															
		$\mathbf{e}_1$	$\mathbf{a}_{12}$	$\mathbf{a}_{13}$	$\mathbf{a}_{14}$	$\mathbf{a}_{21}$	$\mathbf{a}_{22}$	$\mathbf{a}_{23}$	$\mathbf{a}_{31}$	$\mathbf{a}_{32}$	$\mathbf{a}_{33}$	$\mathbf{a}_{34}$	$\mathbf{a}_{41}$	$\mathbf{a}_{42}$	$\mathbf{a}_{43}$	$\mathbf{a}_{44}$	b
III	$\mathbf{a}_{11}$	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	$500 = x_{11}$
	$\mathbf{e}_2$	0	0	0	−0·8	1	$\boxed{1}$	1	0	0	0	−1·2	0	0	0	−0·8	$100 = u_2$
	$\mathbf{e}_3$	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	$700 = u_3$
	$\mathbf{e}_4$	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	$300 = u_4$
	$\mathbf{e}_5$	15	15	15	15	−12	0	0	−10	0	0	0	−8	0	0	0	$-3{,}000 = u_5$
	$\mathbf{e}_6$	0	−12	0	0	0	−15	0	0	−10	0	0	0	−8	0	0	$-6{,}000 = u_6$
	$\mathbf{e}_7$	0	0	−8	0	0	0	−6·25	0	0	−6·25	0	0	0	−5	0	$-2{,}000 = u_7$
	$\mathbf{a}_{24}$	0	0	0	0·8	0	0	0	0	0	0	1·2	0	0	0	0·8	$1{,}000 = x_{24}$
	$\mathbf{e}_9$	16	7	3	20	−11	−13	−12	−5	−9	−12	−8	−1	−5	−7	4	$23{,}000 = z$
		$\mathbf{e}_1$	$\mathbf{a}_{12}$	$\mathbf{a}_{13}$	$\mathbf{a}_{14}$	$\mathbf{a}_{21}$	$\mathbf{e}_{21}$	$\mathbf{a}_{23}$	$\mathbf{a}_{31}$	$\mathbf{a}_{32}$	$\mathbf{a}_{33}$	$\mathbf{a}_{34}$	$\mathbf{a}_{41}$	$\mathbf{a}_{42}$	$\mathbf{a}_{43}$	$\mathbf{a}_{44}$	b
IV	$\mathbf{a}_{11}$	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	$500 = x_{11}$
	$\mathbf{a}_{22}$	0	0	0	−0·8	1	1	1	0	0	0	−1·2	0	0	0	−0·8	$100 = x_{22}$
	$\mathbf{e}_3$	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	$700 = u_3$
	$\mathbf{e}_4$	0	0	0	0	0	0	0	0	0	0	0	1	1	$\boxed{1}$	1	$300 = u_4$
	$\mathbf{e}_5$	15	15	15	15	−12	0	0	−10	0	0	0	−8	0	0	0	$-3{,}000 = u_5$
	$\mathbf{e}_6$	0	−12	0	−12	15	15	15	0	−10	0	−18	0	−8	0	−12	$-4{,}500 = u_6$
	$\mathbf{e}_7$	0	0	−8	0	0	0	−6·25	0	0	−6·25	0	0	0	−5	0	$-2{,}000 = u_7$
	$\mathbf{a}_{34}$	0	0	0	0·8	0	0	0	0	0	0	1·2	0	0	0	0·8	$1{,}000 = x_{34}$
	$\mathbf{e}_9$	16	7	3	9·6	2	13	1	−5	−9	−12	−23·6	−1	−5	−7	−6·4	$24{,}300 = z$

TABLE 13: PART B

APPLYING THE SIMPLEX ALGORITHM TO THE SOLUTION OF THE LINEAR PROGRAMMING PROBLEM: TABLEAUX V, VI, VII, VIII

Tableau	Basis	In each tableau: co-ordinates of each heading vector in relation to the current basis															
		$\mathbf{e}_1$	$\mathbf{a}_{12}$	$\mathbf{a}_{13}$	$\mathbf{a}_{14}$	$\mathbf{a}_{21}$	$\mathbf{e}_2$	$\mathbf{a}_{23}$	$\mathbf{a}_{31}$	$\mathbf{a}_{32}$	$\mathbf{a}_{33}$	$\mathbf{a}_{34}$	$\mathbf{a}_{41}$	$\mathbf{a}_{42}$	$\mathbf{e}_4$	$\mathbf{a}_{44}$	$\mathbf{b}$
V	$\mathbf{a}_{11}$	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	$500 = x_{11}$
	$\mathbf{a}_{22}$	0	0	0	−0·8	1	1	1	0	0	0	−1·2	0	0	0	−0·8	$100 = x_{22}$
	$\mathbf{e}_3$	0	0	0	0	0	0	0	1	1	1	$\boxed{1}$	0	0	0	0	$700 = u_3$
	$\mathbf{a}_{43}$	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	$300 = x_{43}$
	$\mathbf{e}_5$	15	15	15	15	−12	0	0	−10	0	0	0	−8	0	0	0	$-3{,}000 = u_5$
	$\mathbf{e}_6$	0	−12	0	−12	15	15	15	0	−10	0	−18	0	−8	0	−12	$-4{,}500 = u_6$
	$\mathbf{e}_7$	0	0	−8	0	0	0	−6·25	0	0	−6·25	0	5	5	5	5	$-500 = u_7$
	$\mathbf{a}_{24}$	0	0	0	0·8	0	0	0	0	0	0	1·2	0	0	0	0·8	$1{,}000 = x_{24}$
	$\mathbf{e}_9$	16	7	3	9·6	2	13	1	−5	−9	−12	−23·6	6	2	7	0·6	$26{,}400 = z$
		$\mathbf{e}_1$	$\mathbf{a}_{12}$	$\mathbf{a}_{13}$	$\mathbf{a}_{14}$	$\mathbf{a}_{21}$	$\mathbf{e}_2$	$\mathbf{a}_{23}$	$\mathbf{a}_{31}$	$\mathbf{a}_{32}$	$\mathbf{a}_{33}$	$\mathbf{e}_3$	$\mathbf{a}_{41}$	$\mathbf{a}_{42}$	$\mathbf{e}_4$	$\mathbf{a}_{44}$	$\mathbf{b}$
VI	$\mathbf{a}_{11}$	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	$500 = x_{11}$
	$\mathbf{a}_{22}$	0	0	0	−0·8	1	1	1	1·2	1·2	1·2	1·2	0	0	0	−0·8	$940 = x_{22}$
	$\mathbf{a}_{34}$	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	$700 = x_{34}$
	$\mathbf{a}_{43}$	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	$300 = x_{43}$
	$\mathbf{e}_5$	15	15	15	15	$\boxed{-12}$	0	0	−10	0	0	0	−8	0	0	0	$-3{,}000 = u_5$
	$\mathbf{e}_6$	0	−12	0	−12	15	15	15	18	8	18	18	0	−8	0	−12	$8{,}100 = u_6$
	$\mathbf{e}_7$	0	0	−8	0	0	0	−6·25	0	0	−6·25	0	5	5	5	5	$-500 = u_7$
	$\mathbf{a}_{24}$	0	0	0	0·8	0	0	0	−1·2	−1·2	−1·2	−1·2	0	0	0	0·8	$160 = x_{24}$
	$\mathbf{e}_9$	16	7	3	9·6	2	13	1	18·6	14·6	11·6	23·6	6	2	7	0·6	$42{,}920 = z$

Tab-leau	Basis	In each tableau: co-ordinates of each heading vector in relation to the current basis															
		e_1	a_{12}	a_{13}	a_{14}	e_5	e_2	a_{23}	a_{31}	a_{32}	a_{33}	e_3	a_{41}	a_{42}	e_4	a_{44}	b
VII	a_{11}	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	$500 = x_{11}$
	a_{22}	$\frac{5}{4}$	$\frac{5}{4}$	$\frac{5}{4}$	$\frac{9}{20}$	$\frac{1}{12}$	1	1	$\frac{11}{30}$	1·2	1·2	1·2	$-\frac{2}{3}$	0	0	−0·8	$690 = x_{22}$
	a_{34}	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	$700 = x_{34}$
	a_{43}	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	$300 = x_{43}$
	a_{21}	$-\frac{5}{4}$	$-\frac{5}{4}$	$-\frac{5}{4}$	$-\frac{5}{4}$	$-\frac{1}{12}$	0	0	$\frac{5}{8}$	0	0	0	$\frac{2}{3}$	0	0	0	$250 = x_{21}$
	e_6	$\frac{75}{4}$	$\frac{27}{4}$	$\frac{75}{4}$	$\frac{27}{4}$	1·25	15	15	5·5	8	18	18	−10	−8	0	−12	$4,350 = u_6$
	e_7	0	0	−8	0	0	0	$\boxed{-6{\cdot}25}$	0	0	−6·25	0	5	5	5	5	$-500 = u_7$
	a_{24}	0	0	0	0·8	0	0	0	−1·2	−1·2	−1·2	−1·2	0	0	0	0·8	$160 = x_{24}$
	e_9	18·5	9·5	5·5	12·1	$\frac{1}{8}$	13	1	$\frac{50{\cdot}8}{3}$	14·6	11·6	23·6	$\frac{10{\cdot}4}{3}$	2	7	0·6	$42,420 = z$
		e_1	a_{12}	a_{13}	a_{14}	e_5	e_2	e_7	a_{31}	a_{32}	a_{33}	e_3	a_{41}	a_{42}	e_4	a_{44}	b
VIII	a_{11}	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	$500 = x_{11}$
	a_{22}	$\frac{5}{4}$	$\frac{5}{4}$	−0·03	$\frac{9}{20}$	$\frac{1}{12}$	1	0·16	$\frac{11}{30}$	1·2	0·2	1·2	$\frac{2}{15}$	0·8	0·8	0	$610 = x_{22}$
	a_{34}	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	$700 = x_{34}$
	a_{43}	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	$300 = x_{43}$
	a_{21}	$-\frac{5}{4}$	$-\frac{5}{4}$	$-\frac{5}{4}$	$-\frac{5}{4}$	$-\frac{1}{12}$	0	0	$\frac{5}{8}$	0	0	0	$\frac{2}{3}$	0	0	0	$250 = x_{21}$
	e_6	$\frac{75}{4}$	$\frac{27}{4}$	−0·45	$\frac{27}{4}$	1·25	15	2·4	5·5	8	3	18	2	4	12	0	$3,150 = u_6$
	a_{23}	0	0	1·28	0	0	0	−0·16	0	0	1	0	−0·8	−0·8	−0·8	−0·8	$80 = x_{23}$
	a_{24}	0	0	0	0·8	0	0	0	−1·2	−1·2	−1·2	−1·2	0	0	0	0·8	$160 = x_{24}$
	e_9	18·5	9·5	4·22	12·1	$\frac{1}{8}$	13	0·16	$\frac{50{\cdot}8}{3}$	14·6	10·6	23·6	$\frac{16{\cdot}4}{3}$	2·8	7·8	1·4	$42,340 = z$

inequalities "$\geqslant$" by (-1) or increasing the extent of the simplex tableaux by inserting coefficient-columns owing to additional variables (Kreko 1968, 63–65), but if we proceed in this way, the known rules for transforming the basis are to apply with some complementary operation to any case in which we choose an element for pivotal function in a row representing inequality of type "$\geqslant$".

No complementary operation will be necessary if we turn the inequalities appearing originally with the sign "$\geqslant$" into those with sign "$\leqslant$" as a consequence of multiplying them by (-1). Of course, multiplying those inequalities by (-1) implies making the constant term on their right-hand side negative and makes these terms as negative elements $(-10{,}500,\ -6{,}000,\ -2{,}000)$ of **b** enter the Tableau I. Such initial negativities, however, raise no obstacles in the way to finding optimal solution, if any, to a general linear programming problem, in fact, they can't help suggesting the need to replace them by non-negative values.

The sequential construction of the tableaux (for example, deriving Tableau II from Tableau I) in Table 13 takes place by using the known rules of basis transformation, which are put into words through the following four points.

(1) Select in the extablished tableau a co-ordinate (an element) for *pivot* generating the co-ordinates for the next tableau (e.g., see element 8 framed in Tableau I). Interchange in the next tableau the positions of the two vectors, that are designated in the pivotal row and in the pivotal column respectively (e.g., considering framed pivot 8 in Tableau I, replace $\mathbf{e}_8$ standing as a basis vector in the pivotal row by $\mathbf{a}_{24}$ heading the pivotal column, and conversely, in Tableau II). *Replace the pivot by its reciprocal* in the next tableau (e.g., replace pivot 8 by its reciprocal $\frac{1}{8} = 0{\cdot}125$ in Tableau II).

(2) Multiply those remaining co-ordinates along the *pivotal row* by the *actual reciprocal* in order to have the new ones replacing them in the next tableau (e.g., along the pivotal row multiplying the co-ordinate 8,000 of **b** by 0·125 yields $8000{\cdot}0{\cdot}125 = 1{,}000$ for Tableau II).

(3) Multiply those remaining co-ordinates along the *pivotal column* by the *reciprocal of opposite sign* to obtain the new ones entering their positions. E.g., along the pivotal column multiplying the co-ordinate 1 of $\mathbf{a}_{24}$ by $(-0{\cdot}125)$ yields $1(-0{\cdot}125) = -0{\cdot}125$ for Tableau II.

(4) Replace every co-ordinate standing outside the pivotal row as well as the pivotal column by a *difference* obtained by subtracting from the current co-ordinate the product of two numbers: one of them is to be found in the pivotal column when moving from the current co-ordinates as starting-point horizontally, the other one, however, vertically, in the new row computed for the *next* tableau according to point 2. For example, the co-ordinate 1,100 of **b** in Tableau I is replaced by 100 in Tableau II, since we had to subtract from 1,100 the product of 1 (found along row of 1,100 in the pivotal column) and 1,000 (found by moving vertically from 1,100 through Tableau I till $\mathbf{a}_{24}$ –row of Tableau II computed according to point 2), that is $1{,}100 - 1{\cdot}1{,}000 = 100$. The initial value zero of the objective function z had been similarly replaced by $0-(-15)\ 1{,}000 = 15{,}000$ in Tableau II.

Knowledge of the described rules enables us to consider how far the selection of a certain element for pivot will in the next tableau affect the objective function to be optimised and the other requirements to be met.

The necessary elimination of negativities within the co-ordinates of **b** has come successively to an end in Tableaux II through VIII. We could have achieved it in fewer iterations as well, but that necessity was not the only requirement controlling the sequential selection of elements for pivotal role. By choosing pivot in $\mathbf{e}_8$-row of Tableau I we had two aims in view: (1) to remove $\mathbf{e}_8$ once for all from the basis, thus drive the value of u_8 to zero in order to enable constraint (8) to be fulfilled in its form of equality, (2) to increase the value of the objective function in Tableau II. Guiding principles of finding pivot in each of Tableaux II through V have been: (1) to set back negatives in **b**-column, (2) to increase the value of the objective function. In order to have completely eliminated the negativity within the co-ordinates of **b**, we had no other choice but to accept such elements for the last two pivots which generated decrease in value of the objective function in Tableaux VII and VIII respectively. *Choosing a pivot, that generates incidental decrease in value of an objective function to be maximised is unusual in programming practice, but mathematically correct without requiring definitely more iterations as at least necessary* (as shown by Tableaux I through VIII).

Since each co-ordinate of **b** in Tableau VIII is non-negative and the value of u_8 from Tableau II on equals zero, Tableau VIII involves one of the feasible solutions of our extended model. In addition to this, each of the co-ordinates (18·5, 9·5,..., 1·4) in row of the objective function is non-negative, therefore the first feasible solution involved in Tableau VIII is simultaneously the *optimal solution* as well.

The optimal programme which consists of values of the primal variables in Tableau VIII, is arranged in Table 14 in the same way as these variables in Table 11 are. In addition, the optimum hectare-values, considered within species according to sites, have been multiplied by the appropriate data of the average annual increment taken from Table 11, then the computed products have been shown columnwise

Table 14

The Optimum Solution

Site	Superficial extent (in ha) of the species				Total area available per site
	spruce	fir	larch	beech	
Number	1	2	3	4	hectare
1	x_{11} = 500	x_{12} = 0	x_{13} = 0	x_{14} = 0	500
2	x_{21} = 250	x_{22} = 610	x_{23} = 80	x_{24} = 160	1,100
3	x_{31} = 0	x_{32} = 0	x_{33} = 0	x_{34} = 700	700
4	x_{41} = 0	x_{42} = 0	x_{43} = 300	x_{44} = 0	300
	Average annual increment (m^3 per ha)				
1	15	12	8	6·4	
2	12	15	6·25	8	
3	10	10	6·25	9·6	
4	8	8	5	6·4	
	Sustained yield programmed per year (m^3)				
1	7,500	0	0	0	
2	3,000	9,150	500	1,280	
3	0	0	0	6,720	
4	0	0	1,500	0	
Total	10,500	9,150	2,000	8,000	

each and summed as volumes of sustained yield programmed per year specified by species.

If we substitute the variables of the originally formulated model for the corresponding hectare-values of the optimal programme, every constraint will be satisfied and the objective function will have the greatest value equal to 42,340 obtained for z in Tableau VIII of Table 13. Besides this manner of checking the calculations there is a similar method by which *the accuracy of co-ordinates obtained can be checked not only at the end but also in any phase of computing the simplex tableaux*. It is mathematically necessary that any vector designated in heading of any simplex tableau can be reproduced as the sum of products obtained by multiplying its current co-ordinates by the along-row respective basis vectors (i.e. to be able to reproduce any heading vector as resultant of the linear combination pertaining to it). E.g., before selecting pivot in $\mathbf{a}_{34}$-column of Tableau V we have checked the numerical accuracy of co-ordinates of $\mathbf{a}_{34}$ related to the corresponding basis ($\mathbf{a}_{11}$, $\mathbf{a}_{22}$, $\mathbf{e}_3$, $\mathbf{a}_{43}$, $\mathbf{e}_5$, $\mathbf{e}_6$, $\mathbf{e}_7$, $\mathbf{a}_{24}$, $\mathbf{e}_9$), as follows:

$$-1{\cdot}2\overset{\mathbf{a}_{22}}{\begin{bmatrix}0\\1\\0\\0\\0\\-15\\0\\0\\\hline -13\end{bmatrix}}+1\overset{\mathbf{e}_3}{\begin{bmatrix}0\\0\\1\\0\\0\\0\\0\\0\\\hline 0\end{bmatrix}}-18\overset{\mathbf{e}_6}{\begin{bmatrix}0\\0\\0\\0\\0\\1\\0\\0\\\hline 0\end{bmatrix}}+1{\cdot}2\overset{\mathbf{a}_{24}}{\begin{bmatrix}0\\1\\0\\0\\0\\0\\0\\8\\\hline -15\end{bmatrix}}-23{\cdot}6\overset{\mathbf{e}_9}{\begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\0\\\hline 1\end{bmatrix}}=\begin{bmatrix}0\\-1{\cdot}2+1{\cdot}2\\1\\0\\0\\18-18\\0\\9{\cdot}6\\\hline 15{\cdot}6-18-23{\cdot}6\end{bmatrix}=\begin{bmatrix}0\\0\\1\\0\\0\\0\\0\\9{\cdot}6\\\hline -26\end{bmatrix}=\mathbf{a}_{34},$$

where the terms in which vectors were to be multiplied by zero could be and hence have been omitted. Since the resultant-vector of the above linear combination is exactly the same as $\mathbf{a}_{34}$ with the meaning defined in Table 12, all co-ordinates of $\mathbf{a}_{34}$ related to the basis in Tableau V of Table 3 are exactly calculated. If the resultant-vector had differed in any component from $\mathbf{a}_{34}$, at least one co-ordinate of $\mathbf{a}_{34}$ in Tableau V would have been miscalculated.

It is obvious that deviations induced by mistake, if any, are to be rectified. Deviation from the correct value of any co-ordinate can occur, however, because of rounding-off as well, when computations are to be performed with decimal numbers (as is the case with an electronic computer). Errors of this kind are reducible by extending the number of calculated decimal places, but cannot be driven absolutely to zero.

We are able to compute the simplex tableaux free from rounding-off errors if and only if every co-ordinate, that figured out with decimal number would require rounding-off, is calculated as a fraction. Making use of this possibility, no co-ordinate shown in Table 13 has been rounded-off in order to enable one in checking to reproduce exactly (similarly to $\mathbf{a}_{34}$) any heading vector as the result of a linear combination constructed from the appropriate co-ordinates and basis vectors.

The available extent of this paper allowed one to give only a comprehensive description of the presented algorithm. Little room has been left for details of computational technique and no room for discussion of the model and optimal solution from the view-point of forestry.

The attempt has been made to represent the algorithm in an interpretation, that is close to forestry, mathematically justified but not demanding more abstraction than necessary and hence it is hoped easily understood. The whole sphere of linear programming allows representation of this kind.

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DISCUSSION OF THE PAPER BY DR. FARKAS

Wardle. This is a presentation of the simplex algorithm for the solution for linear programming problem; what I found of great merit was the extreme clarity of this presentation, for that reason I am glad we are able to publish it. The author has also written at greater length on the linear programming problem and he has translated this work into German while on a visit to Professor Frauendorfer's department in Vienna. I understand that copies of the German translation are available from Professor Frauendorfer.

Paper 5

ECONOMIC PLANNING OF THE FARM FOREST OPERATING UNIT

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Introduction

The planning model I am going to present in this paper is in no way new or unique. The basic model was presented originally in 1967 in my dissertation and an abbreviated version in English was published in 1969 in *Studia Forestalia Suecica* after a two year struggle for funds. The technique used, linear programming, is more or less common goods in planning, and as far as I know it was used in forestry planning for the first time by Coutu-Ellertsen in 1960. Since then it has been widely used in planning models of different kinds.

During the last two years we have developed the basic model further and tried to make it at the same time more realistic and more flexible, while still keeping it general, which was one of the basic ideas behind the originally developed model. And I believe our model may become a useful tool for management planning in forestry.

What I intend to do here is to outline the model and discuss its practical application and possible usefulness, but before that I must give you a general view of the background and the intended use of the model.

Farm forest structure

In Sweden roughly half of the forest area is owned by private persons, most of them farmers but with an increasingly large portion of others. The conditions are similar in the other Scandinavian countries, except that the private forests dominate still more among our neighbours.

Most of the private forests are small. More than 90 per cent of the holdings comprise less than 100 hectares. 75 per cent of the holdings are combined with agriculture.

Co-operation

Farmer co-operatives—mostly for marketing—are common in Sweden and have a strong and established position. The Forest Owners Associations have gone one step further than the associations within other branches. They assist their members not only in marketing their products, but also in the production phases, mainly logging. Because of the small holdings they strive for a closer co-operation between the forest owners also on the production side. To that aim the farm forests within a local region are organised in what is commonly called Farm Forest Areas, the size of which varies between 5,000 to 15,000 hectares. The owners of these separate farms are of course free to manage their forests individually, if they want to, but they have the possibility to utilise the Area-organisation for logging and other kind of work. Head of the Area-organisation is an Area-ranger and at his side is a council of a few forest owners elected by the members.

This set-up has of course many advantages, but there are some difficulties. The main work of the Area-organisation is logging, and it is the responsibility of the ranger to make these operations efficient considering *the Area* as an operating unit. The individual farmers on the other hand are primarily interested in the efficiency of their *separate farms*, which constitute *their* operating units. Consequently, there is a possibility or probably a certainty of conflicting interests between the collective represented by the Area-ranger and the individuals.

One condition for solving this conflict is that the individual owner has a long-run management plan for his forest, which will set the limits for the short-run management plans made by the ranger for the logging operations. These long-run plans should—as a principle—comprise the production on the individual farms as a whole, that is they should include agriculture and other possible production.

Management planning system

Planning is a procedure going on at different levels within and outside of the enterprise. I have tried to give a very rough outline of a planning system in figure 3.

The outer framework is given through the policy adopted by government. Within that framework the individual enterprise is free to make up its own plans. The first step here should be to establish an over-all plan for the total enterprise, if possible (strategic-tactical planning).

This in turn will serve as framework for the management plans for the different branches of the enterprise. Limiting ourselves to forestry, the first step will be a long-run plan (strategic-tactical planning), the second step a short-run plan (tactical-operational planning) and the third and final step logging plans (operational planning). These different

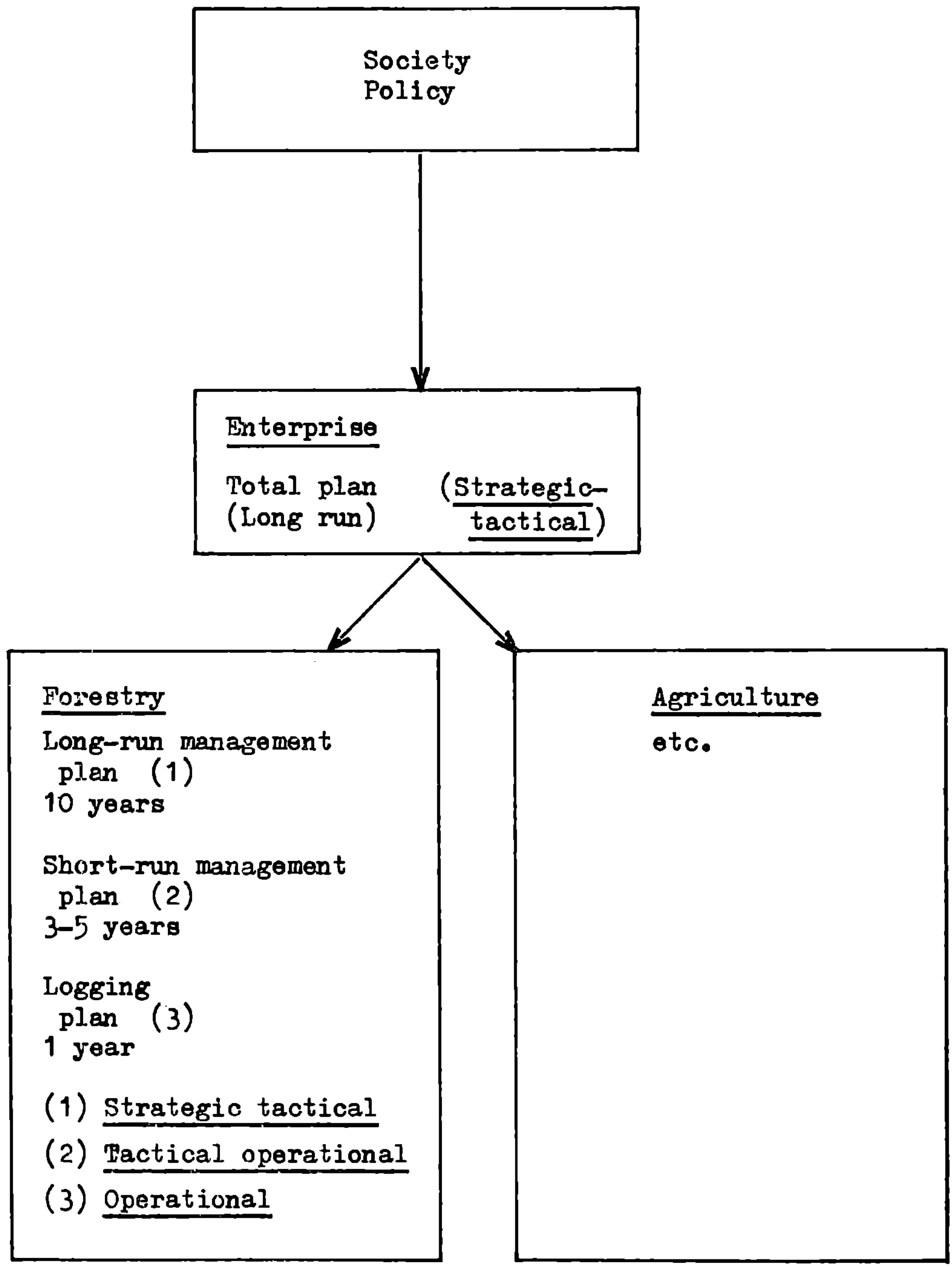


Figure 3. The planning system.

steps can be planned separately or in combinations.

It may seem advantageous to combine several steps into one single plan comprising long-run as well as short-run aspects. On the other hand there are certain disadvantages. For example:

—The kind and amount of data needed may not be the same

—The kind of information sought is partly inconsistent

—The model will become more complicated and more expansive to use

—The resulting plan will be less flexible

—Short-run changes, which may not affect the long-run plan, may invalidate the short-run plans completely.

Personally, at the present state, I prefer a system where the different steps are clearly distinguishable and separate. And that is one of the basic ideas behind our model construction.

Purpose

The model described here is designed for long-run planning of forestry, separately or in combination with agriculture or some other production. The resulting plans can be total enterprise plans or long-run management plans referring to figure 3. If it is a total plan the long run forestry plan is obtained simultaneously as part of the total plan.

The resulting plan is optimal from the point of view of the individual owner and functions as a steering instrument for the following planning steps, whether they will be conducted by him or by the Area-organisation. There are certain limitations here, which are touched upon in the next paragraph.

As for the Area-ranger, the long-run plans will serve as a basis for his planning, both the short-run plans and the final logging plans. If enacted it will mean a considerable advance and enable more efficient logging-operations compared with the present situation. One of the problems facing the ranger will be the combination of the plans for the different farms into efficient logging plans. It will obviously necessitate compromises and certain changes in the original long-run plans. Such compromises are inherent in all co-operation, where you always should consider the goals of the individuals and the goals of the collective simultaneously. However, in this case conflicts between these goals may be more seeming than real. For example separate logging may be so much more expensive, that it outweighs other advantages. Anyway, as you know linear programming gives information about shadow prices, etc. and this information can be used for estimating losses encountered because of deviations from the optimal long-term plan.

To summarise: The model is meant to be used

—as an instrument for long-term, survey planning and analysis of individual farms

—for establishing forestry long-run management plans on a large scale

—for model studies of different kinds and for various aims.

I have stressed here the importance of the planning model as a steering instrument of the individual owner for the management of his woodlot according to his individual goals but within the framework of the Forestry Areas. Considering the existing social and economic conditions as well as the basic ideological philosophy in the Scandinavian countries this approach seems most relevant. However, the model is applicable also to long-term planning of for example a Forest Area as a whole, without any special considerations of the individual owners, but solely to optimise the Area activity as such. This may be relevant in certain parts of Sweden, where the woodlots are extremely small, scattered, and impossible to manage individually.

Description of the model

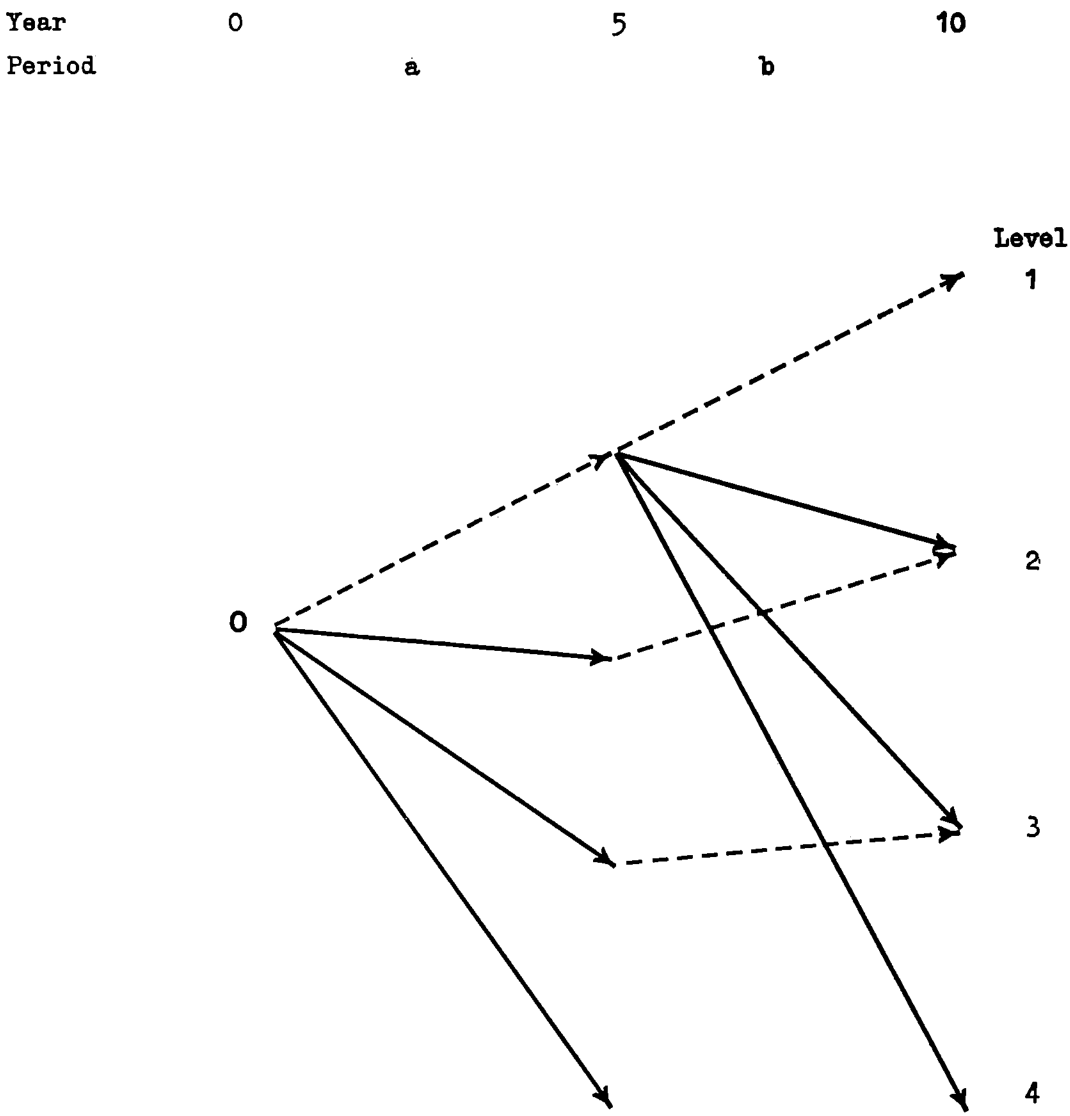
The model is designed for long-term planning in farm forestry, based on the economic situation of the individual owner and goals specified by him with respect to income from forestry, forestry job opportunities etc.

Thus the plan gives the same information as the traditional management plans, but recommendations with respect to cutting policy are based solely upon objectively recorded technical, biological and economic data. Using these data in the calculations, which form a part of the model, the plan is obtained without using the subjective recommendations, on which conventional forestry management plans are based.

The model can be used for making separate forestry plans or combined plans for the farm as a whole. In the latter case the plan will include, besides the production plan, a complete investment and financing plan. In both cases the implications of taxation are taken into account.

A total planning period of 10 years divided into two or alternatively three sub-periods is suggested. Other lengths of period can be used, but 10 years is commonly used in forestry. In the basic model a total planning period of 30 years was used. That length was chosen because it corresponded roughly with the active entrepreneurial period of a farmer.

Technical and biological field data are mainly the same as those collected for traditional forestry plans. In addition, data for appraising harvested volume and evaluating the stands at the end of the



Level

1. No cutting
2. Light thinning (large volume)
3. Heavy " (minimum allowable volume)
4. Clear cutting

- - - - - - - - - No cutting during the sub-period

__________ Cutting " " " "

Figure 4. Forestry activities.

period must be collected. These data refer to timber quality and logging conditions as well as prices for timber, labour costs and so on.

Alternative estimates of timber output are made for each separate stand according to certain rules, which are schematically outlined in figure 4. The figures on the right hand side denote different levels of growing stock. The level 1 means that no cutting will take place during the period. Level 2 means a fairly high level corresponding, in the example we have been working with so far, to the so called "better half" of the stands as measured by the National Forest Survey. Level 3 means a lower level. In our examples corresponding to 60 per cent of the volume at level 2. Level 4 finally means clear-cutting.

This construction corresponds with what in Sweden is known as "thinning models". These are constructed in the same way and are meant to serve as a guide for how much to cut in a certain stand. The difference is that the thinning models are to be used as a decision-guide by the forester in the forest. In our system the data are collected but the decision is left to the planning model. In that way the decision is made first when you can consider all the stands simultaneously.

One part of this construction is the yield function on which the development of the stand is based. A few years ago we had many difficulties in obtaining suitable functions, but much work is now being done in this field, and we are hopeful to get more reliable functions in a near future. The same is true with respect to data for labour productivity.

All these data as well as prices and the expected development of prices and labour productivity during the planning period should probably be collected and analysed centrally.

The different cutting alternatives are evaluated as well as the alternative values of each stand at the end of the period.

A special computer program has been made up to perform the calculation for the alternative cutting programs as well as for their evaluation. Before, these calculations were made manually, which is a very cumbersome procedure and besides almost impossible to do without calculating errors. A condition for the use of the model in practice therefore has been such a computer program. We also have found that the calculations using the computer is much cheaper than even preparing the data cards after manual calculations. According to the limited experience we have up to now, those costs are about 40 to 50 times as large. In addition, we save all the manual work and avoid computational errors.

Linear programming technique is used for solving the combination problem and the resulting plan will be presented in a form suitable for practical use. The complete cost for solving the LP model seems to vary between $20 and $40 for a farm forest with about a hundred stands. Including the agriculture part. Schemes depicting the various steps of the calculations is presented in figures 5 and 6.

The goals of the farmers can be stated in different ways within the framework of the general model. However, certain basic assumptions have been made.

(*a*) Regulations defined in forest laws, etc. are to be met. Partly this is done already in the defining of cutting alternatives.

(*b*) The objective function is based on the capital value of the forest, disposable funds, newly constructed buildings and—as negative entries —funds borrowed during the period.

(*c*) A certain annual amount — individually defined—for living expenses and certain fixed outlays must be met and is taken up as a restriction in the LP-model.

In addition other demands can be taken into account, for example, a certain amount of forest work, minimum level of growing stock and so on.

DISCUSSION OF VON MALMBORG'S PAPER

Glück. Should you not add more possible levels of cutting besides no cutting and clear cutting so as to be able to show correct response to market price variation. *von Malmborg*. This would be possible but I would rather go the other way. The farmer either needs money or he doesn't. *Bjora*. The objective function was based on present value, to what extent do you take account of levels of living expenses, availability of own labour, and so on. *von Malmborg*. Young men do a lot of work, old men less—we build in the assumptions of the particular farmer. *Jackson*. You mentioned getting a sequence of farms modelled simultaneously, would these be linked together. *von Malmborg*. I can only get a percentage of farms in any one forest area. *Frauendorfer*. You mentioned the total plan and long-term and short-term plan for co-operating farms. In the total plan you have forestry and agriculture, is it possible to optimise income by altering the area under forestry and agriculture? *von Malmborg*. This is possible but it is rather difficult if you are dealing with such long-term questions as afforestation. *Paillé*. In terms of forestry problems the planning period you speak of is rather short-term. *Wardle*. I wonder whether it is not important to draw a distinction between the time horizon for planning and the period or term of the plan. *von Malmborg*. This long-term problem of afforestation is hardly a problem in Sweden since you must replant and likewise you must plant up land which is not in agricultural use. *Risvand*. What

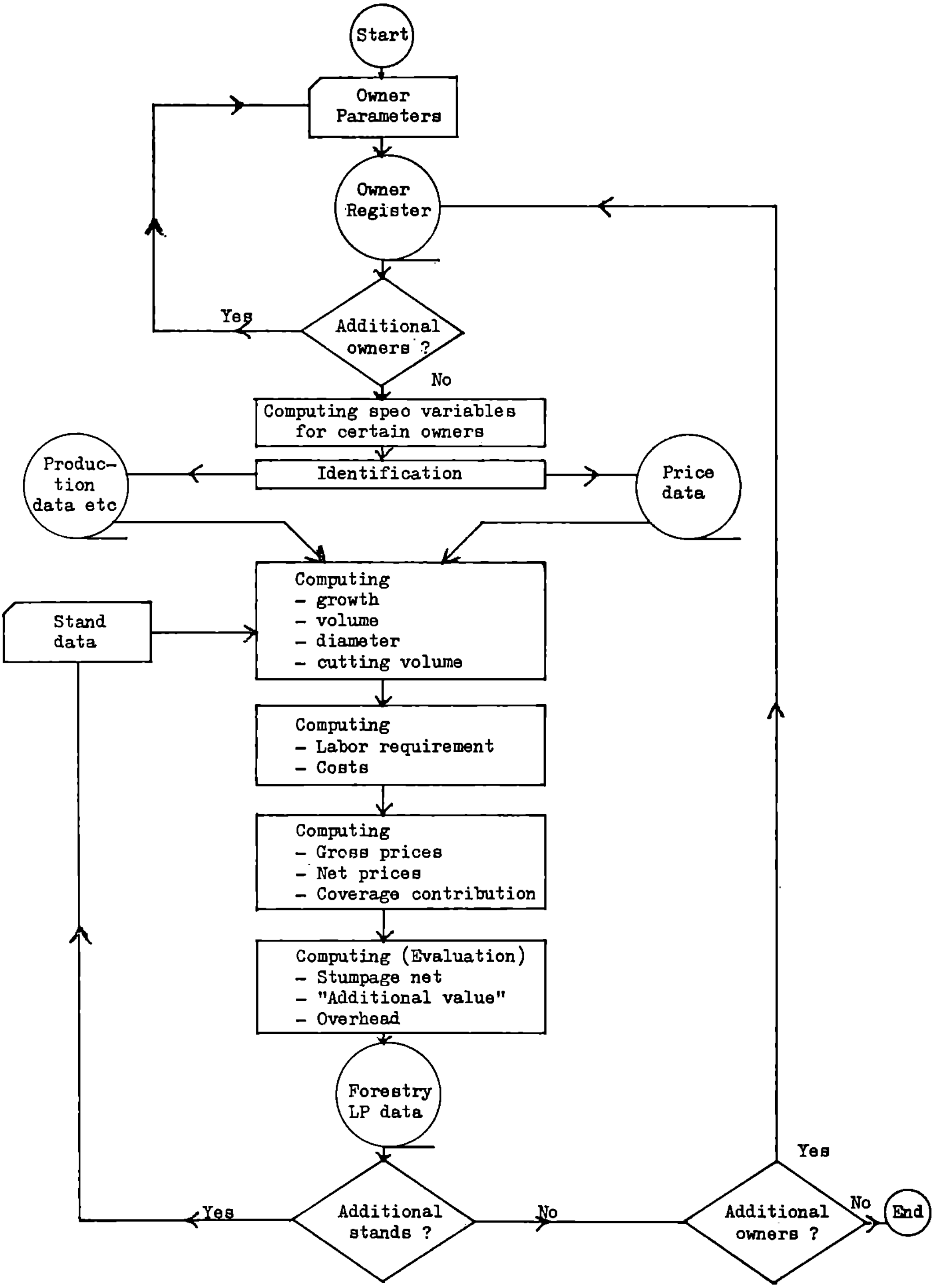


Figure 5. Outline of the planning model as a whole.

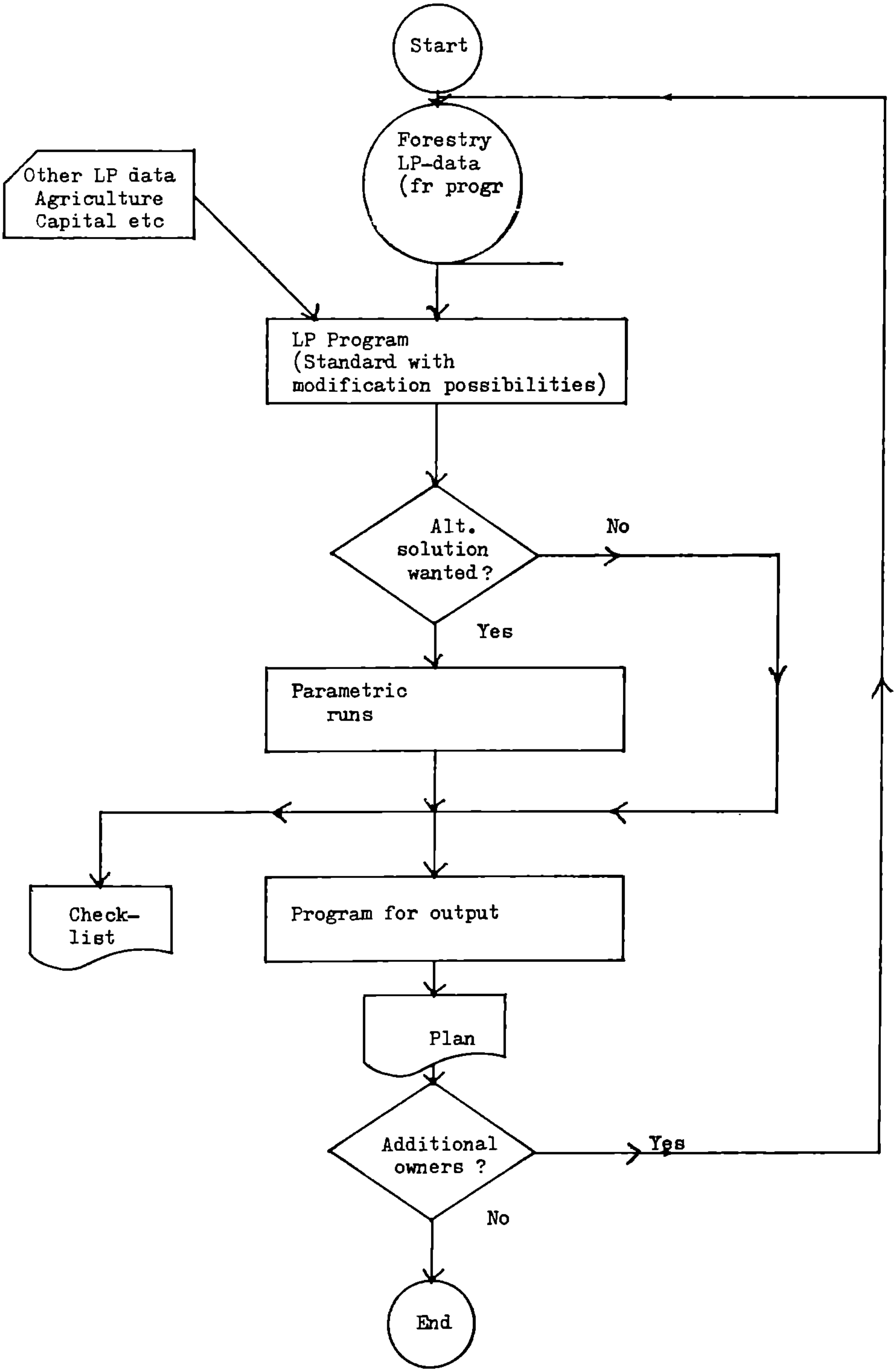


Figure 6. Outline of the computer program for forest calculations.

about the interaction between short and long-term plans. *von Malmborg*. You will never get through a long-term plan without changes by the time you get to the short-term plan. *Bjora*. The short-term plan will affect possibilities, not the plan itself. *Wardle*. I should be interested to know to what extent you have used these models and whether they gave rise to surprising results—different from what one might have expected before doing the work. *von Malmborg*. We have three farms with which we have worked and it has worked out very well, but the resulting plans have not been put into effect. As to surprise, I was very surprised and scared because one owner, according to the results, should not cut anything but in real life he cut quite a lot. I think the model was right, because he was very well-off. The reason for the difference was that in real life he would not borrow money, whereas in the model we allowed borrowing which was better business. In another case I was confused—we changed taxes—we lowered taxes and the model said cut more. We then increased taxes and expected we would cut less, but in fact the model said cut more in order to pay the taxes.

DYNAMIC PROGRAMMING FOR DETERMINING OPTIMUM CUTTING POLICIES FOR A FOREST ENTERPRISE

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Introduction

The aim of this paper is to present an economic model, providing a basis for planning the operations in a forest enterprise. To illustrate the model, an example is described, in which the problem is to determine the cutting policy, when the purpose of the plan is to maximise the present value of the forest. The quantities to be decided are the distributions of cutting volume between and within management blocks; the stands to be clear cut, and to be thinned, and the thinning grade. A planning period of five years is used in the example. One may, however, use planning periods of different lengths, or plan for more than one period. It is also possible to use restrictions other than the cutting volume, as well as a greater number of restrictions.

Special problems concerning planning in forestry

One characteristic of forestry is that the production is spread over large areas. This fact involves difficulties as regards survey, organisation and transport.

The cost when operating in a stand is not independent of the locations in which operations are going on elsewhere in the forest. The reciprocal distances between the units, on which the model is built, may, therefore, be of great importance for the choice of operational plan. Hence, a realistic and effective model can hardly be obtained, if the location of the individual stand is not taken into account.

The production time is very long in forestry. Many of the decisions made will have consequences far into the future. In addition the growing stock partly has a double significance, representing a means of production as well as a product.

The long perspective of the forest production, in addition to the problems of co-ordination arising because of the large areas involved, is the reason why the models constructed have a tendency to be complicated and unwieldy. These problems are difficult to avoid, because the perspective of time as well as the short-term problems of co-ordination, must be considered, if the model is to be tolerably realistic.

Forest resources

Planning must always have a starting point and a more or less well defined purpose. The basis upon which we may construct the plan may roughly be divided into outward conditions, the structure of the enterprise, and the actual state of the forest resources (Seip, 1964). Only the forest resources will be discussed in the present paper.

In order to be able to plan the disposal of the resources, it is necessary to have an account of them. As to forest resources, this involves some problems of estimation, which will not be discussed here.

Usually, in dividing a complex of problems into minor ones, it is sought to do this in such a way that the actual interdependence between the problem groups will be as small as possible.

The plan will be influenced by the location of the stands. By concentration of the cutting some of the advantages of a large-scale operation will be obtained, and the unit costs reduced, compared with cutting spread all over the forest. This is due partly to the saving of costs in connection with transportation, and partly to the fact that administration and inspection are simplified and more effective (Väisänen, 1967).

As a consequence of taking the location of the individual stands into account, the enterprise should be divided into management blocks. A management block is a forest area which owing to its location with respect to topography and means of transportation naturally belongs to a main artery of transportation (Seip, 1964). An application of effective optimalisation methods requires that the management blocks are independent of each other; i.e. the profitability of an enterprise in one block must not affect the profitability of an enterprise in another block.

Ordinarily, a management block will consist of several forest stands. A forest stand is a unit area, comparatively homogeneous as to biological and technical data. The forest stands are regarded as units of management and used as building bricks in the model. Because of this, it is preferable that the size of the stands does not vary too much. A large stand may, therefore, be divided into two minor ones. The size of the stands has also to be considered in connection with the size of the forest.

To make tolerably reliable economic appraisals, it is necessary to have information for every forest stand on:

(1) The state of the stand.
(2) Production possibilities.
(3) Operational circumstances.
(4) Location.

The state of the stand is characterised by volume per decare (daa), mean diameter, mean height, and age. In addition, information about the state of health may be relevant.

The production possibilities are stated by site classes (H_{60}).

The operational circumstances are classified by the regularity and the slope of the terrain.

To describe the regularity of a terrain, a classification from Samset, (1957) is used (cf. Opheim, 1969):

(1) Uniform terrain.
(2) Terrain with large stones and other obstacles.
(3) Boulder-strewn and rocky ground.

The slope of the terrain is expressed by the gradient in per cent.

The location of the stand is classified by the management block and the transport distance. In addition, information about the quality of the transport artery will enable a higher precision of the calculations.

From such a description, it is possible to indicate the costs involved by operations within a unit with reasonable accuracy. It will also be possible to appraise the economic consequences of investments in transport arteries or an alteration of the operating method.

The forest used in this example consists of three management blocks (see Table 15). Clear cut areas and young wood (development classes I and II) have been omitted, since in this example, the calculations are intended to cover only stands where thinning or clear cutting is in question.

Restrictions

In analysing an attack on "the narrow sections" has often proved effective. The attention is concentrated on the production factors that seem to be limiting to or determinative of the production. In forestry, the growing stock is often the restricting factor, but an influx of workers, and financial possibilities, etc. may also be relevant restrictions.

As to growing stock, it is natural to start the analysis with a more or less coarse account of the production possibilities. The long perspectives of forest production require a long-term analysis. This forecast may be of value when appraising the size of the cutting quantity during the planning period. It may be relevant to use the value of the cut instead of the quantity.

Methods for such long-term projection were prepared by Nersten and Delbeck, (1965) and Nersten, (1965). The calculations have been programmed and can be accomplished by an electronic computer.

These projections are intended to clarify the consequences which the alternative dispositions of

TABLE 15
INFORMATION ON THE STANDS

Management block No.	Stand No.	Area daa	Site class H_{60}	Volume m^3 per daa	Mean diam. cm	Mean height m	Age years	Transport dist. km	Slope of terrain %	Regularity of terrain classes
1	1	12·2	8·0	14·2	20·0	17·0	100	0·9	10	2
	2	9·0	11·0	15·9	16·0	15·0	76	1·0	30	1
	3	7·0	11·0	9·3	10·2	10·9	50	1·3	10	1
	4	3·0	17·0	10·0	12·2	11·6	35	1·2	0	1
	5	10·0	17·0	24·8	22·6	22·6	68	0·8	0	2
2	1	15·2	8·0	6·2	10·8	10·3	60	0·5	10	1
	2	5·3	8·0	17·4	19·2	16·6	110	0·8	20	1
	3	8·5	11·0	16·6	24·8	19·1	90	0·7	0	1
	4	16·0	11·0	10·0	17·3	15·2	71	0·4	10	1
	5	5·9	14·0	8·0	20·5	17·0	60	0·6	10	2
3	1	8·0	8·0	5·0	9·0	8·0	50	1·3	20	1
	2	20·0	8·0	13·2	14·0	13·5	80	1·7	0	2
	3	13·6	14·0	19·1	19·5	16·8	60	1·5	20	1
	4	2·9	17·0	20·9	22·0	21·5	72	1·1	10	1
	5	17·6	11·0	13·8	21·3	18·7	92	0·9	0	2
	6	6·8	11·0	12·5	18·5	17·6	83	1·1	10	1

today may have in the future. It is also of current interest to throw some light upon the future development in domains where preventive efforts may be necessary.

As a result of such projections among others, the owner is supposed to have arrived at the decision that he is interested in choosing a cutting quantity within the range of 100–300 m³ for the planning period. The owner has not taken, and can scarcely take, any decision as to the final cutting quantity at this moment. It is, therefore, natural to carry out calculations for alternative sizes of the cut.

Analysis of individual stands

The determination of the cutting policy for the individual forest stand is based on calculations by Risvand, (1969). The stand is at each age level, or point of time, characterised by the standing volume per decare and the mean diameter. These two state variables are chosen because they are the ones that there is most opportunity in practice to influence by silvicultural methods. The mean height of the stand is also taken into account, but it is not used as a separate state variable in the same sense as volume and diameter.

The thinning grade is used as a decision variable. The problem to be faced is to find the most satisfactory development of the stand for the given purpose. This problem is solved by dynamic programming, which determines the treatment of the stand; time of thinnings, degree of thinning, and time of final felling.

Immature stands

Assuming an optimum pattern of treatment, the present value of a forest stand can be expressed as a function of the state of the stand and the assumptions made as to production, prices and costs. Such a function can be estimated, if optimum cutting policies and present values are calculated for a series of different forest stands under varying circumstances.

Optimum patterns of treatment have been calculated by dynamic programming, and, by a step-wise regression method, the following function for the present value of a forest stand was obtained.

$$
\begin{aligned}
y = {} & 494{\cdot}20 + 19{\cdot}72\,V - 19{\cdot}95\,t + 5{\cdot}90\,vp + 53{\cdot}88\,H_{50} \\
& + 2{\cdot}113\,V{\cdot}d - 0{\cdot}07186\,V{\cdot}d^2 + 0{\cdot}1996\,t^2 \\
& + 2{\cdot}638\,V{\cdot}h + 3{\cdot}791 H_{50} - 0{\cdot}003773\,V^2{\cdot}t \\
& - 0{\cdot}832\,V{\cdot}vp + 0{\cdot}0352\,V{\cdot}d{\cdot}h \\
& - 138881\,V{\cdot}\sqrt{vp}/(d{\cdot}h{\cdot}H_{50}{\cdot}t) \\
& - 0{\cdot}08825\,V{\cdot}t - 0{\cdot}06326\,H_{50}^2{\cdot}vp - 1{\cdot}2286\,H_{50}{\cdot}t
\end{aligned}
$$

$$R = 0{\cdot}998, \quad S = 3{\cdot}5\%$$

where:

d = basal area mean diameter in centimetres,
h = mean height, basal area used as weight, in meters,
H_{50} = definition of site index: Mean height of the stand, basal area used as weight, in metres at the age of 50 years,
t = total age of the stand, years,
V = Volume, m³ per daa under bark,
vp = costs per unit of volume, kr. per m³,
y = present value, kr. per daa.

The calculations have been based on the same assumptions as those mentioned for treatment within a management block (see page 46 below). It is possible that further analysis may lead to better results.

Mature stands

The determination of present values above presupposes an optimum cutting policy for the stand. When the age of maturity has been passed, the most profitable procedure would be to clear cut the stand immediately. If this policy is not feasible, alternatives should be considered. These may consist in leaving the stand undisturbed till the final felling, some time in the future, or in thinning immediately and leaving the remaining trees for the final felling. To enable a calculation of the economic result of such policies, it is necessary to know how long a time will elapse, before a possible final felling can take place. This point of time can be estimated on the basis of the time interval between each operation taking place within a management block. Next time operations take place, a new appraisal of each stand will be relevant, and it will have to be decided whether the stand is to be clear cut or left for future growth.

In the present example, it is assumed that a clear cutting may take place in the next planning period. The state of the stand, therefore, has to be estimated at this point of time. The present value of such a policy will consist in the value of the potential thinning removal at the point of appraising, in addition to the value of the final cut and the soil value discounted to the same point of time.

Determination of optimum cutting policies

As previously stated, a planning period of five years will be used in this example. All cutting is supposed to take place in the third year, that is in the middle of the planning period. The task now consists in deciding from which of the management blocks and from which of the stands within the blocks, the cutting quantity should be taken, when the objective is to maximise the present value of the forest.

The algebraic formulation of the problem is:

$$\text{Maximise} \quad Z = \sum_{j=1}^{k} \left[\sum_{i=1}^{nj} f_{ji}(x_{ji}) - h_j(\sum_{i=1}^{nj} x_{ji}) \right]$$

$$\text{subject to} \quad c_1 \leqslant \sum_{j=1}^{k} \sum_{i=1}^{nj} x_{ji} \leqslant c_2$$

$$x_{ji} \geqslant 0 \quad j = 1,2,...k \quad i = 1,2,...n_j$$

where:

x_{ji} = removed volume, stand i, management block j,

$f_{ji}(x_{ji})$ = present value* for stand i, management block j, when the removal is x_{ji},

$h_j(\sum_{i=1}^{nj} x_{ji})$ = costs depending on the concentration of the cutting, management block j,

c_1 = lower restriction of cut,

c_2 = upper restriction of cut,

n_j = number of stands, management block j,

Z = present value of the forest.

*Note** The value of the cutting quantity in the stand in addition to those of the remaining stand and the soil.

The problem is solved by dynamic programming. The calculations should be carried out in two steps. First, the treatment of stands within a management block should be determined for the different requirements made on the cutting quantity in the block (see below). The results of these calculations, together with the costs, depending on the concentration of the cutting, are used in determining distribution of the cutting quantity among management blocks (see page 50). This method may be used, since the costs depending on the concentration of the cutting, will not influence the distribution among stands of a given cutting quantity within a block.

Calculations for treatment within management blocks

The calculations will be illustrated for management block 1. The following, among other information, is available for stand 1 (see Table 15):

H_{60}	V	d	h	t
8·0	14·2	20·0	17·0	100

This information, together with the functions described by Risvand, (1969) is the basis for estimating the development of the stand. In the third year, the calculations will turn up these figures:

V	d	h	t
15·2	20·6	17·3	103

At this point of time, it is assumed that it will be possible to choose among five cutting policies:

Policy 1: Leaving the stand undisturbed.

Policy 2: Thinning, removal of 20 per cent of standing volume.

Policy 3: Thinning, removal of 30 per cent of standing volume.

Policy 4: Thinning, removal of 40 per cent of standing volume.

Policy 5: Clear cutting.

Traditional thinnings will increase the mean diameter and the mean height of the remaining trees, because the average diameter and height of trees removed are below those of the whole stand prior to thinning. This increase is determined by the thinning grade, the thinning ratio, and the ratio between the heights of removed and remaining trees.

The price calculations are based on separate price charts for thinning and final felling. This is because the costs per tree depend on the type of cutting. Fig. 7 shows a price chart for final fellings. The thinning costs per tree are estimated to be kr. 1·29. Otherwise the thinning price chart is the same as at final fellings.

To enable an estimation of the values, the costs per m³ must be taken into account. These costs are estimated partly by help of work by Svendsrud, (1968) and partly by an investigation by Opheim, (1969). The following functions are obtained:

Transport distance $<$ 0·3 km:

$$vp = 23{\cdot}80 + 16{\cdot}70\, l + 0{\cdot}20\, sl + 2{\cdot}15\, rt - 0{\cdot}027\, sl{\cdot}rt$$

Transport distance $\geqslant$ 0·3 km:

$$vp = 27{\cdot}85 + 5{\cdot}00\, l + 0{\cdot}20\, sl + 2{\cdot}15\, rt - 0{\cdot}027\, sl{\cdot}rt$$

where:

l = transport distance, kilometres,

rt = regularity of terrain, classes,

sl = slope of terrain, per cent,

vp = costs per unit of volume, kr. per m³.

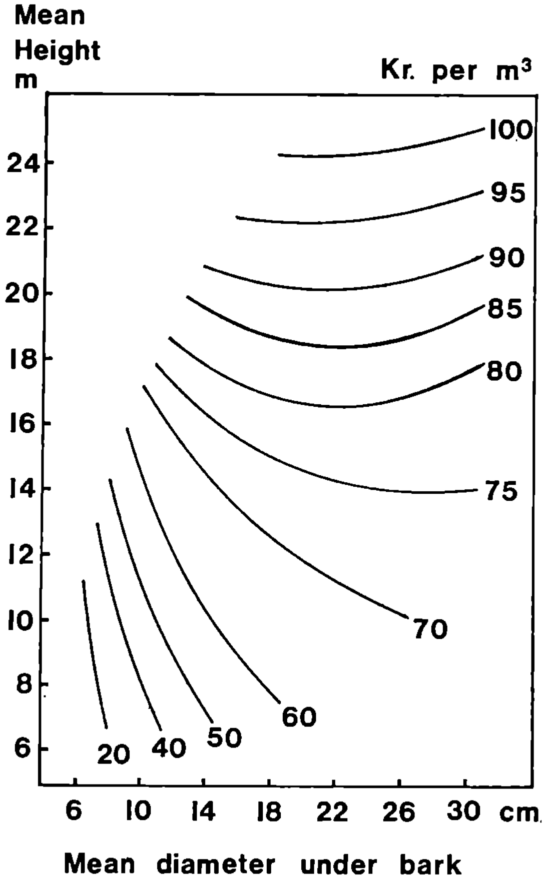

Figure 7. Price chart for fellings.

This is a mean price chart for spruce showing reduced gross value in kr. per m³. The underlying assumptions are in price of pulpwood kr. 90 per m³, of sawlogs kr. 105 per m³. Cost per tree kr. 1·03 and per log kr. 0·20.

The value of the removed trees can be calculated by taking the reduced gross value from the price chart and subtracting the costs proportional to the volume. In thinning, the extra costs involved in this type of cutting should be considered. The rising costs are due to the obstacles represented by trees left standing and a smaller removal per decare. Costs per decare are estimated at kr. 15·30 and kr. 5 for thinning and final felling, respectively. If the stand has not reached the age of maturity, the remaining stand should be appraised by means of the function described above for immature stands (see page 45). Otherwise, the principles for mature stands page 45 should be used. The most imporant results of the calculations for stand 1 are shown in Table 16.

Corresponding calculations have been carried out for the other stands in the management block. The total figures for removed volumes and present values are recorded in Table 17.

Calculations have been carried out to show the results, to which the different policies will lead in each stand. Now a total optimalisation should be carried out, to define the optimum policy for each stand, when certain requirements are made on the cutting quantity in the block. It will be shown how this problem can be solved by dynamic programming. The forest stand will be regarded as a unit in the model. (A policy, therefore, cannot be partially accomplished in a stand.) Algebraically the problem may be formulated as follows for management block j (see also determination of optimum cutting policies, page 45 above).

$$\text{Maximise} \quad z_j = \sum_{i=1}^{nj} f_{ji}\,(x_{ji})$$

$$\text{subject to} \quad \sum_{i=1}^{nj} x_{ji} \leqslant c^2$$

$$x_{ji} \geqslant 0 \qquad i = 1,2,\ldots n_j$$

where: z_j = present value for management block j.

TABLE 16
RESULTS OF COMPLETED CALCULATIONS FOR STAND 1

Policy	Remaining trees: Volume m³ per daa	Remaining trees: Mean diam. cm	Remaining trees: Mean height m	Removed trees: Volume m³ per daa	Removed trees: Conversion surplus kr. per daa	Present value kr. per daa	The total stand: Removed volume m³	The total stand: Present value kr.
1	15·2	20·6	17·3	0	0	692	0	8441
2	12·2	21·4	17·7	3·0	84	652	37·1	7957
3	10·7	21·8	17·8	4·5	137	640	55·7	7809
4	9·1	22·2	18·0	6·1	191	628	74·3	7663
5	0	0	0	15·2	685	701	185·7	8547

The other symbols are the same as those used previously (p. 46). For a solution by dynamic programming the following relation is used as a recurrence formula for $n = 2,3,...n_j$ (cf. Bellmann and Dreyfus, 1962: 14–18):

$$F_n(c_2) = \max_{0 \leqslant x_n \leqslant c_2} \left[F_n(x_n) + F_{n-1}(c_2 - x_n) \right] *$$

where:

$F_n(c_2)$ = optimum result for n stands when the removal is c_2,

$F_{n-1}(c_2-x_n)$ = optimum result for $n-1$ stands when the removal is c_2-x_n,

$F_1(c_2-x_2)$ = present value of stand 1 when the removal is c_2-x_2.

*Note** The indexes for management block (j) have been omitted.

The procedure is first to determine the optimum policy for one stand at different levels of cut (Table 18, stage 1). The levels, not filled in, cannot possibly be reached by the alternatives of action chosen. However, when more stands are included in the calculations, the variation possibilities increase rapidly, and most of the levels will be filled in.

The policy chosen for stand 2 has been recorded in stage 2, together with the figures for the total cut and the present value for stands 1 and 2. An example will illustrate how the figures for a cutting level of 20–40 m³ are obtained.

First, it is determined which of the policies can possibly be used for stand 2, when the removed volume is not to exceed 40 m³. Table 17 shows that policies 1 and 2 are feasible.

Policy 1:

Removed volume 0 m³, present value	6,474
Left for stand 1:	
Lower bound 20 m³, upper bound 40 m³	
Removed volume 37·1 m³, present value	7,957
Total removal 37·1 m³, total result	14,431

Policy 2:

Removed volume 31·6 m³, present value	5,989
Left for stand 1:	
Lower bound 0 m³, upper bound 8·4 m³	
Removal volume 0 m³, present value	8,441
Total removal 31·6 m³, total result	14,430

Policy 1 gives the best result and is chosen for stand 2 at this level of cut (Table 18, stage 2). The calculations proceed in this way. In the third stage, only the results from stage 2 and Table 17 are needed. The further procedure is based on the results from the preceding stage. Only optimum solutions are allowed to proceed further.

In the fifth stage, the calculations are finished. A survey of removals and economic results for different levels of cut has been obtained. The policy to be chosen for each stand, can be determined by working backwards through the calculations.

Example: What is the optimum policy when the total cut is to be between 160–180 m³?

At this level of cut, Table 18, stage 5, shows the total removal to be 165·8 m³

The following policy should be chosen:

Stand 5, policy 1, removed volume	0·0 m³
Left	165·8 m³
Stand 4, policy 2, removed volume	7·8 m³
Left	158·0 m³
Stand 3, policy 1, removed volume	0·0 m³
Left	158·0 m³
Stand 2, policy 5, removed volume	158·0 m³
Left	0·0 m³
Stand 1, policy 1, removed volume	0·0 m³

The precision of the calculations depends on how many levels of cut are examined, and the size of these (cf. Hadley, 1964: 368–373). This should be considered in relation to the size of the forest, and the principles used in dividing the forest into stands.

TABLE 17

REMOVED VOLUMES AND PRESENT VALUES FOR STANDS 2 TO 5

Policy	Removed volume m³				Present value kr.			
1	0	0	0	0	6474	3334	3638	19975
2	31·6	15·6	7·8	55·1	5989	3100	3489	19433
3	47·4	23·4	11·7	82·7	5799	3016	3436	19217
4	63·2	31·2	15·6	110·3	5603	2921	3382	18987
5	158·0	78·0	39·0	275·6	6126	1641	1930	19588
Stand No.	2	3	4	5	2	3	4	5

TABLE 18

SUMMARY OF CALCULATIONS FOR BLOCK 1

Stage No.	1			2			3			4			5		
Level of cut m³	Policy	Removed volume m³	Present value kr.	Policy	Removed volume m³	Present value kr.	Policy	Removed volume m³	Present value kr.	Policy	Removed volume m³	Present value kr.	Policy	Removed volume m³	Present value kr.
0	1	0	8441	1	0	14915	1	0	18249	1	0	21887	1	0	41861
0– 20							2	15·6	18015	2	7·8	21738	1	7·8	41712
20– 40	2	37·1	7957	1	37·1	14431	3	23·4	17931	1	23·4	21569	1	23·4	41545
40– 60	3	55·7	7809	1	55·7	14283	1	55·7	17617	1	55·7	21255	2	55·1	41319
60– 80	4	74·3	7663	1	74·3	14137	1	74·3	17471	1	74·3	21109	2	62·9	41170
80–100				2	87·3	13798	2	89·9	17237	2	82·1	20960	3	82·7	41103
100–120				2	105·9	13652	4	105·5	17058	1	105·5	20696	4	110·3	40873
120–140				3	121·7	13462	1	121·7	16796	4	121·1	20440	4	133·7	40555
140–160				5	158·0	14567	1	158·0	17901	1	158·0	21539	1	158·0	41513
160–180							2	173·6	17667	2	165·8	21390	1	165·8	41364
180–200	5	185·7	8547	1	185·7	15021	1	185·7	18355	1	185·7	21993	1	185·7	41967
200–220				2	217·3	14536	2	201·3	18121	1	201·3	21759	1	201·3	41733
220–240				3	233·1	14346	1	233·1	17680	1	233·1	21318	1	233·1	41292
240–260				4	248·9	14150	3	240·7	17552	1	240·7	21190	2	240·8	41425
260–280							4	264·3	17267	1	264·3	20905	5	275·6	41474
280–300							4	280·1	17071	1	280·1	20709	5	283·4	41326

Distribution of cutting quantity among management blocks

The kind of calculations carried out for management block 1, should be carried out for the other management blocks. When this is done, the economic results for various levels of cut have been calculated for each management block (see Table 18, stage 5). The last step is to determine how the total cutting quantity should be distributed on management blocks. This may, in principle, be solved in the same way as the optimalisation within the blocks. Now, however, the results of the calculations in the last stage for each management block form the basis of the appraisement.

At this point it is possible to include the costs, depending on the concentration of the cutting in the model. This may be effected, by adjusting the results of the calculations in the last stage for each management block.

If the starting of cutting in the different management blocks involves high costs, this will result in a concentration of the cutting. In this way, the costs can be reduced. The problem is specially important, when planning for one year, but the model may also be used to decide in how many and in which management blocks cutting is to take place during a planning period of e.g. five years.

No costs dependent on the concentration of the cutting have been considered in this example. Table 19 shows how the total cutting quantity should be distributed on the management blocks for levels of cut in question. The value of the cutting quantity and the present value of the forest are also shown in the table.

An example of the more detailed transcripts for each management block is shown in Table 20. The level of the cut is 240–260 m³ and the whole cut should be removed from management block 2.

The time consumption connected with the solving of this problem, consisted mainly of the time required for the reading of data and the writing of results. The calculations do not take long. An

TABLE 19

DISTRIBUTION OF TOTAL CUTTING QUANTITY ON MANAGEMENT BLOCKS

Level of cut m³	Cutting quantity m³				Value of cutting quantity kr.	Present value kr.
	Total	Management block				
		1	2	3		
100–120	108·0		98·2	9·8	4074	121976
120–140	128·6		128·6		5656	121765
140–160	151·9		151·9		8135	122192
160–180	161·7		151·9	9·8	7832	122129
180–200	185·7	185·7			8360	122073
200–220	201·6		98·2	103·4	8447	121862
220–240	222·2		128·6	93·6	10028	121651
240–260	250·1		250·1		12512	122264
260–280	261·0		261·0		12817	122064
280–300	283·9	185·7	98·2		12737	122145

TABLE 20

CUTTING POLICY FOR MANAGEMENT BLOCK 2

Stand No.	Policy	Remaining trees				Removed volume m³ per daa	The total stand		
		Volume m³ per daa	Mean diam. cm	Mean height m	Age years		Removed volume m³	Value of removed volume kr.	Present value kr.
5	1	9·2	21·6	17·6	63	0	0	0	3996
4	1	11·2	18·1	15·7	74	0	0	0	9438
3	5	0	0	0	0	17·9	151·9	8135	8547
2	5	0	0	0	0	18·5	98·2	4377	4457
1	1	7·2	11·4	10·8	63	0	0	0	4276

increasing number of stands, however, will result in a higher time consumption, as the work connected with the calculations will be a linear function of the number of stands.

One advantage of this model is that the location of the stands can be taken into consideration. The short-term problems of co-ordination, therefore, can be included in the model. Moreover, it is not necessary to commit the planning to a fixed cutting quantity, or a certain value of the removal. It is often difficult to decide on a cutting quantity in an early stage of the planning, and calculations carried out may give information of importance for this appraisement.

Otherwise, it should be noted that the stand is always regarded as a unit. Hence, it is not possible to divide a stand in such a way for instance that one half of it is thinned and the rest clear cut. A heavy drawback of this method is that the calculations increase rapidly with an increasing number of restrictions.

Solution by linear programming

On certain conditions, the problem may be solved by linear programming (cf. Kilkki, 1968; Lundell *et al.*, 1969). The algebraic formulation of the problem for solution by linear programming is:

$$\text{Maximise} \quad Z = \sum_{j=1}^{N} \sum_{i=1}^{5} a_{ji} x_{ji}$$

$$\text{subject to} \quad \sum_{i=1}^{5} x_{ji} = b_j \qquad j = 1,2,\ldots N$$

$$c_1 \leqslant \sum_{j=1}^{N} \sum_{i=1}^{5} y_{ji} x_{ji} \leqslant c_2$$

$$x_{ji} \geqslant 0 \quad j = 1,2\ldots N \qquad i = 1,2\ldots 5$$

where:

x_{ji} = part of stand j treated with policy i, daa,

y_{ji} = removed volume, m³ per daa, in stand j when policy i is applied,

a_{ji} = present value, kr. per daa, for stand j when policy i is applied,

b_j = area of stand j, daa,

c_1 = lower restriction of cut, m³,

c_2 = upper restriction of cut, m³,

Z = present value of the forest, kr,

N = number of forest stands.

y_{ji} and a_{ji} are calculated in the same way as for the dynamic programming model (see Table 16). The solution of this linear model may differ from that of the dynamic programming model. This is due to the fact that the use of linear programming presupposes divisibility. Then the possibility arises that the different parts of a stand should be treated differently in the forthcoming solution. In order to avoid such solutions, integer programming may be used.

A great advantage of the linear programming model is that a large number of restrictions can easily be considered, without the calculations becoming insuperable. In addition, information can be gained as to the way in which alterations in the preconditions influence the plan. This is very valuable, but it should be realised that the shadow prices apply only to the present solution. Minor deviations from this solution may lead to considerable changes in the shadow prices.

It is difficult to take the short-term problems of co-ordination into consideration in this linear model. This may be regarded as a drawback. Usually, the operational costs in a stand will depend on where other operations are going on in the forest. The importance of this fact will depend on the purpose of the plan.

Another thing is that little information emerges about the manner in which the plan is influenced by the size of the cutting quantity. The shadow prices may offer some information, but it may often be of limited value. Of course, calculations may be carried out for different values of the restriction, but this is a time-consuming procedure.

Conclusions

The models outlined, give an optimum result based on the available data. The requirements on the quantity and the quality of the data depend on the desired intensity of planning. Especially the forest data may often be inaccurate and unreliable. In addition, the models are always a simplified representation of the actual situation. The results of the calculations, therefore, should not be regarded as conclusive. They may, however, prove helpful when decisions are being made.

Further study of mathematical models may offer valuable incentives for practical planning. The reasoning behind the construction of a model is valuable in itself. Besides, experiments with models may give information concerning important and less important matters in the planning.

The problem described has been simplified since the appraisal applies to forest resources, only. This has been done intentionally, as the forest resources often cause most inconvenience in the planning. The models, however, can be easily expanded, so that they become more realistic in survey planning for a forest property, or a combined forestry-agricultural firm.

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DISCUSSION OF RISVAND'S PAPER

Vornstad. You have shown that your problem can be tackled by both dynamic programming and linear programming. When should DP be used and when LP, and what size of problem can you handle with dynamic programming. *Risvand.* With cutting plans location problems are very important then I prefer to use dynamic programming. If co-ordination problems are central then linear programming. Perhaps the location of stands can be tackled with linear programming using decomposition, but I don't know how to do it. *Wardle.* It seems to me that the problem of location you have raised is very similar to Dr. Höfle's, use of capacity of machinery and related to economies of scale. I wonder if this could be handled in a linear programming model by parametric programming using stand location as the basis for the series of constraints. *von Malmborg.* In the model I described earlier for each stand costs are determined according to the location of the stand. If one is worried about the problem of integer solutions, only a few marginal stands will be partially cut in the solution and they don't matter very much. I might mention at this point work by Stephen Anderson in Sweden on the application of integer programming to the treatment of areas. The question of rates of interest has always confused me, and we have got rid of almost all of that by including only the value at the end of the period. Do I understand that in your case the amount of cut is given and that one is not hoping for the optimum cut. *Risvand.* The owners should decide what the cut should be and the constraints included are over a very wide range. There is a restriction on the minimum quantity removed from management block. For the rest costs should determine what should be cut and it should not be necessary to insert many restrictions. *Bjora.* The model being discussed appears to be dependent on the conditions of terrain in which we are working. The same approach might not be necessary in the conditions of another country. The approach being discussed seems to concern operational planning within the overall cutting policy of the long-term plan.

Paper 7

A SIMULATION MODEL FOR COMPARING PLANS OF MANAGEMENT ON PRIVATE FORESTRY ESTATES IN SCOTLAND

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Introduction

Most owners of private forestry estates accept the need to plan the use of their resources and predict the output of wood from their forests several years in advance. A forest is a very valuable asset; it is also a long-term investment and decisions taken now may have consequences, fifty or sixty years hence, which are undesirable because of the conditions then prevailing. Therefore it is important that the results of a plan of management should still be satisfactory if circumstances in the future differ from those assumed for the plan, or, at least, that modifications to the plan made as fresh information becomes available, should reduce adverse changes in the results.

The preparation of plans for the future management of a private forestry enterprise can be split into two parts. First there are the plans for each crop of trees, and in this study a "crop" is defined as a collection of trees of a single age and species in a compartment or sub-compartment, whose present state and future pattern of growth is uniform and can be simply described. Net discounted revenue or some similar measure provides a satisfactory criterion of "profitability" when a crop is considered in isolation from the rest of the forest and permits the statement of general rules to guide the manager to the "best" decision about that crop. Secondly there are the plans for the entire forestry enterprise and it is evident that the whole cannot be considered as merely the sum of the individual crops. There may be restrictions on the capital available for planting, the acreage to be cut each year, the permitted level of operating surplus or loss, the capital value of the forests, or restrictions imposed by such subjectively-valued factors as amenity. All of these form part of the policy of the owner and as a result the problem of selecting the best plan of management can be so complex that few owners have the resources to attempt a full analysis of the alternatives open to them. In some cases of course, the choices will be few and the solution simple but in others the choice made by the owner may be far from the best attainable.

With the advent of electronic computers the situation has changed because computers make possible detailed analysis of the problems encountered in forestry policy making and planning. The manager is relieved of the more tedious computations, and can concentrate on defining precisely the objects of management for his forest, ensuring that the information available for planning is as comprehensive and accurate as possible and on the choice of the plans themselves.

The analysis of alternative plans may be done using a model that simulates the real situation, and whose changing patterns defined and predicted in the computer copy the real choices in the forestry enterprise. So far, few such mathematical models have been built and used in forest management. This paper describes the progress made since 1968 in constructing and using such a model, the project being financed by the Natural Environment Research Council.

Requirements from the Model

Alternative Strategies

Before formulating details of the model we must decide what it has to do, for this to a large extent determines its structure and mechanism. The most important requirement is that the significance of the output information should be easily appreciated by the forest owner and manager; it is necessary also that they should understand the methods used to obtain the output information. These requirements weigh rather heavily against dynamic or linear programming, both of which have disadvantages despite their efficiency in indicating the best solutions to problems. The problem met by forest owners and managers would have to be simplified to reduce the number of variables to a manageable size. There would be difficulty in defining the relationships between the variables. Furthermore dynamic or linear programming would not provide directly all the information needed about the forest enterprise, particularly annual accounts and detailed analyses of the growing stock.

One alternative is simulation. Starting with the initial state of the forest enterprise, we can then form and use rules of change to predict the subsequent state of the enterprise as it moves through a given period of time. This copying or "simulation" of the behaviour of the forest enterprise makes possible the avoidance of most of the difficulties of a mathematical analysis. Simulation does not seek the optimum solution for a given set of circumstances and this suits the forest owner and manager quite well. Conditions fifty or sixty years hence are so uncertain that no single plan is likely to be the best in all possible or even likely situations. Provided the investigator uses the simulation model to predict the results from a wide range of alternatives, the results will form a useful basis for discussion between the planner and the owner.

Simulation has other advantages. The change in state of the model can be made to correspond directly to real-life changes in the forest and it is therefore easier to convince forest managers of its usefulness. As much detail about the forest as is desired and available can be included in the model, subject only to the limitations of the size of the computer and the cost of computer time and data preparation. The simulation model is also very flexible. If the computer programme is written in sections corresponding to the main parts of the model, then alterations and improvements to particular parts, and the addition of new ones may be done without affecting the remainder. Thus simulation seems the most suitable method for our investigation. It must be stressed however, that the model does not provide an absolute measure of the financial return from forestry for comparison with other fields of investment such as agriculture, which use different criteria of profitability.

Treatment of Uncertainty and Risks

There are of course limits to simulation. Some features of a private forest enterprise cannot be reproduced with accuracy because they depend on factors about which very little is known. It is sometimes possible to deal with the uncertainties by assigning probabilities to the events and their outcomes and simulating their occurrence by sampling at random from these probability distributions. Forest fires and windblow have been simulated in this way (Gould and O'Reagan, 1965). Subjective estimates can also be made of the probabilities of the occurrence of particular rates of growth, costs of forest operations and the prices obtained for standing or felled trees. However the number of simulation runs required to judge the average effects of these factors would considerably increase the cost of computer simulation and quickly become prohibitive. In our case it seems preferable to study certain situations and their outcomes under specified conditions, rather than relinquish control to sequences of random numbers.

Type of Simulation

There are two general methods for advancing the simulation model of the forest enterprise through time on a digital computer.

(1) **Fixed-time Increment Method.** A clock is simulated by the computer to record real time in the forest enterprise and maintain the correct sequence of alternations in its state. The clock moves on at uniform intervals and the forest enterprise is re-examined at each unit of clock time to determine whether any events should occur.

(2) **Variable-time Increment Method.** The clock time is advanced by that amount necessary to cause the next event in the forest enterprise to take place.

It is not necessary to specify the timing of forest operations decades ahead more precisely than to the nearest year and in some situations this period is shorter than is needed. It is however desirable to examine the economic state of the forest enterprise annually and so the use of the fixed-time increment method with a time period of one year becomes the natural choice.

General Description of the Model Now in Use

The flow diagram in figure 1 illustrates the sequence of steps taken in the model. The individual crops that make up the growing stock are described and the management plan for each year of the period to be examined appears as an ordered list of forest operations and their costs, to be done on the individual crops. It is possible to specify that some of the forest operations, including thinning, are done automatically at set ages of crops with constant unit costs and revenues. This reduces the task of preparing the data, but causes a loss in flexibility, for the costs and revenues cannot be varied to suit the conditions of the individual crops. Forestry Commission planting and management grants are calculated, and all fixed costs are included under the heading of overheads. The revenues, expenditures and yields of produce for each year are printed out in detailed and summary form. The annual surplus or deficit on the forest operations account is accumulated in a bank account. The output of the model includes also a valuation of the growing stock, while the capital account records the expenditure on roads, buildings, and machinery. The descriptions of the individual tree crops are updated according to the operations done on them. The accounts are presented for each year of the simulation run, to show the financial state of the enterprise when the schedule of operations for the year has been completed. Finally at the end of last year, the whole plan having been completed, the growing

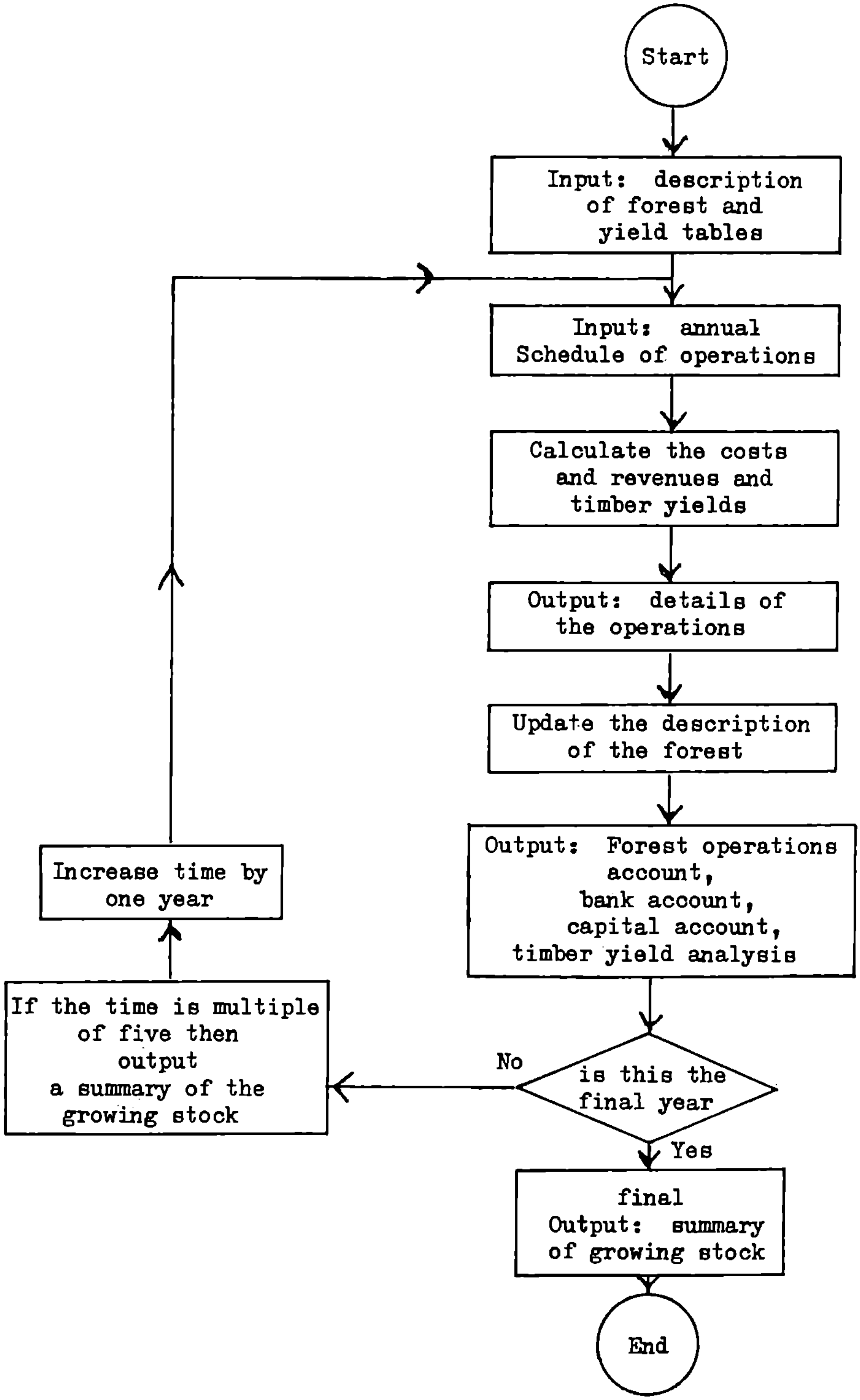

Figure 8. Simplified flow diagram of the computer program.

stock is summarised in a table giving areas by species and age class.

Although the main purpose of the simulation model is to compare alternative plans of management made using similar policy assumptions, it is also possible to use the model to forecast yields of produce or investigate the effects of different discount on interest rates, maximum overdrafts, and different prices and costs, any one of which may be critical in determining the most suitable plan for the private forestry enterprise.

Components of the Model

The Representation of the Forest

On most private estates in Scotland, the basic unit of management is the sub-compartment. Many sub-compartments contain only a single crop of trees but some may include bare ground and several tree crops thus forming a complex mixture of species and ages. If natural regeneration is present this further complicates the task of describing a complex structure. Management practices also differ from estate to estate. Individual crops of trees within sub-compartments may be treated separately or, at the other end of the scale, several sub-compartments may be grouped together to form larger units. To encompass this range of practices the individual crop of trees has been chosen as the basic forest unit in the simulation model. The condition of large areas of land can be built up from the descriptions of the tree crops growing on them and a list of these descriptions together represent the entire private forest.

The parameters used to describe the tree crops must, of course, include age and species. The density of stocking is also important because of the very wide variation encountered on private estates. But most important of all are the parameters used in the model of tree growth. The growth model must predict intermediate yields throughout the life of a tree crop and the yield at final felling. These predictions are essential not only for assessing future production of wood and the associated revenues, but also because a valuation of the tree crop is an integral part of the simulation model and must take account of future expected yields of produce.

The Forestry Commission Yield Tables satisfy these requirements and have the added advantage that only one parameter is required to determine the growth pattern of crops of a given species and age. The "Yield Class" as it is called, is based on a relation of height to age. However this economy of description is accompanied by some drawbacks. The Yield Tables present average figures based on sample plots established in even-aged crops throughout Great Britain and so are not always directly applicable to particular tree crops. Nevertheless, by adjusting the Yield Class and the stocking to bring them into line with local conditions it is possible to describe adequately the standing volume and potential growth of most even-aged pure crops and simple mixtures from the thinning stage onwards. Young crops not yet at the age of first thinning present a more difficult problem because the trends of height growth are difficult to determine and the estimates of Yield Class are therefore less precise. The use of local experience and especially of the performance of older crops in a similar environment are of great assistance in solving these problems.

Two-storied mixtures consisting of a mature crop which has been underplanted are treated as two separate crops and a crop must also be sub-divided into two parts whenever an operation results in two ages of trees—as for example when part of a crop is felled or part of an area previously occupied by a single crop is planted. The occurrence of each sub-division is recorded in the output from the simulation model but care has to be taken to ensure that subsequent operations apply only to the crop intended.

The problem of making a suitable model of the growth of complex mixtures of crops has not yet been solved and requires special investigation. The Forestry Commission Yield Tables are not applicable and so special yield tables must be constructed. Estimates of the present standing volume and rate of growth for each constituent of the mixture and the assumption of a linear increase in volume may predict the increase in volume accurately enough for short periods but the results of different treatments of the crops over longer periods will be little more than guesswork. Consequently it is difficult to make valuations on the same basis used for simpler types of forest.

Because of this lack of knowledge of patterns of growth in uneven aged mixed crops it is not yet possible to make a standardised provision for them in the simulation model and each case has to be considered individually. If no suitable growth model can be constructed then these crops must for the present be excluded from the simulation.

The predictions made from Yield Tables assume a standard thinning regime and a five year thinning cycle. The consequent inflexibility is not as serious as may appear because little is known of the response of tree crops to lengthened or shortened thinning cycles or to no thinning at all. The Yield Tables are believed to be accurate enough for most of the thinning regimes encountered in practice. Some allowance for extremely heavy thinnings might be made by lowering the stocking and/or Yield Class of a crop at the time of first thinning. But if it is desired to assess the effect of these treatments throughout the forest enterprise it will be easier to incorporate their estimated effects directly into Yield Tables.

To sum up, the parameters used to describe the tree crops comprising the forest growing stock are:

Area of the forest unit containing the crop, including roads, rides and unproductive areas.
Age.
Species.
Stocking—expressed as a percentage and including an allowance for roads, rides and unproductive areas.
Yield Class.
The number of the forest unit in which the crop is situated.

Each crop is also recorded as being Dedicated or Approved* under the Forestry Commission scheme for planting and management grants (Forestry Commission 1956).

(NOTE*

Dedicated Woodland: The owner enters into a legally binding agreement with the Forestry Commission, under which he undertakes to manage his woodlands for the main purpose of timber production in accordance with an agreed plan of operations and to ensure skilled supervision.

Approved Woodland: The woodland is managed in accordance with a plan of operations approved by the Forestry Commission.)

Forest Operations

The following operations are incorporated into the model:

Preparation of ground for planting, including ploughing, draining and similar operations.
Fencing.
Planting.
Weeding and beating up.
Cleaning.
Brashing.
Thinning.
Felling.
Road making.

All these operations appear by name on the output from the model. Other activities which can be called forest operations and which are not included among the categories listed above are assigned to one of them. Thinning and felling may result in revenue as well as expenditure whereas only expenditure is anticipated for the remainder of the operations, but it is simple to vary this.

As already noted the tree crop is the basic unit of the forest enterprise and those forest operations which create or eliminate a crop and alter or make use of the parameters used to describe the tree crop must be applied to the appropriate crops. Thus a thinning or felling in a mixture of two species is specified as two separate operations, one to each crop. On the other hand fencing is allocated to a particular tree crop only to identify the expenditure in the output from the model.

In a large private forest enterprise there may be several hundred tree crops and the preparation of a schedule of operations over periods of ten or more years would be a considerable undertaking. To reduce this task, the operations in the simulation model have been divided into two parts.

Optional forest operations are specified individually in the annual input instructions and are done only if sufficient money is available. Mandatory forest operations are done automatically to the whole of the tree crop at stated ages. It is assumed that enough money will always be available for these operations, if necessary by withdrawals from the bank account.

The two categories of operations are not exclusive. Weeding and beating up may be mandatory for crops two and three years old, but might also be specified optionally for a four year old crop. If it is desired to exempt several crops from this latter operation, it may be specified optionally with the equivalent negative cost to neutralise the cost of the mandatory operation in the forest operations account.

(*a*) *Optional Forest Operations*

Four main parameters are used to specify each operation:

(1) Operation code—each type of operation is assigned a numerical code.

(2) Unit cost—the unit for most operations is area in acres, but for fencing and road-making the unit is length in yards and for thinning and felling it is the Hoppus cubic foot. The cost per unit includes all expenditure directly attributable to the operation.

(3) Size—this states the magnitude of the operation in terms of the chosen unit.

(4) Crop number—the number of crop in which the operation is to be performed.

For planting the species, stocking and estimated Yield Class must be specified, and for felling and thinning the method of sale of the produce.

Total operation expenditure: the total expenditure E is calculated as follows:
for fencing, preparation of ground, road making, $E = \text{Size} \times \text{Unit Cost}$.
for planting, weeding, brashing, cleaning, $E = \text{Size} \times \text{Unit Cost} \times \text{Stocking}$.
for felling and thinning, $E = \text{Yield of Wood} \times \text{Unit Cost}$.

Operation priority: the optional forest operations are assigned to three sections 1, 2 and 3 which are attempted in order within a year's schedule. Within each section the operations are also arranged in their order of priority. The optional forest operations are assigned to their sections as follows:
Section 1—operations during the current year which it is expected will provide a net income, that is felling and thinning. Each operation must be preceded by any associated road-making or other capital expenditure.
Section 2—operations which have been postponed from a previous year due to lack of capital. All types of operations may appear in this section.
Section 3—operations in the current year (other than felling or thinning) which are expected to result in a net expenditure.

This system of ordering the selection of the section is intended to ensure first that the order which the operations are put into the model is maintained when desired (Mode 1) and second that when a change in the order of the operations is allowed (Mode 2), the sequence of operations for each crop is unaltered.

(*b*) *Mandatory Forest Operations*

In the model all types of forest operations except felling, planting, road-making and fencing can be specified as mandatory. A revenue and expenditure account is calculated for thinnings, depending on the type of sale, but the other operations will result only in expenditure. Two main parameters are used to specify each mandatory operation:

(1) The unit cost—has already been defined on page 57. The unit cost remains unaltered throughout a run of the model except for a fixed percentage change annually.
(2) The age of crop at which the forest operations is to be performed. For thinning, this is the age at which thinning begins, and a different age is allowed for each crop. In the model crops can be exempted from mandatory thinnings by making their ages greater than the ages of all the crops at the end of the run. The Yield Tables assume a regular five year cycle and this is also assumed for the crop valuations. If a different thinning cycle is chosen then yields from thinnings are estimated as proportions of those given in the Yield Tables.

If a tree crop is to be felled before the end of the run of the model its age at felling is recorded so that the mandatory thinnings may cease within ten years of the date of felling.

Total Operation Expenditure: the expenditure E is calculated as:

crop area $\times$ unit cost $\times$ stocking.

The Calculation of Revenues from Yield of Produce

As already noted the estimates of yields of wood from thinning and felling are based on the Yield Tables of the Forestry Commission which give thinning and felling yields in Hoppus cubic feet per acre to top diameter of 3, 7 and 9 inches (now m^3 per hectare to 7, 18 and 24 cms top diameter). The values given in the Yield Tables are multiplied by the area of each crop and the stocking given in the description of the crop to provide the forecast of yield. Yields at intermediate ages are obtained by linear interpolation from the values given in the Yield Tables.

For each of the three size categories of produce there are three prices per hoppus foot corresponding to sale standing, at roadside or delivered to the customer.

These prices are the estimated revenue to the estate from the whole harvesting operation but they exclude any large capital outlay on special machinery or road improvements made before a particular harvesting operation commences, and the cost of which should not be attributed entirely to that operation.

Overheads

Overhead costs are calculated annually as a flat rate per acre and may also be varied annually by a fixed percentage. They include general estate maintenance, office expenses, supervisory charges, the provision of housing and pensions for permanent estate workers, travelling expenses, and other items of expenditure which are incurred whatever the policy for the forest.

In certain circumstances, additional expenses may cause further increase in the basic rate for overheads. (See below.)

Forestry Commission Grants

Areas "dedicated" to forestry are eligible for the planting grant and it is assumed that this is paid at the end of the year of planting. It is also assumed that the management grant is paid annually.

Areas of "approved" woodlands are eligible only for the planting grant and it is assumed that this is paid in two parts, three quarters at the end of the year of planting and the remainder five years later.

Labour Requirements

As a guide to the manager of the labour requirements, an estimate is made each year of that portion of the total expenditure on forest operations attributable to labour charges. This is done by assigning to the labour charges a fixed proportion of the cost for each type of operation and accumulating a total charge over all the operations done in a year. This helps the manager to decide when it is necessary to increase or reduce the labour force or engage contractors to fulfil the schedule of operations.

Valuation

The crops are valued annually by calculating discounted net revenues using the prices for a standing sale and assuming that tree growth and thinning accord with the Forestry Commission Yield Tables. The age of the final felling is varied according to the discount rate chosen. To prevent abrupt changes in the annual balance sheet, the income from the removals of the year is subtracted from the total valuation before it is entered in the balance sheet. This method of valuation has several desirable properties. The value of a crop increases steadily with age. A planting which is unprofitable by the criterion of net discounted revenue eventually appears as an overall loss in the balance sheet because the future revenues accounted for in the valuation will be more than offset by the future costs. Furthermore, adherence to the optimum felling date will maximise the overall profit over a rotation by the criterion of net discounted revenue.

Accounts

The annual accounts give the information necessary for the comparison of management policies and also detailed lists of items of expenditure and income.

(*a*) *The Forest Operations Account*

The revenues and expenditure from the forest operations, overheads and government grants are combined to produce an operating profit or loss, and the bank account is amended accordingly.

(*b*) *The Bank Account*

This represents the cash available to the forest enterprise. If there is a positive cash balance interest is added annually and if the balance is negative, the interest on the overdraft is subtracted.

The initial cash balance and the maximum permitted overdraft are put into the model. The overdraft may be positive or zero (so that no overdraft is available) or negative (which fixes a minimum lower limit for the cash balance).

The bank balance at the end of the year of simulation of a given plan of management is one of the criteria used to judge the success or failure of a management policy.

(*c*) *The Growing Stock Account*

This records the annual changes in the valuation of the forest crops.

(*d*) *The Capital Account*

The making of new forest roads increases the capital value of the forest. An appropriate entry is made at cost in the capital account and remains unaltered in subsequent years. The costs of maintaining new roads is calculated as a fixed percentage of the capital value and added to the charge for overheads.

Similar reasoning is applied to expenditure on new machinery (as distinct from the renewal of existing equipment). Such items are added to the capital account at cost and the running changes are included in the costs of operations together with a fixed annual percentage for depreciation of the capital value.

(*e*) *Taxation*

Taxation is not included in the model at present. If an estate has woodlands under both the Schedule B and Schedule D categories of taxation* they will have to be treated separately in two different runs of the simulation programme. The inclusion of provisions for the effects of taxation is part of the future development planned for the model.

(NOTE *

Schedule B Woodland: Tax is paid at a flat rate/acre/annum irrespective of the amount of timber sold.

Schedule D Woodland: The profits are calculated for assessment in the same manner as for a trade. Tax relief on income from other sources may be claimed on losses. Relief is allowed in respect of capital expenditure in some cases.)

The Model in Operation

It is now timely to describe how the components of the model are linked together to form the whole. There are two alternative modes for operating the model which differ in the action taken if current activity requires an expenditure greater than the amount of money available. Mode 1 assumes that the current and all remaining activities for the year are postponed until sufficient income is available. In Mode 2, operations for a specified number of years in the future are held in the computer store and a search made among them for one which can be brought forward to the current year and produce the required income. This feature and the consequent slight changes in the input data are the only ways in which Mode 2 differs from Mode 1. It is therefore sufficient to describe the first mode of operation.

Mode 1

The major items in the first group of input data are the description of the forest and the data on outturn of produce contained in the Yield Tables. Using the wood prices for a standing scale, the programme calculates for each Yield Table a table of discounted revenues which forms the basis of the crop valuations. The second group of data specifies the operations for the first year and includes a list of crops to be "Dedicated" or "Approved" during the year. If any planting or ground preparation is to be done in a compartment not previously under forest, the area under management is increased accordingly.

The forest operations account is opened with an estimate of fixed annual costs and the overheads (see page 58) followed by the cost of the mandatory forest operations (see page 58). Then the optional operations are done section by section the position of an operation within a section indicating its priority. The output from the model consists of the "crop" number, the size of each operation, the serial number of the thinning, the species concerned and the income and expenditure. The yields of wood are presented by the three top diameter classes of the Yield Tables and by species.

If the cost of an operation is greater than the money available (including that in the bank overdraft), then one of two paths is taken according to which section the current operation belongs.

Path A for Section 1. The remaining operations in Section 1 are transferred to the end of Section 2 and an attempt made to carry out the less costly operations. If a capital shortage recurs then Path B is taken.

Path B for Sections 2 and 3. No more operations may be done during the year because the order of priority must be strictly followed and so the operations in Section 3 are transferred to the end of Section 2 and control is transferred to the next part of the programme.

A running total is kept of that part of the expenditure attributable to labour costs so that an estimate may be made annually of the manpower required to achieve the plan of operations. The purchase of machinery increases capital assets and is recorded in the capital account. The building of new roads is treated similarly and this also automatically increases the overheads to cover the additional cost of maintenance.

The programme of operations for the year ends when either all the specified operations have been completed or a shortage of capital makes necessary the steps described under Path B above. The income from Forestry Commission planting and management grants is now entered in the forest operations account since the money is paid at the end of year and is not available to finance the operations of the current year.

To summarise the effect of the year's operations and permit comparisons with alternative policies, the following outputs are provided:

(1) The balance of the forest operations account.

(2) The income from thinnings.

(3) The value of the increment of the growing stock (which is the increase in crop valuation less the income from thinnings). This calculation is not made in Year 1.

(4) The balance of the bank account.

(5) The balance in the capital account under the headings of Growing Stock Valuation, Roads and Machinery.

(6) The cost of labour for the year.

(7) A summary of the total yields of wood for the year by three top diameter classes and groups of species.

The valuation tables are now updated by the fixed annual change in prices and those input values which change annually by fixed percentage are amended ready for the next year.

The cycle of operations beginning with the second group of input data is repeated for each year of simulation. Every five years the growing stock is summarised by species and age classes using a ten year class-interval, so that the impact of the plan of management on the age–class structure of the growing stock may be examined.

Mode 2

The input data for the first year differs from that in Mode 1 because the operations for the first N* years are fed in. (*N is the number of years by which an operation may be brought forward to pay for current expenditure. See page 58.) In subsequent years the operations for the year $N-1$ ahead of the current year are needed to replace the completed year and ensure that N year's operations remain in store. Mode 2 also differs from Mode 1 in the action taken if for a particular operation the overall expenditure exceeds the capital available. As already noted, a search is made among the Section 2 lists for each of the next $N-1$ years in turn for some felling or thinning which can be brought forward to the present to provide the necessary income. There must of course, be sufficient money available to cover costs of roads or machinery required for a felling. If the search is successful a return is made to the current year's operations where they were broken off. Otherwise no more operations may be done during the current year and the sequence of events is the same as that described when this situation occurs in Mode 1.

An example of the printed output for a year is shown in figure 9.

The Comparison of Management Plans

The starting point for all plans of management is the present state of the enterprise and when comparing

Cpt	Item	Income	Expenditure	Size	Stocking	Species	Volumes 3 - 7	7 - 9	>9
36.2	Overheads		12700						
	Fire insurance		1200						
36.2	Weeding 1		50	25	0.85				
51.0	Cleaning 1		65	7	0.65				
86.0	Weeding 2		30	15	0.85				
169.0	Weeding 3		84	42	0.85				
61.3	Thinning 3	2424	0	24	0.85	SP	7038	102	0
62.0	Thinning 4	2145	0	141	0.85	SS	40469	587	0
71.4	Felling	8561	0	25	0.65	SP	2775	13825	49700
80.0	Felling	4869	0	12	0.85	NS	2014	10572	26856
71.4	Planting		660	25	0.85	SS			
80.0	Planting		317	12	0.85	LP			

Man. grants	3421
Planting grant	857
Total income	22277
Total expenditure	15106
Profit	7151

Crop value	894321
Income from thinning	4569
Value of increment	2164
Bank Account	·47210
Capital Assets	
Roads	8740
Equipment	1200
Labour cost	3100

Summary of Yields		3 - 7	7 - 9	>9
Thinning	Larch	0	0	0
	Spruce	40469	587	0
	Pine	7038	102	0
	Other	0	0	0
Felling	Larch	0	0	0
	Spruce	2775	13825	49700
	Pine	2014	10572	26856
	Other	0	0	0
Felling + Thinning	Larch	0	0	0
	Spruce	43244	14412	49700
	Pine	9052	10674	26856
	Other	0	0	0

Figure 9. Example of annual output from the model.

alternative plans it is necessary to take account only of those changes brought about during the same period of time. Other changes, such as increases in land values, will be common to all management plans and need not be considered.

Each owner has his own policy criteria for judging the success of alternative plans of management on his estate, and these criteria are rarely solely economic. Nevertheless the detailed output information provided by the simulation model is sufficient to enable comparisons to be made, whatever the criteria used.

Progress to date

The simulation model has been used for three large private Forestry estates in northern Scotland. On two of these the records of the growing stock were not sufficiently detailed to be used directly in the description of the forest and sample surveys were made to provide the necessary information. A linear growth model was used to construct Yield Tables for the complex crop mixtures on one of the estates but this procedure was satisfactory only because predictions over the short time period of fifteen years were required.

Several felling and planting schedules were compared using different discount rates. The owner's policy in all three cases was to sustain a surplus on the forest operations account without decreasing the capital value of the growing stock.

Future Developments

We now intend to use the simulation model for other forest estates in Scotland. However, because complex mixtures of crops are likely to be encountered again, it is desirable to attempt to replace the present general growth model by one which will represent these mixtures and be capable of predicting responses to a wider range of treatments than has been possible hitherto.

The simulation model will be modified so that each crop can be allocated to either Schedule B or D taxation. Separate accounts will then be kept for crops under the different taxation schemes.

A possible extension of use of the simulation model is in forecasting yield of wood from a group of private forest enterprises within a region taking account of the varied objects of management of the different owners. The achievement of a sustained or increasing yield would encourage more favourable marketing conditions, but would require considerable co-operation between estates. This could be the most useful line of development in the application of computer-based methods to aid the private sector of forestry. Not only would the forest owners themselves benefit but so would the whole country.

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DISCUSSION OF SAYERS' PAPER

Jackson. You mention that roads construction increases the capital value of the forest. I am not quite clear where this stops or whether it is a good idea. Someone with a maximising type of mind might decide to maximise the quantity of roads. *Sayers.* It seems to me that if you do put in roads for extraction you increase the capital value and it is assumed the increase in value equals the cost. *Wardle.* We have concluded in the Forestry Commission that the value increase resulting from roading is the change in present value of the crops, resulting from the provision of that facility. *Morgan.* What degree of sensitivity have you found to changes in assumptions? *Sayers.* Surprisingly little, it turns out that it doesn't very much matter what plans are carried out. *Vornstad.* You defend the use of simulation, provided the investigator uses the model to predict the results from a wide range of alternatives, the results form a useful basis for discussion between the planner and the owner. It seems to me that the operational researcher has an obligation to make the owner ask the right questions to formulate policy. *Sayers.* In this case the policy is taken as given. *Vornstad.* The owner may not be very ready to formulate policy. Insofar as it is given the model may give absurd answers and this is valuable output forcing people to re-consider policy. *Sayers.* It is possible that the owner has no financial policy. *Vornstad.* Is the owner aware of what he wants and that he may be acting irrationally. *Sayers.* We make the owner aware of cost. For example, an estate with mixed ages and species may be an attractive forest, we try to make the owner aware of the financial result of alternatives such as cutting it down and planting up with Sitka spruce. *Vornstad.* Using linear programming the dual values show the cost of imposing such constraints as maintaining the mixed forest. *Duerr.* The argument can surely be questioned that managing forests for the sake of having forest or making the trees look prettier is necessarily irrational unless one defines rationality as the effort to maximise present value and this can be questioned. I see nothing irrational about the estate owner Robin Sayers is describing; if he sees it that way presumably he has a set of values against which he is weighing various things. *Vornstad.* I agree it is perfectly rational too. If, however, he thinks he is maximising capital value at the same time as he is beautifying his estate then he is acting irrationally. It is most important that the operational research worker makes the position clear. *Duerr.* The operational research worker has to fit his model to the owner's circumstances. He doesn't have an optimising model here as he doesn't have a system for weighing the values that must be taken into account. He has, however, simulation, letting the owner look at possible consequences of alternative courses and allowing him to decide for himself on the basis of his own set of values what is best. *Vornstad.* I fear the simulation model makes it too easy for him while the optimising model forces him to look at the values. *Warren.* Is there a danger in looking at individual estates in isolation. Surely there is going to be interaction between the various estates which will upset the values calculated. *Sayers.* It is very unlikely that all or many of the 900 estates will be dealt with. We have the capacity for only a few estates so there is little danger of interaction. *Bjora.* We are trying to make long-term investigations of the consequences of short-term decisions. The simulation might be used to investigate these consequences. The short-term optimum depends on assumptions about the future, the future will depend on short-term decisions. *Von Malmborg.* You say it is easy to explain this type of model compared with linear programming. This really isn't a difficulty since we present our plan made using LP in the traditional way. The difference is only how you formulate the plan, but it doesn't matter how you arrive at the result. I enjoy TV every night, but don't have to worry about how the TV is constructed. A possible difference from the linear programming model is that when you choose an alternative implicitly you are deciding policy. *Vornstad.* But you don't realise what policy you have chosen. *Wardle.* What seems to me important in this simulation model is the attempt explicitly to draw pictures where previously there was only mist. In some cases the optimising component is not very important and one may be able to use the simulation more or less on line without introducing an optimising process as in the case of the simulation of nursery production we are using. The danger is that the picture is drawn arbitrarily. Likewise, though linear programming selects from a number of possible states relevant states may have been left out. Though one may question the adequacy of these models I suggest the thing that is important is that someone has started an explicit exploration of the decision space and started to draw boundaries. *Paillé.* I should like to raise a question of scale. My model dealt with the tree, Risvand's with the stand, von Malmborg and Sayers' with the small enterprise, all essentially small scale. Supposing we have an optimum for each of the 900 then we have to have another planning tool or perspective over the whole area which brings together all the components. This sort of model doesn't seem to exist. I doubt if we can take the same approach, and one questions whether our efforts are as useful as they could be to government for example. *Jackson.* In my view, the choice of simulation or linear programming is more a question of what you can explain by independent variables. If you cannot get the information, you cannot simulate. *Sayers.* The thing that led me to choose simulation was the amount of detail I wanted to carry.

Paper 8

METHODS OF OPERATIONAL RESEARCH AND PLANNING SELECTED OPERATIONS IN FORESTRY

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From a cybernetic point of view a forest enterprise may be considered as a relatively isolated system. Even if inputs to this system are much more extensive and differentiated than outputs, the system proper forms a rather homogeneous whole of a dynamic character. After simplification this system may be represented both graphically and with the help of matrices in a manner generally valid for forest management purposes.

Dynamism of the model through time may be adjusted in a way used frequently in forest management, namely by relating the construction of the model to a limited period. This simplification is not prejudicial to simulation of economic phenomena, for the necessity of proportionality of production processes has been and obviously will remain one of the main principles of forest management.

A general scheme of a forest enterprise is described in the form of matrices in figure 10. The model shows clearly that after a suitable simplification of both elements and structure the inner structure is not the main problem. A much greater problem is the determination of technical and economic coefficients of the structure. It has to be stated openly that at present most of these coefficients are not known and the objective construction, first of all, of inputs and outputs of elements such as "stand tending" and "thinning", is very dubious having regard to the low degree of knowledge in the field of biological characteristics of forest growth. Not the whole forest production processes, however, requires long-run investigation. Results of analysing inputs and outputs over a short period, which may be easily checked, may be provided for such elements as "production of seedlings" and "stand formation" where the time difference between inputs and outputs is only about 5 years.

Working out technical coefficients for existing production and economic conditions in a desirable number of variants would make it possible by methods of operational research and with the help of a computer to solve many problems. Thus one might find the optimum technical and economic plan for an enterprise by means of structural analysis, or the optimum organisational structure by means of simulating organisational patterns.

In literature dealing with forestry economics the approaches to applications of methods of operational analysis vary considerably. Our approach to this problem is based on the following principles:

(1) first of all to optimise, by means of selected methods of operational analysis, individual production activities,

(2) to devise, on the basis of preoptimum solutions established in this way, the first variant of a plan of technical and economic activities and to optimise the whole system having regard for mutual relations of individual activities and the aims which are to be attained,

(3) to modify the detailed scheme of partial activities according to the results stated under point 2,

(4) to draft the final proposal for an optimum plan of development of a particular organisation for the next cycle extending for 3–10 years,

(5) to prepare a detailed plan of technical and economic activities of forest management organisation for the next year,

(6) to evaluate at the end of each year by means of economic analysis both the model and its practical application in the last period and to prepare a revised plan for the rest of the long-run planning cycle.

For the individual activities it is expedient to use straight forward methods of operational analysis, mostly linear programming, even if it is necessary to reckon with simplifications. These simplifications concern mostly the form of a function and the problems of determination. With regard to the present state of information linear functions may be substituted for functions which have non-linear form and many phenomena may be treated as deterministic, even if they show stochastic features. The present state of application of methods of operational analysis in forestry gives evidence that this approach has been realistic, though one can only welcome attempts to apply other programming methods to forest management, especially dynamic programming, theory of games and non-linear programming.

According to the practical experience of the applications of operational analysis which have been tested in this country, a rather good effect may be

Building activities 62	Forest techn. ameliorations 61	Hunting production 55	Agricultural production 54	Other associated prod. 53	Associated wood prod. 52	Associated forest prod. 51	Expedition 43	Cross-cutting OS 42	Cross-cutting OM 41	Haulage 32	Skidding 31	Other transportation 30	Logging 24	Thinnings 23	Stand tending 22	Stand formation 21	Production of seedlings 12	Seed production 11		
												$a_{30,11}$	$a_{24,11}$					$a_{11,11}$	Seed production	11
$a_{62,12}$						$a_{51,12}$						$a_{30,12}$	$a_{24,12}$				$a_{12,12}$	$a_{11,12}$	Production of seedlings	12
												$a_{30,21}$	$a_{24,21}$			$a_{22,22}$	$a_{12,21}$		Stand formation	21
						$a_{51,22}$									$a_{22,22}$	$a_{21,22}$			Stand tending	22
														$a_{23,23}$	$a_{22,23}$				Thinnings	23
													$a_{24,24}$	$a_{23,24}$					Logging	24
																			Other transportation	30
$a_{62,31}$			$a_{54,31}$										$a_{24,31}$	$a_{23,31}$	$a_{22,31}$				Skidding	31
$a_{62,32}$			$a_{54,32}$						$a_{41,32}$		$a_{31,32}$								Haulage	32
											$a_{31,41}$								Cross-cutting OM	41
$a_{62,42}$										$a_{32,42}$									Cross-cutting OS	42
								$a_{42,43}$											Expedition	43
						$a_{51,51}$						$a_{30,51}$	$a_{24,51}$	$a_{23,51}$	$a_{22,51}$	$a_{21,51}$			Associated forest prod.	51
$a_{62,52}$					$a_{52,52}$					$a_{32,52}$		$a_{30,52}$							Associated wood prod.	52
												$a_{30,53}$							Other associated prod.	53
			$a_{54,54}$			$a_{51,54}$						$a_{30,54}$							Agricultural production	54
		$a_{55,55}$	$a_{54,55}$			$a_{51,55}$						$a_{30,55}$							Hunting production	55
				$a_{53,61}$								$a_{30,61}$							Forest technical ameliorations	61
				$a_{53,62}$								$a_{30,62}$							Building activities	62
	Y_{61}	Y_{55}	Y_{54}	Y_{53}	Y_{52}	Y_{51}	Y_{43}			Y_{32}			Y_{24}				Y_{12}	Y_{11}		Y

Figure 10. General scheme of a forest enterprise.

obtained by means of the above mentioned procedure without imposing requirements for a higher computing technology or special data collection. For application of methods described below, from the point of view of computing technique, in most cases a small automatic computer is sufficient and from the point of view of information flow the normal data on production, costs and yields, used in the state forests in C.S.S.R. A decentralised way of treating the basic data enables application of methods of operational analysis at the level of a forest enterprise with area from about 10,000 to 300,000 ha without having to use centralised computing facilities.

Seed Production

In production of seed we are mainly concerned with optimisating the linear function

$$f(x) = \sum_{j=1}^{n} c_j x_j \tag{1a}$$

on condition that

$$\sum_{j=1}^{n} a_{ij} x_j \begin{Bmatrix} \leqslant \\ = \\ \geqslant \end{Bmatrix} b_i \tag{1b}$$

and

$$x_j \geqslant 0 \tag{1c}$$

where x_j = amount of seed produced of j-th species.
c_j = production costs or price of j-th species per unit quantity of seed.
a_{ij} = consumption or production of i-th source of seed per unit of j-th variable.
b_i = amount of production available at i-th source.

We are concerned with maximisation of yields.

Production of Seedlings

In the case where the sale of seedlings is possible linear programming may also be used (see equations 1*a*, 1*b*, 1*c* above). Apart from the possibility of utilising manpower and purchase of material the following factors should be considered the main limiting factors: acreage of forest nurseries, the supply of first-rate seed and the possibilities for sale of seeds.

If we produce seedlings for our own consumption we are only concerned with minimising the linear cost function. The problem of optimum acreage of forest nurseries may also be solved by means of linear programming method worked out by V. Novotny.

For transportation of seedlings the algorithm for solving two dimensional transport problems, i.e. minimisation of linear function, may be used, namely:

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{2a}$$

on condition that

$$\sum_{j=1}^{n} x_{ij} = a_i \tag{2b}$$

$$\sum_{i=1}^{m} k_{ij} x_{ij} = b_j \tag{2c}$$

$$x_{ij} \geqslant 0 \tag{2d}$$

where x_{ij} = amount of seedlings transported from i-th place of production to j-th place of consumption.
c_{ij} = transportation costs of unit quantity of seedlings from i-th place of production to j-th place of consumption.
a_i = amount of seedlings destined for transportation from i-th place of consumption.
b_j = amount of seedlings destined for transportation to j-th place of consumption.
k_{ij} = coefficient indicating preference for supplies from i-th place of production to a certain consumer at j-th place of consumption.

As places of consumption we consider either stands or foreign consumers. If costs c_{ij} are replaced by total costs and the solution of the transportation problem may be extended to all sorts of seedlings and both location of forest nurseries and the serviceableness of individual deliveries from the point of view of producer may be optimised. It is possible to use even maximisation of profit in such a way that we replace c_{ij} by a unit profit z_{ij} the difference between unit costs and yields according to sorts of seedlings, localities for production and consumers.

Stand Formation

Linear programming methods may be used in determining optimum species composition where the main limitations are typologic characteristics of stands such as maximum and minimum representation of individual species and sources of afforestation material. We are concerned with maximisation of a linear function where c_{ij} is the yield of individual species. This may be extended to handle the problem of optimising costs by including costs of afforestation and protection.

For optimum timing of all operations in stand formation the method, programme evaluation and revenue technique (PERT) may be used as we are concerned with time limited tasks for optimisation from the biological point of view. It is expedient to work out some more variants of the plan so that various variants differing in the timing of critical process and usually in costs too, are available to be used according to the real state of weather.

Analogically methods of linear programming and graphic nets may be also used in further activities, having the character of production of standing timber, i.e. in *stand tending, thinning and exploitation.* In this we are concerned only with that part of forest activities which influence production of standing timber and not timber harvesting or assortment. The question only is what system of cultivating measures to choose in order to attain, with optimum costs and yields, optimum production of standing timber from both quantitative and qualitative points of view.

Even if the methods of linear programming and graphical nets provide us with directions for optimising some activities in production of standing timber, they cannot give us an answer to the question what complex system of cultivation measures to choose in order to achieve the aim in relation to the production of standing timber. Solution of this problem is, in my opinion, the most relevant and complicated of all problems in forest management. We are faced here not only with the fact that the production process in forestry is a long-term one, but also with difficulties in determining the optimum combination of assortments to produce in relation to species composition. It is a question of an extremely long-termed prognosis, with problems of determining acceptable cost and yield calculations for a long time in advance and at the same time lacking objective information concerning natural factors influencing the production of standing timber.

Although we have not enough data at our disposal for exact mathematical formulation of a given problem I shall at least try to outline generally the way to its solution.

From the point of view of these problems, their biological, technological and economic character and aims to be met we are evidently concerned with a *bionic system* which is relatively closed, dynamic and non-deterministic.

Considering all the known methods of operational analysis dynamic programming may be used in solving these problems, as it has the following characteristics:

(1) it is a decision process with many stages and with different initial stages and with a relatively limited strategic space,

(2) the process has a stochastic character influenced primarily by biological and climatic factors,

(3) it is possible to use multichain Markov processes in determining serviceable function.

It is possible for stands formed by artificial or natural regeneration, to choose according to criteria of the serviceable function, a set of acceptable strategies—cultivation systems—in rather unambiguously determined stages up to the exploitation stage, namely in several independent Markov chains. Mathematical formulation will be generally determined as a set of equations:

$$\left[g_i = \sum_{j=1}^{N} p_{ij}\, g_j \right] \tag{3a}$$

$$\left[V^n{}_i + g_i = q_i + \sum_{j=1}^{N} p_{ij}\, V^n{}_j \right] \tag{3b}$$

on condition that maximum profit is determined as a serviceable function (as a simple difference between reproduction yields and reproduction costs). In this system:

g_i ...expected profit in stage i in time t.

p_{ij} ...probability that the given system which in time t is in stage i will be in time $t+1$ to $t+n$ in stage j.

q_{jn} ...decision for transition into j-th stage.

V^i ...total expected profit from n-th process starting from stage i with the use of chosen strategy k.

q_i ...decision for i-th stage.

$V^n{}_j$ ...total expected profit from n-th process after transition to j-th stage.

The set of equations is solved for all V^i and g. For each stage i we solve an alternative to maximising function:

$$\sum_{j=1}^{N} p_{ij}^{\;\;k}\, g_j \tag{3c}$$

where k is a constant which corresponds to the chosen or accepted strategy in a given chain and g_j is expected profit in the following (j-th) stage.

This decision is considered as a new decision in stage i.

In case that (3c) is equal for all variants the chosen decision has no real alternative decision we shall use another criterion than profit (e.g. costs). Non-negativeness of p_{ij} is not a condition; in the first stage of the production process in forestry p_{ij} will be negative.

If we attain, by the new decision, only a minimum improvement, which is manifest in the whole chain, the old decision will remain valid.

If we carry out this procedure for all stages we shall determine, in this way, new strategy and new matrices $[p_{ij}]$, $[q_i]$. If the new strategy is the same as the preceding one, it is optimum.

This procedure can be justified theoretically but there is, for the time being, not enough real and objective data for determining components of the submitted mathematical formulations. In spite of this it is possible to suppose that theoretical formulation of the problems will make it possible, by purposeful modification of information flow, to ensure that the problems may be solved with the help of objective methods in the near future.

Logging

Here we are usually concerned with optimising the introduction of necessary means of mechanisation and optimising the organisation of work from the point of view of time and costs. The whole complex of logging (cutting, skidding, cross-cutting and haulage) may be, from the point of view of time, optimised by the method PERT.

The optimisation of introducing the necessary working means is also concerned with the whole range of logging activities. It is supposed that even the structure of working means (e.g. power saws, tractors, cableways, lorries and other devices) is optimised by finding the minimum function:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{t=1}^{k} c_{ijt}\, a_{ijt}\, x_{ijt} + \sum_{i=1}^{m} p_i\, x_i \qquad (4a)$$

on condition that

$$\sum_{j=1}^{n} x_{ijt} \leqslant x_i \qquad (4b)$$

$$\sum_{=1}^{k} \sum_{i=1}^{m} a_{ijt}\, x_{ijt} = b_j \qquad (4c)$$

$$x_{ijt} \geqslant 0,\ x_i \geqslant 0 \qquad (4d)$$

$$\sum_{i=1}^{m} p_i\, x_i \leqslant P \qquad (4e)$$

where x_{ijt} ...amount of i-th machine used for j-th work in time t.

a_{ijt} ...performance of i-th type of machine in doing j-th work in a given time t.

x_i ...amount of machines of i-th type.

c_{ijt} ...cost of i-th machine while performing j-th work in a given period.

p_i ...price of i-th machine.

b_j ...volume of j-th work.

P ...available financial means for buying new machines and devices.

Optimisation of utilisation of machines will obtain when the following function is minimised:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, k_{ij}\, x_{ij} \qquad (5a)$$

on condition that:

$$\sum_{j=1}^{n} x_{ij} \leqslant a_i \qquad (5b)$$

$$\sum_{i=1}^{m} k_{ij}\, x_{ij} = b_j \qquad (5c)$$

$$x_{ij} \geqslant 0 \qquad (5d)$$

where

x_{ij} ...time extent of utilisation of i-th machine for j-th work.

b_j ...amount of j-th work which must be done.

a_i ...cost for i-th machine while performing a unit of j-th work.

k_{ij} ...performance of i-th machine while performing j-th work.

This optimisation can be carried out only with those machines and devices which are able to perform some more sorts of work, or the same work in different production conditions (motor saws for logging or cross-cutting, tractors in various field conditions etc).

In this connection we are also faced with the problem of deciding whether a machine should be, after some time of operation, maintained and repaired, or replaced. The general formulation is that it is necessary to decide when to buy new devices because it does not pay any more to maintain the old one. It is a characteristic problem of dynamic programming, a non-determinist one with a limited horizon. Optimum strategy in this case is:

$$g_{n,N}(y_n) = x\int_{n=1}^{\infty} \left[\min A + g_{n+1,N}(B),\ x_{n+1} + g_{n+1,N}(y_{n+1})\right] dH(x_{n+1})(y_n) \qquad (6a)$$

on condition that $N > n \geqslant 0$ (6b)

where

$g_{n,N}(y_n)$...mathematical expectation of optimum strategy values from y_n, to $n = N$.

y_n ...age of device at the beginning of a period (year).

N ...number of periods (years).

$x+1$ ...maintenance costs at the end of a period (year).

A ...purchase value of new equipment (devices).

H ...set of a number of phases where costs play some role.

Survey and calculations are to be carried out every year throughout the whole period N and we get optimum decisions in each period.

While constructing a continuous flow of wood from the forest to depots we are faced with the problem of *optimising the supplies*. If we neglect complicated cases of big expendition depots the question is the minimisation of the function costs:

$$N(n) = \frac{n^2}{2r} G + t.\frac{n}{2} G + \sum_{k=1}^{n} x_k\, p(x_k)\, H - H_{un} \quad (7)$$

where

$p(x_k)$ ...distribution of probability of unsatisfiable demands of random variable x_k from the set x_k ; x_k represents wood supply covering probable unsatisfied demands of customers.

n ...amount of auxiliary supply (it balances irregularities in delivery of wood from available sources—stands and transportation places—to depots).

r ...amount of delivery per day in time of forming the supply.

G ...costs for keeping a unit of supply in a unit of time.

H ...losses in yields caused by failing to meet the demands of customers per unit of products (plm, prm).

t ...average length of critical period.

x_k ...unsatisfiable demands.

$N(n)$ ...costs for keeping the supply n.

This optimisation must be carried out for all groups of assortments delivered to one type of customer. In case of big wood distribution depots further limitations and probability factors are included in the solution—possible restrictions on the supply of cars, defects in equipment of depots, variants in technology of cross-cutting etc.

For complex solutions of problems of cross-cutting in depots, cross-cutting technology and expedition of wood even the theory of mass attendance the theory of queues may be used; in C.S.S.R. this problem was solved experimentally by simulation of a system with waiting and with a limited number of production places.

In optimising all sorts of *transportation*, i.e. first of all haulage from a production place to distribution depot and to customers' wood depots and delivery of wood from depots to customers, a solution of transportation problem is used. As it is quite often in forest management that wood from a forest is transported through own depots to customers depots, the three-dimensional transportation problem may be advantageously used as a minimisation function:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} c_{ijk}\, x_{ijk} = \min \quad (8a)$$

on condition that

$$\sum_{j=1}^{n} \sum_{k=1}^{p} x_{ijk} = a_i \quad (8b)$$

$$\sum_{i=1}^{m} \sum_{k=1}^{p} x_{ijk} = b_j \quad (8c)$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} = d_k \quad (8d)$$

and on condition

$$x_{ijk} \geqslant 0 \quad (8e)$$

where

c_{ijk} ...costs for transportation of wood from i-th transportation place via k-th depot to j-th customer,

x_{ijk} ...amount of transported wood.

a_i ...amount of wood on i-th transporting place.

b_j ...demands of j-th customers.

d_k ...capacities of k-th depot.

In optimising this transportation problem it is advantageous to use a combination of an approximation method by means of dissociation into two two-dimensional problems and the MODI method.

Solution of transportation problems in forest management itself may be simplified, without detriment of precision, by means of calculating the so called "transportation places" the association of several supply areas into a gravitationally homogeneous transportation regions. For the optimum distribution of depots we may use either the solution of a classical transportation problem with the use of dual variables or the distribution tasks formulated as finding the minimum of the function:

$$i, \sum_{r=1}^{m} c^m{}_{ir}\, x^m{}_{ir} \quad (9a)$$

on condition that

$$\sum_{r=1}^{m} x^m{}_{ir} + m^m{}_i = a^m{}_i \quad (9b)$$

$$\sum_{i=1}^{m} x^m{}_{ir} = k_m\,(\overline{K}_r - y_r) \quad (9c)$$

$$k_m \sum_{q=1}^{m} b_q \leqslant \sum_{i=1}^{m} a^m{}_i \quad (9d)$$

$$x^m{}_{ir},\ x^m{}_i \geqslant 0 \quad (9e)$$

where

$a^m{}_i$ ...capacity of i-th source of m-th raw material.

b_q ...amount of production demanded by q-th customer.

$c^m{}_{ir}$ ...costs for transportation of a unit of m-th raw material from i-th source to r-th production place (depot).

x^m_{ir} ... amount of m-th raw material which is to be transported from i-th source to r-th production place (depot).

y_r ... amount of products to be transported from r-th place of production.

$\overline{K_r}$ ... upper limit of capacity in r-th place.

k_m ... limit of consumption of m-th raw material.

The problem in question is the transformation of transportation problem into a transit task. In logging and building activities we are often faced with the necessity of transforming individual *means of mechanisation* to another workplace; this transformation should be, of course, as economical as possible. In solving this task we use optimisation of transportation problem by means of "Hungarian optimisation method" which solves highly degenerate transportation problems. In optimising *cross-cutting* to assortments the simplex method is commonly used and the system of linear inequalities includes limitations concerning raw wood sources and marketing. Eventually limitations on capacity and transportation may be included. The problem concerned is either maximisation of yields and profits or minimisation of costs. If we are concerned with the substitution of only two assortments we may effectively combine optimum assortment with the solution of transportation problems (e.g. in optimising production and transportation of pulpwood and wood for mining purposes).

For optimum time planning of *all logging activities* the PERT method may be advantageously used or in simpler cases CPM the critical path method. With regard to standardisation in programming these tasks on computers, the utilisation of these methods is very effective from the point of view of rationalisation of management.

From other activities which are in principle very near to industrial (associated production, building and ameliorations) and agricultural types of production, as far as optimisation methods are concerned I exclude hunting (hunting production) where for optimisation of effectivity of hunting grounds or enclosures the solution of the so called "alimental problem" may be advantageously used. It is the problem of linear programming where the number of game is maximised and limitations are concentrated on available individual components of nourishment in the controlled area. For solution simplex method is used in most cases.

Optimisation of Medium Term Plans

The above described methods of operational analysis contain the most important applications to forest management on the level of forestry organisations. In C.S.S.R. some of the simpler methods are used practically on a large scale, others are used only partly.

Further stage in the process of optimum management is based on utilisation of "structural analysis" as a method for optimising medium-termed plans of forestry organisations.

$$[X] = [A]\,[X] + [Y] \tag{10a}$$

from this

$$[Y] = [X] - [A]\,[X] \tag{10b}$$

$$= ([E] - [A])\,[X] \tag{10c}$$

where

$[A]$ is a matrix of technical (technological) coefficients expressed in costs.

$[X]$ is a vector of the total amount of production whose nomenclature corresponds with nomenclature A.

$[Y]$ is a vector of final production.

$[E]$ is a unit matrix.

By means of inverse matrix

$$[X] = ([E] - [A])^{-1}\,[Y] \tag{10d}$$

we get, with the determined limitations (concerning capacity and consumption), volumes of total desirable production.

If there is no good evidence of real costs in a desirable extent at our disposal we use a "value forming model". This model is based on material and working costs according to types of cost and types of working costs (live and materialised labour).

If we want to follow the course of production and consumption in technical units we use the "natural model". The two types of models may be combined or used separately on various levels of management.

After working out the first variant of the model we proceed further in the way outlined in the introduction. Selected methods of operational research, however, are commonly used in operational management (fulfilling plans).

This survey is concerned mainly with our approach to the use of methods of operational analysis and to optimisation of economic and technical activities of forestry organisations. It is, therefore, a methodical outline whose components partly have been and, above all, will be practically verified.

Finally I should like to draw attention to the fact that a lot of data due to many research workers were used in this work. The names of authors whose works were used more frequently are stated at the end of this paper because quotations from individual works or a list of used literature would disproportionately enlarge the scope of this work. They are: Beer, Cooper, Cruon, Chetyrkin, Doodkin, Frisch, Grevatt, Charnes, Kadlec, Kaufman, Klein, Korda, Koten, Lange, Malmborg, Makower, Nemchinov, Newnham, Novotny, Prokhorov, Rosenberg, Schreuder, Wardle and others.

DISCUSSION OF RUPRICH'S PAPER

Von Malmborg. In the main Ruprich's paper deals with different planning models for different phases of the forestry operation. But he takes up the question of the general approach to planning as well in his introduction and in discussing the complete management plan. What interests me most are the steps in planning he mentions in his introduction which fits in very well with the approach described by Wardle. He starts by optimising individual activities from the technical point of view, he then uses them in developing an integrated plan. Finally, he has the very important points about control which are also taken up by Wardle.

Paper 9

"GUIDES TO LAND MANAGEMENT", AN INTEGRATED OPERATIONS-RESEARCH PROJECT WITH A SYSTEMS VIEWPOINT

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I propose to give you the highlights of a research and development project, now commencing its seventh year of life, sponsored by the Bureau of Land Management in the United States Department of the Interior. The name of the project is "Guides to Land Management." Its aim, in briefest terms, is to produce aids to the making of wise land management and planning decisions. In pursuit of this aim, the research problems chosen for study are those which have arisen in the work of the Bureau.

The studies themselves under the Guides to Land Management Project are being made by a small group of faculty members and postgraduate students at the University of California, Berkeley, and the State University of New York College of Forestry at Syracuse. Professors Dennis E. Teeguarden and Neils B. Christiansen and I are the faculty persons principally involved.

My purposes in this paper are as follows: First, I should like to acquaint you with an interesting program of research. Second, I want to comment on the lines of study that show special promise for the future. Third, I hope, as I go along, to make clearer some of the merits and difficulties of the systems approach to resource management research.

Before I talk about the research, I should say a word about the Bureau of Land Management. This agency manages more land than does any other entity in the United States. It has responsibility for all the federal lands, ranging from forest to desert, which have not been placed in special categories such as national park, forest, wildlife refuge, military reservation, and so on. The aggregate is some 725,000 square miles, or about 190 million hectares. In addition, the Bureau has responsibility for mineral leasing on all federal lands, including submerged lands of the outer continental shelf. The Bureau's goals and values respecting its lands run across the widest conceivable spectrum of individual products and of product combinations. Thus the Guides to Land Management Project has ultimately a strong multiproduct orientation.

In some parts of the western states, the Bureau is a major forest-land manager. In the Douglas-fir region of Oregon, between the coast and the Cascade Mountains, the Bureau manages some 3,600 square miles (950,000 hectares) of generally productive forest lands. Here its aim is to produce, not only wood, but also water, forage, wildlife, and a variety of recreational opportunities. It is in Western Oregon, in the forest, that the Guides to Land Management Project has maintained its principal headquarters over the years.

Employee Training

However, one of the first breakthroughs in the Project was not in the forest, but in range management. It was early in the Project, and I had been working for some time in an effort to make a model of the Bureau's land-management system: a general model that would outline the system as a whole and yet would be reasonably simple and understandable. I had sheets of paper with lists of variables, and the sheets were accumulating, but I was not finding the answer. I then sought the temporary assistance of a range-management specialist from one of the eastern Oregon grazing districts. I was hoping that a fresh subject might give me a fresh view. The man joined me. I explained, not very clearly, what I was trying to do, and for some days we sat, brooding and attempting to communicate. Then quite suddenly, when we were in the midst of our question-asking and our drawing of little boxes and arrows, we hit upon a solution:

Range resources can be depicted as a stock at a point of time. They are changed from one point of time to the next through the flows of influences which are exerted upon them in the interim. Range management is the process of controlling these flows so that resources and resource outputs will be changed in desired directions or maintained at desired levels. This concept of management can readily be put into a dynamic, recursive model. By changing just a few of the names of variables given in the boxes, the model can be converted to one of timber management. A different set of changes would make it a model of wildlife management. The timber-management form is shown in Figure 11.

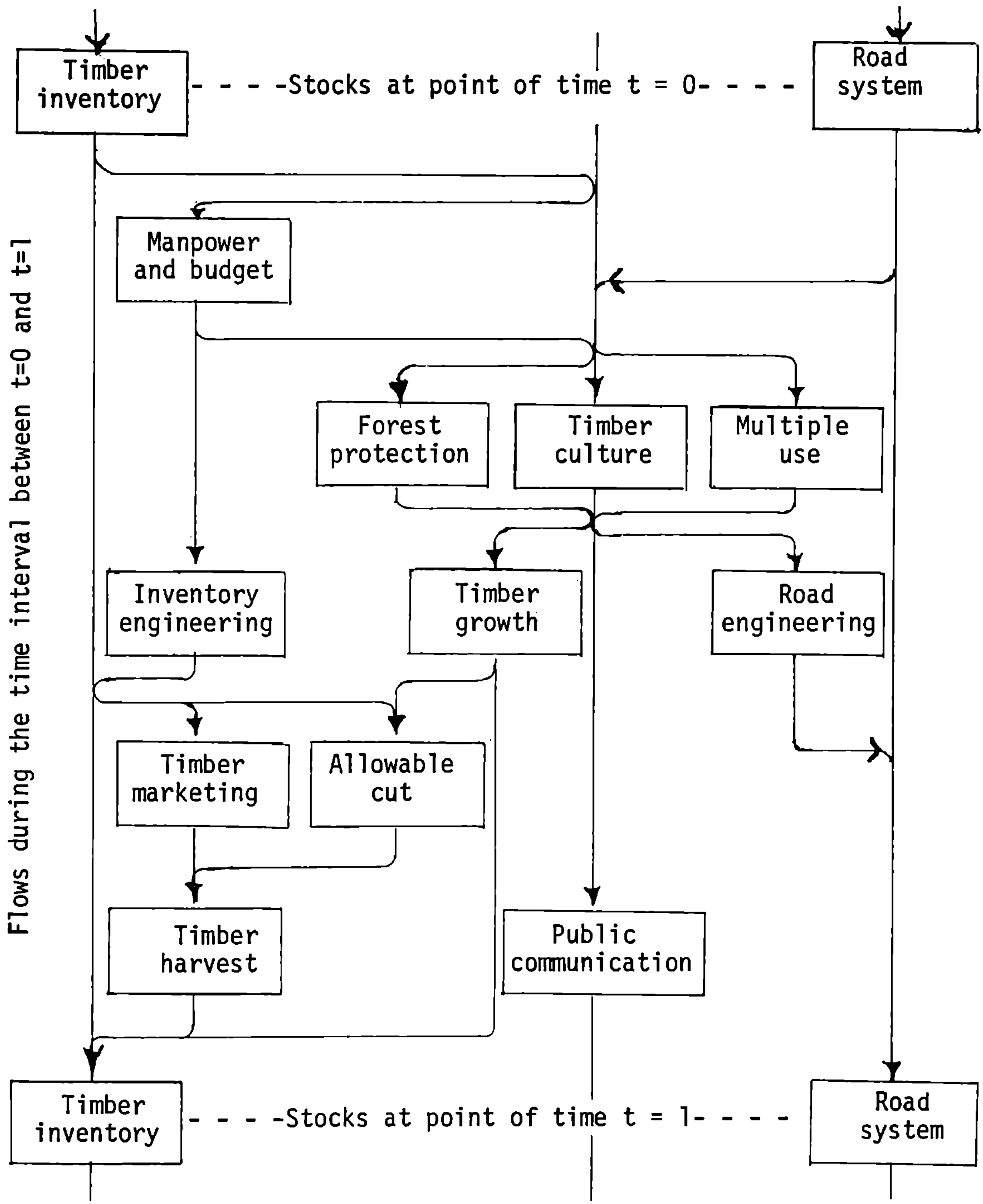

Figure 11. Bureau of Land Management's timber management system and the principal sub-systems.

Inventory engineering
- Methods of inventory
- Equipment
- Supplementary information, such as volume tables
- Carrying out inventories
- Storing and retrieving inventory data

Timber growth
- Growth and yield by site, species, age, stand density, and method of treatment
- Timber losses from destructive agencies

Road engineering
- Objectives for the road system: location and standards
- Road building, maintenance, retirement
- Planning and constructinq alternative transportation systems

Timber marketing
- Planning sale locations
- Sale preparation on planned areas
- Timber appraisal
- Advertising and conducting sales
- Administering logging and related operations

Allowable cut
- Regulatory policy
- Procedures for estimating allowable cut
- Calculation of the cut

Timber harvest
- Inventory reduction by site class, tree species, stand age and density, and location

Public communication
- Interpreting forestry to the general public
- Weighing general public sentiment and wants
- Administrative and Congressional hearings

Figure 12. Composition of the principal sub-systems.

Timber inventory
- BLM land acreage by class of use
- Timberland acreage by site quality, accessibility, location, blockage, aspect, slope, soil, forest type, age, stocking
- Quantity of timber by species, age, stocking, location
- Inventories of the forest resources other than timber

Road system
- Road mileage and location by class and other specifications
- Other transportation inventories

Manpower and budget
- Manpower by location, skill, and experience
- Training programs and opportunities
- Funds by location and function

Forest protection
- Insect control
- Disease control
- Fire control
- Wind-damage control
- Protection against other destructive agencies

Timber culture
- Site improvement, as by fertilizing or irrigating
- Tree improvement
- Regeneration
- Thinning: precommercial and commercial
- Pruning
- Salvage cutting
- Harvest cutting systems, including pre- and re-logging
- Stand conversion

Multiple use
- Systems for producing the various forest values together, on the same acres

Figure 12—*continued*

Shortly after we finished the first range-management model, my colleague left to go back to the grass country. Before he set out, he made a summary comment: "I want you to know," he said, "that during these last few days I have learned some valuable things about range management which I never knew before." This was an interesting comment. How had he acquired this learning? Not from me, surely, for I knew nothing of the subject. Evidently he had acquired it through the approach we had taken to the subject: the systems approach, with its orderly display of the interconnections among events and among programs.

Out of my colleague's parting remark grew one of the major objectives of the Guides to Land Management Project. This objective is employee training: the development, from the research models, of training materials which will help the employee to see his work as a system, to see the system in relation to those of other employees, and to see it also in the whole context of Bureau of Land Management aims and activities. Here is an approach to teaching which can be fruitful. I have since tried it in other connections.

Large-System Models

The employee-training phase of Guides to Land Management is, in a way, merely a by-product of the Project. A main product is a model itself of the Bureau's system, such as the model in Figures 11 and 12. After building this sort of model, if the research worker can apply it to a given region or area, quantify the relationships among the variables, and program the result for the computer, he then has a device for predicting the consequences of alternative courses of action. That is to say, he has, for the given region or area, precisely the sort of guide to land management which the decision maker and planner can use in order to make choices consistent with the Bureau's goals.

In the course of our work upon large-system models, we have come to recognise three types of such models, distinguished primarily on the basis of their geographic scope and detail. The three types we have designated economy models, Bureau models, and tract models (this last being illustrated by Figure 11). Let me characterise each of the three.

The economy model has the broadest geographic scope and the least geographic detail. The form that we have worked with is the Leontief input-output, or inter-industry, model. We have developed three detailed alternative plans for an input-output study of western Oregon. The Bureau's activities would be spelled out as a number of special sectors. The matrix of interdependency coefficients would help Bureau managers to predict the general impacts of Bureau policies and programs under consideration.

The second type of large-system model, the Bureau model, is conceived as representing the Bureau's operations within an administrative unit—notably a district, which is the ultimate unit. Different forms of models, such as simulation and linear programming, have been considered. A particularly interesting form upon which we are now at work is an adaptation of the Leontief model in which each Bureau function—wood, water, and deer production; protection activities; budget and finance; and so on—is represented as a separate sector. Transactions among sectors are the trade-offs necessary to operate the system. Prices are marginal rates of substitution. The interdependency matrix will, it is hoped, offer interesting possibilities, not only for predicting inter-sector impacts, but also for testing the consistency of managerial decisions and for analysing the workings of a so-called "multiple-use" system.

The third and last type of large-system model, called a tract model, is geographically the most restricted and specific. Simulation, after the pattern of Figure 11, has consistently been our approach. We have a model under way of the timber–wildlife interactions on a small watershed. Our most sophisticated tract model is one of timber management on a 16-square-mile (4,000-hectare) forest unit. The computer is programmed to accept information about such a tract in the form of inventory observations at each point in a rectangular 260-foot (80-metre) grid. This is an inventory such as might be taken on the ground or on aerial photographs. The computer is programmed then to find the boundaries of forest stands of a specified degree of homogeneity respecting forest site class and tree species, age, and stocking. Whereupon the manager specifies his assumptions about the regeneration, protection, fertilising, and thinning programs, and the computer accordingly projects the forest into the future, a decade at a time and a stand at a time, allowing for growth, losses, and cutting. The cutting proceeds at a computer-determined level consistent with sustained yield. The cutting sequence from stand to stand is based on appraisals and road-building decisions made each decade within the computer. The manager may call for maps, inventories, and accounts for points of future time and thus have a basis for deciding which of his alternative cultural programs he prefers.

Small-System Models

So far I have discussed two phases of the Guides to Land Management Project: employee training and the building of large-system models. Let me mention next a third phase, concerned with answering specific operational questions by means of smaller-system models. Such work has been the particular assignment of Professor Teeguarden and his assistants at the University of California.

Two operational sorts of question have been chosen for study. One of these is the question faced by every district manager who has cut-over areas to be reforested and who at the same time has a limited budget and supply of seedlings. Which areas shall be planted?—which seeded?—and which left in the hope of getting natural regeneration? It is a complex question. Linear programming has offered the most satisfactory answers.

The other operational sort of question arises in the forests of southwestern Oregon, where sites, tree species, and silvicultural circumstances are varied and complex. Considering both the costs and the probabilities of success, what procedure for timber harvesting and regeneration recommends itself in each of the forest situations? The question is difficult, not only silviculturally, but also because its implications extend outward from the timber stand at issue, to encompass the road system, logging plan, and other management aspects of the whole forest.

Problem Analysis

Finally, there is a fourth phase of the Guides to Land Management Project. Like the first one which I described, it is a sort of by-product of the model building and of the application of the models to management questions. This phase is problem analysis.

In the course of our work with large-system and with small-system models, we have found ourselves in continual need of data. In some cases, the data were readily available, but in most cases, a greater or lesser amount of improvisation was required in the absence of satisfactory data. Quite soon it became apparent that we were acquiring lists of information and some basis for arranging the items in the lists in accordance with their importance and their availability. That is to say, we were acquiring the basis for a problem analysis identifying and ranking the research needs of the Bureau of Land Management.

The potentialities of the Project for problem analysis are now being exploited. An explicit analysis is under way—and, in fact, nearly completed—for timber management and related questions in western Oregon. An effort is being made, consistent with the system approach, to see regional timber problems in a broad context geographically, temporally, and in terms of the forest functions.

Some Conclusions

What lessons have we learned from our work on the Guides to Land Management Project? Let me mention some which have impressed me.

(1) Even in a research project housed, as it were within an operating agency and directed specifically to the problems of the agency, great effort is required from all concerned to communicate and to understand. If the project is to succeed, members of the agency must convey their wants clearly to the research people. The latter, in turn, must explain their results clearly and even offer training in the use of the results. Frequent meetings to promote communication and mutual trust and understanding can be as valuable as the research itself.

(2) In a project such as the one at hand, which makes varied and extensive use of large computers, a well-equipped and well-administered computer centre is an essential. The Project workers need ready access to computer technicians who are familiar with the local system and with the Project. The workers can make good use of quiet, uncrowded office space at the computer centre where they can review and develop their program. Ideally, the computer facilities should be those of the sponsoring agency itself. The same agency technicians who help process the research can then work with the results in application to operational problems.

(3) To operate a simulation model of any complexity for sizeable forest tract puts heavy demands upon computer memory and time. It is the constant concern of the research worker to devise means for enlarging memory capacity, for increasing speed, and at the same time for holding down memory and processing-time requirements. As the size and complexity of models grow, the ideal form may well prove to be analytical representation, as by means of linear programming, of certain subsystems within a simulation model of the system as a whole.

(4) Experience suggests with increasing insistence that the Bureau model is the most promising single type of model and that the ideal guide to on-the-ground operations may well be the tract model into which key Bureau-model features have been incorporated. The issue here is the criterion for decision making, and the point about the Bureau model is that this is the one of smallest scope which still encompasses the major internal criteria for choosing a course of action. Take an example. Suppose that the criterion for deciding among alternative silvicultural programmes is their effect on the allowable cut. Since the allowable cut is properly determinable only for an entire administrative district, a Bureau model is the only appropriate means for evaluating the programs. If geographic detail such as provided by the tract model is required, then it will be necessary to devise a model of the administrative district in which events within the tract can be traced in their impact upon output from the district as a whole.

DISCUSSION OF DUERR'S PAPER

Von Malmborg. Policy formation was not your concern in this project. *Duerr*. One of the first things we tried to learn was what were the objectives. We did not succeed. We don't know what the objectives are, which is somewhat of a handicap. This has led us to simulate rather than to optimise as we don't know what to optimise—to show the manager the consequences of alternative courses of action in such terms as he can decide which he likes best. *Vornstad*. Is there a connection between your work and cost benefit analysis. *Duerr*. There is a relationship to watershed planning. The PPBS system (Planning, Programming and Budgeting System) was devised to make government agencies account in cost benefit terms for the funds they asked of Congress. The work we are doing could contribute to the presentation of the justification in cost effectiveness terms. Such studies as those of the optimum method of regeneration would be invaluable supporting evidence in putting the case for regeneration funds, but they have not been used in this way. What we have not done is to produce any way of telling what total should be spent—only supporting information on priorities. We had this in mind in the Leontief model. Through this the Bureau can show broad rationale for the policies that it proposes in terms of impacts on the economy at large. This would be a very orderly approach to requests for funding. *Risvand*. You say you could use the model to test the consistency of the decision maker. Perhaps the model is too simplified and perhaps the tests should be directed at the validity of the model. *Duerr*. The data we put into the Leontief model comes from the decisions made in the past. We interpret the decisions in terms of intersector flows. For example, suppose that when timber is cut a 50 yard strip is left to protect a stream from siltation, this can be interpreted as a flow from the timber sector to the fisheries sector. One may derive a matrix of coefficients from a series of such decisions and indicate inconsistencies in past decisions. This is without any knowledge of the motivation behind the decision or any intention to direct. *Bjora*. We might ask instead of "do we know the objectives" "in what field do we expect to find objectives", e.g. profitability, timber supply, employment and then try to describe or predict future development—what profitability, what timber supply, what employment do we get. *Duerr*. Yes. Maximisation of allowable cut is an aim—within certain restrictions and there are aims connected with employment. Still we haven't been able to get anyone to say what the objective in the field of forest management is specifically. *Bjora*. Of course one has experience of the decisions made before. *Duerr*. Difficulty attaches to fields other than timber as no value system exists. They know they want to take beauty into account but have no means of being precise about it.

Paper 10

OPERATIONAL RESEARCH AND THE DESIGN OF A MANAGEMENT CONTROL SYSTEM FOR A FORESTRY ENTERPRISE

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Introduction

Over the past two years a small team in the Forestry Commission has been concerned with the design and introduction of a revised system of financial control within the enterprise. In this paper the approach to the design of the system and some features of the design itself are discussed. It is thought appropriate to the discussion of this group, first because the key to the construction of the system was the establishment of a model to which the components of the system could be related as they were developed. This was particularly important as the components had to be developed as they were required to be introduced, rather than it being possible to develop all the components and to view them as a complete system before the system was introduced. The paper is thus relevant because it concerns the application of operational research. It is also relevant because one of the aims in the design of the system has been to conform with requirements of managerial economics.

The project was set up because it was believed that management could be more effective in achieving the objectives of the forest enterprise, or at least that it could be better equipped to demonstrate how effectively it was achieving those objectives. The assumption was made that if the roles and objectives of managers were clearer to them and the information system designed to support them in performing their roles, they would be likely to be more effective in pursuing the objectives of the enterprise and would certainly be better able to demonstrate the results achieved. The aim and outcome of the project has been to establish a system of channels for the information flows necessary to support effective decision making. The quality of the decisions made depends on management and the use made of the system.

The remainder of the paper divides into two parts, The first concerning the approach to design deals with the establishment of the model system, establishment of the functions that had to be performed and identification of these functions with particular managers in the organisation, and particular features that had to be borne in mind in the performance of these functions. The second part concerns the system established and describes certain of the reports to show how features from the models have been incorporated in the formal procedures.

Functions Performed by Management

The first step in developing the design was to identify the functions that had to be performed by management. These fell into three groups, namely strategic, tactical and operational planning and decision making. Strategic planning is generally long-term and in broad outline, tactical planning medium-term and specific, while operational planning is short-term and detailed. The boundaries between them are arbitrary and not rigid.

Strategic Planning

Strategic planning and decision making involves examination of the environment and consultation with the interests the enterprise serves, to determine what activities it should be involved in, the extent of these activities and the criteria by which the performance of the enterprise is to be judged and by which it should judge between alternatives open to it. This is the task of establishing the objective of the enterprise and the constraints imposed on the pursuit of this objective. In the case of the Forestry Commission the essential decisions have been on the rate of new afforestation, the location of afforestation programmes, the level of investment in State forestry and in recreational services provided by the forestry enterprise and lastly the financial criterion for judging between alternative commercial investments. An outcome of strategic planning is the presentation of objectives and constraints in the introduction to the corporate plan of the Forestry Commission.

Tactical Planning

Tactical planning and decision making is performed within the framework of strategic planning, taking as its starting point the objectives and constraints established by the strategic planners. It is concerned with selecting courses of action which fulfil the strategic requirements and achieve the objective to the greatest possible extent. The outcome of tactical planning is a collection of programmes and specifications of work on the main operations and the resources and services necessary to carry those programmes out. Tactical planning is concerned with specific investment programmes and with the assessment of their long-term consequences for the enterprise, as well as their short-term results in terms of production and resource requirements. Investment appraisal is an essential component of this type of planning. The operations of the enterprise are carried out by an organisation consisting of a number of inter-dependent parts and an essential function of the tactical planning is to ensure that the services required by the operational part, are provided for in the service part of the organisation at the appropriate level.

The following examples illustrate further the functions of the tactical planner. He has to decide the specification of silvicultural treatments such as ploughing, the choice of species, spacing and protective measures in carrying out the planting programme, the timing and intensity of thinning and felling in plantations and the type of markets to be supplied. He has to plan the resources, finance, labour and materials to carry out the operations. He has to plan the services; staff, buildings, machinery and repair organisations, to manage and support the operations. His appraisal activity is directed towards determining the optimum specification and the optimum timing of operations, the optimum level of staff and services taking account of the long-term effects of his decisions. In the Forestry Commission tactical planning and decision making is represented by the corporate plan, the Conservancy plans, the basic budgets and 5-year programmes and the allocation of programmes to operational managers.

Operational Planning

The tactical plan having established the programme of operations, the specifications and the level of services, operational planning is concerned with deciding the particular locations where operations will be carried out and the techniques, men, materials and machinery that will be employed. Thus particular compartments will be selected to make up the programme of planting, specific types of ploughing equipment will be requisitioned for preparing the ground, men deployed to the operation, plants and fencing materials ordered to carry it out. Staff will be posted to particular jobs, training programmes set up, machinery repair facilities organised and machine transport arranged. These are all examples of operational planning. It is represented by the detailed budgeting of operational managers and the management of performance during the year.

The Organisation

An important practical problem in developing the system of financial control in the Forestry Commission was to align these functions with persons, or groups of people, in the organisation. It is not my intention to describe the organisational arrangements in detail but I would like to point out that the relationship between position in the organisation and planning function is not entirely arbitrary. The following guide lines are available.

Strategic planning requires information about the environment, an overall view of possibilities open to the enterprise and the implications of broadly defined courses of action to the whole enterprise. It is therefore an activity for central headquarters organisation.

Tactical planning requires knowledge of detail of particular circumstances in which operations are to be carried out of the technical problems involved, knowledge of the objectives and constraints defined in strategic plans, and an overall view of the inter-relationships between parts of the system. It also requires the service of people technically competent in planning. The way in which tactical planning is aligned with the organisation depends on the relative importance of the technical complexity of the operations and of local circumstances. Highly technical operations might tend to be dealt with centrally by a specialised functional team, while cases where local factors play a major part in determining the best course of action would be dealt with on a regional basis. Where co-ordination of inter-related activities is important, planning will be centralised. Finally the degree to which tactical planning is distributed will depend on the availability of competent staff and the possibility of informing staff involved effectively about strategic objectives and constraints.

In the case of operational planning and decision making, immediate knowledge of local circumstances and of the technical characteristics of the operation being carried out are of primary importance. The role of the operational planner and decision maker is to take the programmes provided by the tactical planner and work out in detail how, where, when and with what these are going to be performed and to keep up from day-to-day with the interaction between the circumstances that develop and these plans.

Given these guide lines there remains a wide range of feasible organisations and probably of optimal ones. In the Forestry Commission where there has seemed to be a choice the tendency has been to delegate as far as possible. The alignment between planning function and organisation adopted in the Forestry Commission is depicted in Figure 13. This shows strategic planning to be at the level of government and headquarters. Tactical planning for the Forestry Commission as a whole is carried out at the headquarters level and is embodied in the corporate plan. Detailed planning of a number of

MANAGEMENT LEVEL	PLANNING FUNCTION	PLANNING: STRATEGIC	TACTICAL	OPERATIONAL
GOVERNMENT		TYPES OF ACTIVITY CRITERIA		
HEADQUARTERS	STRATEGIC	BROAD STRATEGIC FRAMEWORK OF F.C. WORK	CORPORATE PLAN MARKETING PLANT SUPPLY SEED SUPPLY PERSONNEL H.Q. BUDGET	MARKETING PERSONNEL
CONSERVANCY	TACTICAL		CONSERVANCY PLAN INCLUDING FIVE — YEAR PLANS. LONG — TERM PRODUCTION PLANS. CONSERVANCY BUDGET.	MARKETING CIVIL ENGINEERING MECHANICAL ENGINEERING ESTATE MANAGEMENT PLANT PRODUCTION PERSONNEL
DISTRICT	OPERATIONAL		ASSIST IN ALLOCATING CONSERVANCY PLANS AND ALLOCATE TO FORESTS	DISTRICT BUDGET ALLOCATION OF RESOURCES SUPERVISION OF STANDARDS AND BUDGETS
FOREST				JOB BUDGETS IMPLEMENTATION

Figure 13. Planning function and organisation in the Forestry Commission.

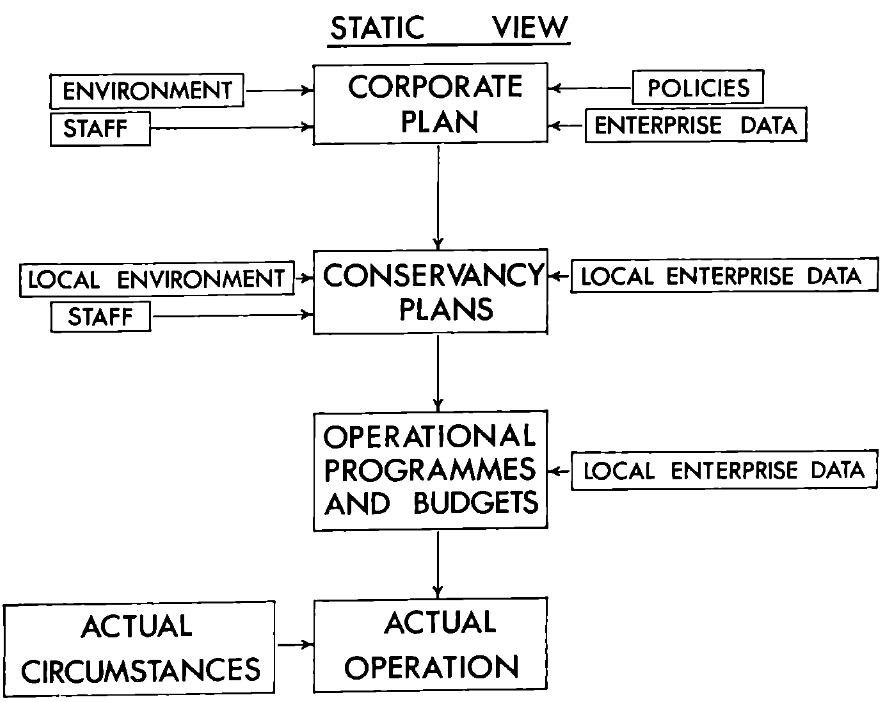

Figure 14. The system of planning.

central services such as marketing and plant supply is at the headquarters level. Detailed tactical planning is mainly, however, embodied in the conservancy plan, the conservancy 5-year programmes and basic budgets and the programme allocation made to districts by conservators. Operational planning and decision making is mainly the concern of local managers, the district officers, foresters, area civil engineers and superintendents of works. Certain operational planning is reserved to headquarters; namely in respect of centrally paid staff and central workshops and the management and control of central timber sales. Similarly operational planning of the deployment and repair of machinery and of timber sales is dealt with centrally in the conservancies.

Features of the System

At this stage let us review the basic structure that has been established (see Figure 14). The essential feature of this structure is that the operations are carried on within the framework of the operational programme and budgets which are made within the framework of the conservancy tactical plans, which in turn are made in the framework of the tactical and strategic plans represented by the corporate plan. The aim in this design is to bring the actual performance of the operational organisation into correspondence with the strategic intentions of the Forestry Commissioners.

The process of adjusting the plans of the various levels to bring about such correspondence is iterative. Each level reviews the plans submitted by the next and indicates adjustments necessary to bring about correspondence. The process is dynamic in that the plans of one level may have been made in ignorance of circumstances known to another, or ones which have emerged since the earlier plans were made. Thus the process of review may result in revision of plans at either level. The objective of this process is to obtain correspondence between performance and strategic plans, but to make sure at the same time that the best use is made of knowledge of developing circumstances and of the abilities of planners at all levels. Putting it another way the aim is to obtain the best result but avoiding surprise (see Figure 15).

The process of controlling performance which involves reporting back on actual performance and comparing the results of plans, is essentially an extension of the review process from the planning into the operational stage. Figure 16 represents the review process in respect of the planning stage and in respect of the operational stage.

The system requires the formulation of plans and the appraisal of alternative courses of action. The results are presented in the lists of projects and the

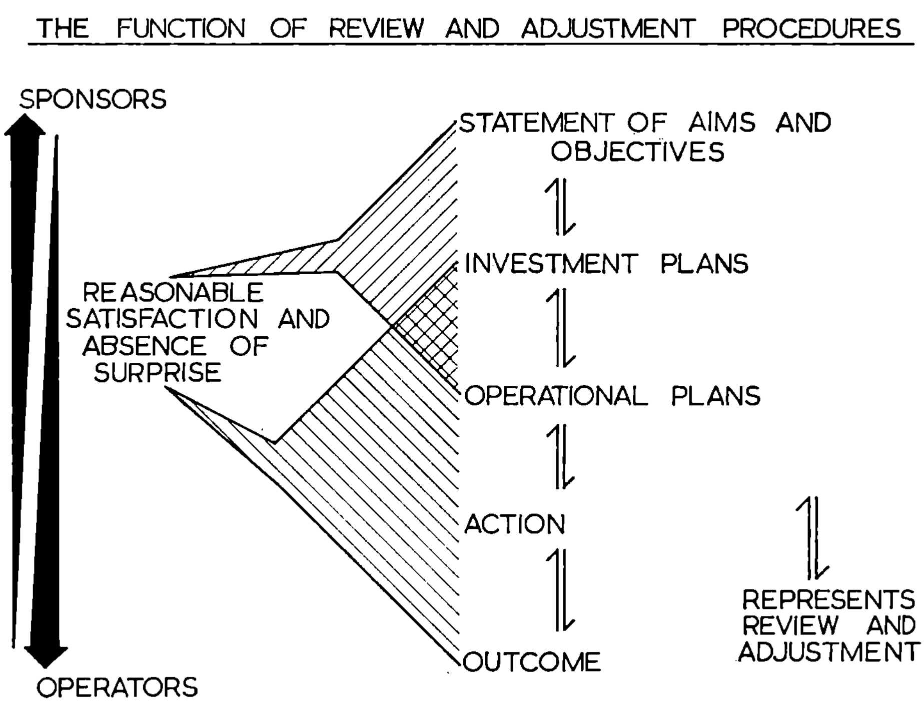

Figure 15. The process of review and adjustment.

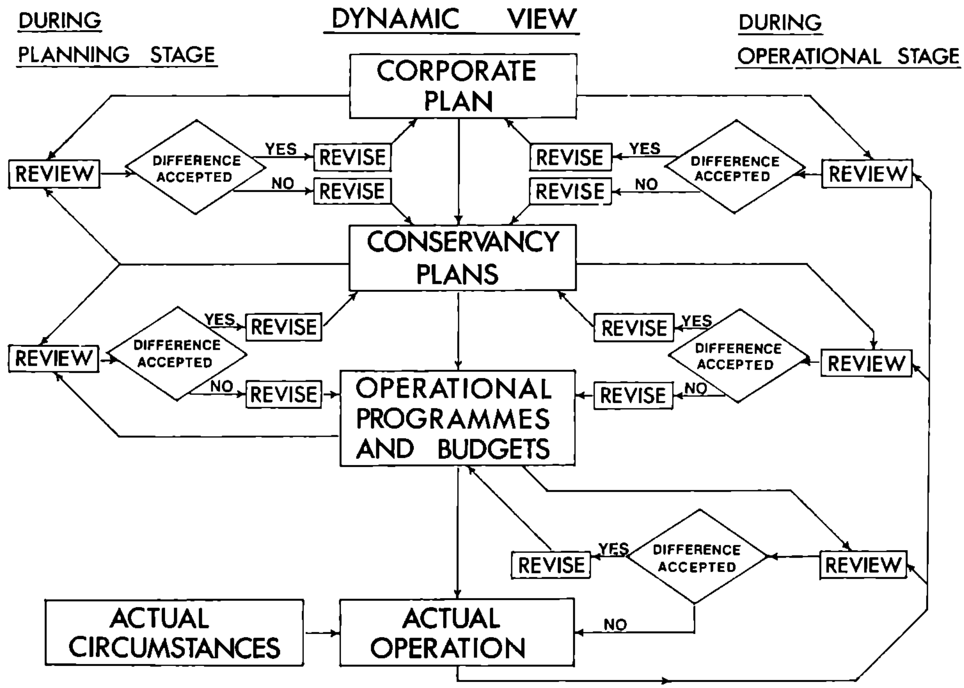

Figure 16. The review process related to the system of planning.

programmes formally provided for. The plans depend for their quality on the management that devises them and involve a much richer network of information flows, perceptions and judgements than is formally provided for. Similarly the progress within the period of the plan is reported on and the statement of adjustment proposals is formally provided for. The process of formulating proposals for adjustment and agreeing revisions of the plan or budget is again in the realm of the richer networks. Figure 17 illustrates the relationship between the formal reports and the decision processes involved in the quarterly review of progress against the budget. Against the decision processes are shown black boxes to indicate the much richer information networks involved, reaching beyond the boundaries of the routine reports. I should say that among the contents of the block boxes one would expect to find the operational research studies such as are discussed in other papers presented to this meeting.

Formal Procedures

In the remainder of the paper a number of formal statements and reports included in the system developed are presented. These have been selected to show (i) the way in which the costs and returns from particular operations are related back to the prospectus of investments checking the correspondence between actual performance and the original plan and (ii) the way in which budget revision is provided for and made explicit in the reporting procedure.

Table 21 shows the presentation of major projects selected after appraisal of alternatives in the conservancy plan. This table indicates the quantities available and includes the annual programme for each project with discounted costs and revenues (benefits) and the benefit/cost ratio.

Table 22 shows the detailed breakdown of two of the projects into their component operations, with the standards for these operations, namely the treatment type standards, and the composition of the discounted cost and revenue.

Table 23 shows the programme by major operational groups in terms of expenditure. (Expenditure corresponds with the quantity by project multiplied by the treatment type standard costs for the operations involved.)

Table 24 shows the composition of a detailed budget made by the local manager (forester) on the work programmes handed down to him. He evaluates the work necessary to perform those programmes under local circumstances, establishing a number of work type standards for the various circumstances involved.

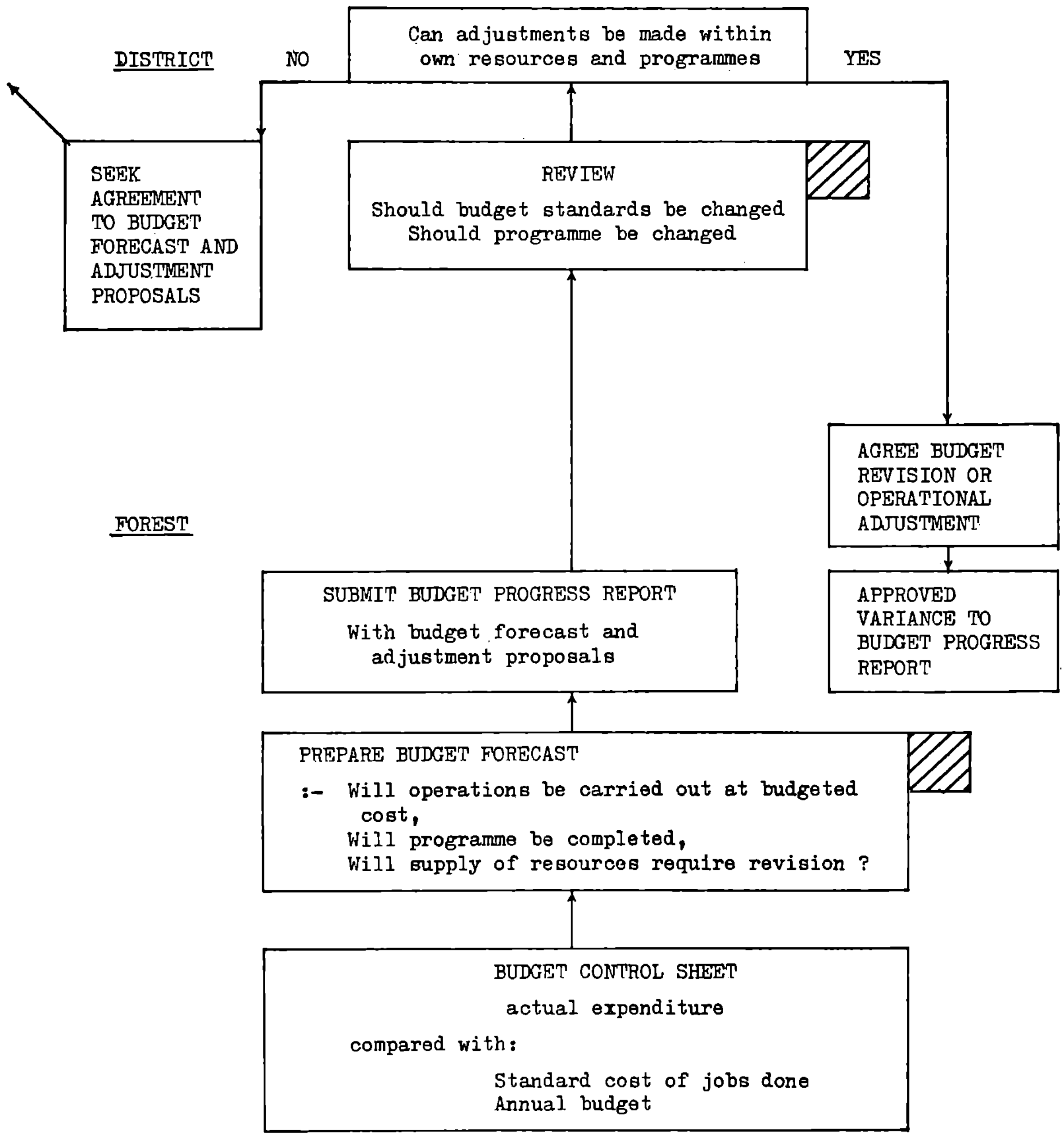

Figure 17. The relationship between reports and the decision process.

TABLE 21

SUMMARY OF INVESTMENT PROJECTS — Conservancy: Midlands

Investment Project	Total Quantity	Annual Quantity	Discounted Costs			Discounted Benefits	Benefit/Cost Ratio
			Initial	Future	Total		
	Acres	Acres	£ per acre	£ per acre	£ per acre	£ per acre	
1. Reafforestation of felled woodlands	2,000	500	62	56	118	185	1·5
2. Afforestation of scrub areas	6,500	1,000	82	58	140	185	1·3
3. Maintenance of established plantations	65,000	65,000	3·5	30	33·5	33·5+	1+
4. Improvement of young plantations by cleaning	20,000	8,000	20	0	20	20+	1+

TABLE 22

INVESTMENT PROJECT COMPONENTS, TREATMENT TYPES AND BENEFIT/COST RATIO

Conservancy: Midlands

£ per acre — FORM F.C.6

Investment Project, Component Treatment Types	Unit Cost	Year or period	Discounted Cost	Unit Revenue	Year or period	Discounted Revenue	Benefit/Cost Ratio
1. *Reafforestation of Felled Woodlands*							
Preparation of ground	10	0	10				
Planting	20	0	20				
Beating-up	5	2	4·5				
Weeding	5	1–4	17·5				
Fencing	10	0	10				
Sub total: initial costs			62				
Maintenance and protection	2·5	1–60	50				
Cleaning	10	10	6				
Sub total: future costs			56				
Thinning and felling				Yield class 160	25–60	185	
Total direct cost and revenue			118			185	1·5
2. *Afforestation of Scrub Areas*							
Preparation of ground	30	0	30				
Planting to fencing (As above)			52				
Sub total: initial costs			82				
Cleaning maintenance and protection (As above)			56				
Harvesting roading	6	25	2				
Sub total future costs			58				
Thinning and felling				Yield class 160	25–60	185	
Total direct cost and revenue			140			185	1·3

TABLE 23

SUMMARY PROGRAMME

Conservancy: Midlands

Activity	1970–71			1974–75		
	Quantity (acres)	Expenditure £000	Income £000	Quantity (acres)	Expenditure £000	Income £000
FOREST MANAGEMENT						
Formation	1,500	100		1,000	60	
Protection	65,000	250		70,000	185	
Crop Improvement	2,000	40		3,000	60	
Total		390			305	

Table 25 shows the aggregation of forest budgets into a district budget and the comparison between the average cost in the district for operations on particular projects with the conservancy treatment type standard. This comparison shows whether operations as planned in that district correspond with the expectations contained in the conservancy plan.

Table 26 shows how a comparison is established between the actual expenditure on work performed during the first quarter and the budget for the year. The aim is to obtain an indication of whether work is going according to plan or whether deviations are appearing which suggest the need for a revision of the plan.

Table 27 is a summary report on performance during the first quarter. It shows in aggregate the deviations from programme so far. It provides for expected deviations from budget by the end of the year to be indicated in the budget forecast, and for proposals for revision of the budget in respect of both operations and resources to put forward. The process of deciding on revision was described above and illustrated diagrammatically in Figure 15, page 82.

Table 28 shows the accounting report for the district at the end of the year in which the actual budget standards are compared with the treatment type standards from the conservancy plan and the actual operational performance is compared with budget. This allows a view of the performance in relation to the original prospectus to be obtained.

The cycle is completed by an accounting report in which cash flows during the accounting period are compared with the change in value. The value is assessed according to the same criteria used in evaluating possible courses of action at the planning stage. In the Forestry Commission these are assessed according to the net present value or net discounted revenue.

Concluding Remarks

The tables selected from the system of formal reports have been selected to show that the presentation of investment plans has been designed in such a way that they can be related to the detailed operational plans and budgets through the treatment type standards. The selected forms also illustrate how lower level plans and budgets are set within the framework of higher level plans and the formal provision made for adjustment. So much then for the translation from model system to the operational design.

In the introduction the aim of meeting the requirements of managerial economics was mentioned. In all decision making the intention is that the objectives of the enterprise should be met to the greatest possible extent consistent with the constraints imposed on the enterprise. In long-term decision making the objective is represented as a financial one—namely to maximise net present value. This criterion is applied both in the decision area of tactical planning and in the value assessments of the accounting reports. In the short term the aim has been to keep reporting in direct cost and direct revenue terms so that judgements are made on the basis of marginal cost and marginal revenue associated with the decision.

DISCUSSION OF WARDLE'S PAPER

Von Malmborg. The presentation is very consistent with intentions of the planning theory group. Particularly noteworthy is the point that planning at different levels requires the constant process of revision.

Duerr. One can think of this series which extends from broad strategic aims at one end to the burning of a certain gallon of petrol at the other as an ends means series. Each item represents the end for the means below it and vice versa. If you have carried out precisely the point, namely that the members of

TABLE 24

FOREST BUDGET: AN EXTRACT

Year 1970/71

Forestry Commission

Forest: Greentrees
FORM F.C.12

A/c No. (1)	T. Type No. (2)	Operation and Work Type (3)	Quantity (Op. Units) (4)	WAGES Unit Cost £ (5)	WAGES Total £ (6)	Details of 'OTHER' (Materials, Contracts) and V.M.E. (7)	Price (8)	OTHER Cash £ (9)	OTHER Non-Cash £ (10)	V.M.E. £ (11)	Total £ (12)	Unit Cost £ (13)	Treatment Type Costs Labour Oncost £ (14)	Total Cost £ (15)	Quantity £ (16)	Unit Cost £ (17)
102		Preparation of Ground														
	1	Beech felling areas	172	1·8	310	Diesel Oil 2490 galls	1/3	156								
						100% 245–T 44 galls	93/–	205								
						Cat 26/1 36 hrs	11/–			20	691	4·0	160	851	172	5·0
	2	Derelict														
		Woodlands (Spray)	58	6·3	370	Diesel Oil 3500 galls	1/3	219								
						100% 245–T 68 galls	93/–	316								
						Cat 26/1 35 hrs	11/–			19	925	16·0				
	2	Derelict														
		Woodland (hand)	10	2·5	25						25	2·5	205	1,155	58	19·8
105	1	Planting	240	6·6	1,590	Plants			2,180							
						Cat 26/1 120 hrs	11/–			66						
						Cat 15 100 miles	1/1			5	3,845	16·0	830	4,670	240	19·5
113	1	Beating up	45	3	140	Plants			180							
						Cat 26/1 10 hrs	11/–			6	325	7·2	75	400	45	8·9
114	1	Weeding (Hand)	55	5·4	300											
		Weeding Spray									300	5·4				
		Gramoxone	275	2·7	750	Cat 26/1 50 hrs	11/–			30						
						Gramoxone 41 galls	106/–	217			995	3·6				
		50% 245–T	90	7·2	650	Cat 26/1 70 hrs	11/–			38						
						Cat 15 50 miles	1/1			3						
						50% 245–T 46 galls	54/–	124			815	9				
		100% 245–T	116	4	470	Cat 15 50 miles	1/1			3						
						Cat 26/1 54 hrs	11/–			30						
						100% 245–T 74 galls	93/–	344								
						Diesel Oil 3823 galls	1/3	239			1,085	9·4	1,130	4,330	536	8·0
					4,605			1,820	2,360	220	9,006		2,400	11,406		

TABLE 25

DISTRICT BUDGET: AN EXTRACT
DETAILED CONSERVANCY/DISTRICT BUDGET
Year 1970/71

Forestry Commission
Activity: Forest Management

Conservancy: Midlands
District: North
FORM F.C.11

Account No.	Tr. Type No.	Operation or Treatment Type Expenditure or Income	Quantity	Direct Cash		Non Cash Transfers			Total Cost	Unit Cost	Standard Cost	
				Wages	Other	Other	V.M.E.	Labour Oncost			Unit Cost	Total
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
102		Preparation of Ground										
	1	Beech felling areas	172	310	361		20	160	851	5·0	10	1,720
	2	Derelict woodlands	58	665	735		30	280	1,710	20·2	30	2,550
105	1	Planting	264	1,660		2,310	75	750	4,795	18·2	20	5,300
113	1	Beating up	71	185		240	10	85	520	7·3	5	355
114		Weeding										
	1	first 5-years	969	4,805	1,580		145	2,160	8,690	8·3	5	4,850
		FORMATION							16,566			14,775

Note. Costs in £'s.

TABLE 26

BUDGET CONTROL STATEMENT
Year 1970/71
Sheet No. 1

Forestry Commission
Unit: Forest, Greentrees.

BUDGET CONTROL CARD

CONTROL SHEET

Unit: Forest, Greentrees
Quarter Ended: June 30, 1970 (First)

A/c No.	T Type No.	Work Type	Budget			Wages	V.M.E.	Other	Cumulative			Quantities
			Quantity	Total Cost £	Unit Cost	Total £	Total £	Expenditure to Date £	Total Expenditure £	Standard Cost £	Variance + or − £	Total
102		Preparation of Ground										
	1	Beech felling areas	172	691	4·0							
	2	Derelict Woodlands (spray)	58	925	16·0							
	2	Derelict Woodlands (hand)	10	25	2·5							
105	1	Planting	240	3,845	16·0	104	8	127	239	144	+95	9
113	1	Beating up	45	325	7·2							
114	1	Weeding (hand)	55	300	5·4	680			680	853	−173	158
		spray Gramoxone	275	995	3·6	423	16	101	540	580	−40	161
		50% 245–T	90	815	9·0							
		100% 245–T	116	1,085	9·4							
		FORMATION		9,006		1,207	24	228	1,459	1,577	−118	

TABLE 27
BUDGET PROGRESS REPORT

Forestry Commission

Period: 1st quarter, 1970

Unit: Forest, Greentrees
FORM F.C.33

Operation Group or Activity (1)	Annual Budget (2)	Exp. to Date (3)	Standard Cost (4)	Variance Due to Cost + or − (5)	Variance Due to Cost % (6)	Programme Variance + or − (7)	Std. Cost % Budget (8)	Budget Forecast (9)	Comments (10)	Approved Variance Cost (11)	Approved Variance Programme (12)
	£	£	£	£		£		£		£	£
Formation	9,005	1,459	1,577	−118	−7½		17½	8,500	Weeding planned with chemicals will have to be done by hand. Materials cost −£500 (Repeat operation next year)		
Protection	2,250	507	545	−40	−8		24	2,350	Additional fire protection, labour cost +£100		
Improvement	3,620	1,202	1,119	+83	+7½		31	4,120	Cleaning planned with chemicals to be done by hand, labour cost +£1000, materials −£500 (This will require a repeat operation later)		
Harvesting	45							45			
Estates	570	125	140	−17	−12		25	630	Increased work required, labour cost +£50 ..		
V.M.E.	260	168	74	+94	+127		28	320	Civil engineering involved increased servicing +£60		
Roads	3,832	2,839	3,158	−319	−10		82	3,330	Expected labour saving −£200. Materials −£300		
Agency and Transfer	150	10	50	−40				150			
Sub-total	19,730	6,310	6,663	−355	−5½		34	19,395			
Labour oncost	3,995	945	1,000	−55	−6		25	4,360	Increased wage cost +£365 ..		
Local supervision	1,405	290	365	−75	−21		26	1,530	Increased wage cost +£125 ..		
Sub total	25,130	7,545	8,028	−485	−6		32	25,285			
S.E.T.	1,930	520	480	+30	+6		25	1,930			
TOTAL	27,060	8,055	8,510	−455	−5½		31	27,215			

	Annual Budget (13)	Exp. to Date (14)	Budget Forecast (15)
	£	£	£
Wages	16,440	4,750	17,900
V.M.E.	2,270	1,050	2,270
Cash	5,970	2,115	4,665
Non-Cash	2,380	130	2,380
Total	27,060	8,045	27,215

(16) Adjustment Proposals
Wage increases involve increase of £500 oncost and supervision. Additional work to carry out cleaning by hand originally to be treated with chemicals will require +£1,000 additional wages .. ..
Saving on the road construction operations and in chemical not to be used −£1,300

Signed A. FORESTER

Approved Variance + or − (17)
£

TABLE 28

REPORT ON PERFORMANCE AGAINST THE BUDGET

Year ended 31st March, 1970

District: North

Operation and Treatment Type or Type of Expenditure or Income	Budget							Actual				Variance				
	Quantity	Treatment Type Standard		Variance from T.T. Standard		Budget Standard		Quantity	Unit Cost	Total Expenditure	Budget Standard Cost of Work Done	Total	Due to			
													Quantity		Costs	
		Unit Cost	Total Expenditure	Unit Cost ±	Total Expenditure ±	Unit Cost	Total Expenditure					±	+	−	+	+
(1)	(2) acres	(3) £pa	(4) £	(5) £pa	(6) £	(7) £pa	(8) £	(9) acres	(10) £pa	(11) £	(12) £	(13) £	(14) £	(15) £	(16) £	(17) £
102 Preparation of ground																
1. Beech felling areas	172	10	1,720	−5	−860	5	860	210	4·5	945	1,050	+85	+190			−105
2. Derelict Woods	85	30	2,550	−9·8	−830	20·2	1,720	50	28	1,400	1,010	−320		−710	+390	

Note: £pa = £ per acre.

the series are simultaneously determined—you do it by a series of approximations, its the same process. The concept is of a system and you are being true to the concept of a system. It doesn't matter where the process commences just so long as you have the feed-back. What if one doesn't have this clear strategic aim then the process may need to come up and the strategic aim is decided per force when the top people come to understand the consequences of the alternatives. *Wardle.* I find that comment extremely interesting. If you look at some of our early papers you will find we said that if strategic plans are made by local managers this cannot be a satisfactory situation. We had our reasons for this and one was that if strategic decisions were made by those several hundred local managers, supposing the organisation had some particular intention it might very rapidly find itself landed in situations that blocked pursuit of these intentions. Thus one starts at the top level to avoid the creation of such blocks. This is not to say that in other situations it may not be appropriate to go up from the bottom as for example in the case of small private ownerships. *Vornstad.* It doesn't matter whether you go up or down so long as you do make the strategic plan but you must not forget the strategic plan and lots do as they are so involved with the operational plan. *Duerr.* If all the levels are embodied in the same individual then surely it doesn't matter. *Wardle.* It is important to recognise that strategic, tactical and operational planning are different processes with different time horizons and must be self-consciously separated. *Vornstad.* In Figure 15 you speak of reasonable satisfaction and absence of surprise. Should you not add efficiency? *Wardle.* Efficiency, maximisation of net present value or whatever it happens to be is part of the "satisfaction" function. Satisfaction is a conscious optimisation process. *Vornstad.* Standard cost is compared with actual but the important thing is how good are your standard costs. *Wardle.* Standard costs are the cost planning base. They have no permanence beyond the budget period or status, they are merely an index of the last agreed plan. *Vornstad.* For efficiency you must be satisfied that differences are real and not stochastic. *Wardle.* Actual expenditure is compared with standard expenditure in aggregate showing whether there has been an overall departure from the original plan. *Vornstad.* The important thing is that the standard should be operational and that one should know what it means. *Wardle.* The standards are established by local managers getting them agreed as practicable and acceptable for the particular job. *Vornstad.* For planning you need reasonably stable elements of costs. If you aggregate you get reasonable accuracy, as you go down, however, they become less stable. *Wardle.* Is this not an estimating problem? *Vornstad.* This is a question of whether the standards are set at a high or a low level. *Wardle.* An important job we were trying to do was to make the local manager feel responsibility for the work he was carrying out, so we wanted to make sure that a contract was established at the beginning of the period against which performance of the contract could be judged. A separate problem was to encourage people to adopt improved techniques and to change the standard.

Paper 11

EXPERIENCE GAINED WITH NETWORK TECHNIQUE IN AVALANCHE CONTROL

By B. BITTIG and F. PFISTER

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Introduction

With regard to the following explanations, it is supposed that the method of Network Technique is sufficiently known. We would like to state furthermore, that this paper does not include amendments of the method, but that it is dealing purely with the use of the Network Technique.

Researches about the use of the Network Technique in forest engineering showed that it is of advantage to apply the Critical Path Method (CPM) (Bittig, 1966). The reason for this is that it is possible to estimate the time of activities with certainty. This estimation is possible due to the absolute correlation between activity and technical steps. The minimum duration of one activity should not be less than 3% of the total time of the project (Brandenberger and Konrad, 1965). Other risks such as accidents, weather conditions etc. cannot be statistically foreseen. This is the reason why other methods as for instance PERT, the "Programme Evaluation and Review Technique", cannot be successfully applied in this field.

The following is a list of situations where it is most profitable to apply the Critical Path Method in forest engineering (Bittig, 1966 p. 49).

Facts	*Examples*
Projects with limited time.	Avalanche control, torrents control, construction of cable cranes, urgent repairs of roads after catastrophes etc.
Projects which cannot be surveyed by the contractor.	Construction of roads on difficult ground, construction of roads and bridges within the same project.
Projects with difficulties regarding preliminary calculation.	Application of new construction methods.
Projects with special problems regarding disposition.	Limited amount of working machines and/or manpower.
Projects which exceed the yearly limit of SFr. 500,000 (=US$ 125,000).	Huge development projects.

Problems in avalanche control

The following problems will arise when constructing avalanche controls: The estimated time limit depends on climate, exposition and altitude. Construction is possible only after melting of the snow in spring time and before heavy snow-fall in autumn. Even during summer time a slight snow-fall can hamper or even stop the work, because it can become dangerous for the workers on steep snow covered hills.

Big losses can arise when construction is not completed on time. Unfinished projects under construction can get damaged by snow, expensive machines vanish or are slipping down-hill.

The site for construction and transport of material depends entirely on the given terrain. Repairs or loss of material which may cause extreme loss of time can only be avoided by excellent organisation. Workers, not accustomed to high altitudes and not experienced to live under conditions prevailing in the Alps, as we have them these days when constructing avalanche controls, cause quite some human problems.

Economical considerations regarding avalanche controls

Up to a few years ago construction firms for avalanche controls had good control over their work. The usual methods of planning (as line diagrams) were sufficient. This situation changed entirely. More and larger projects had to be constructed because of the development of new territory and the construction

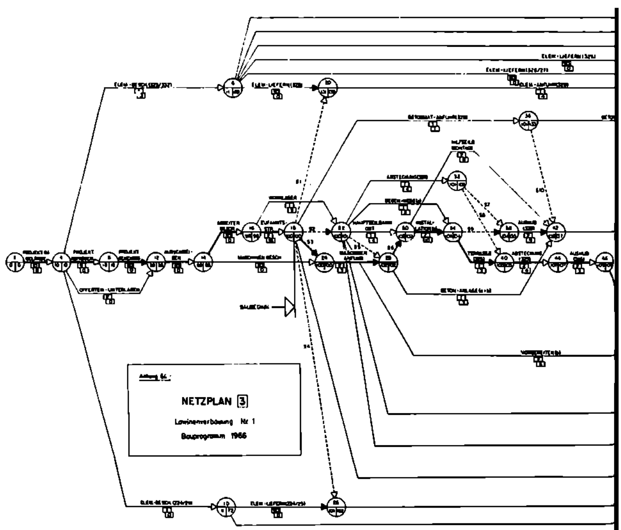
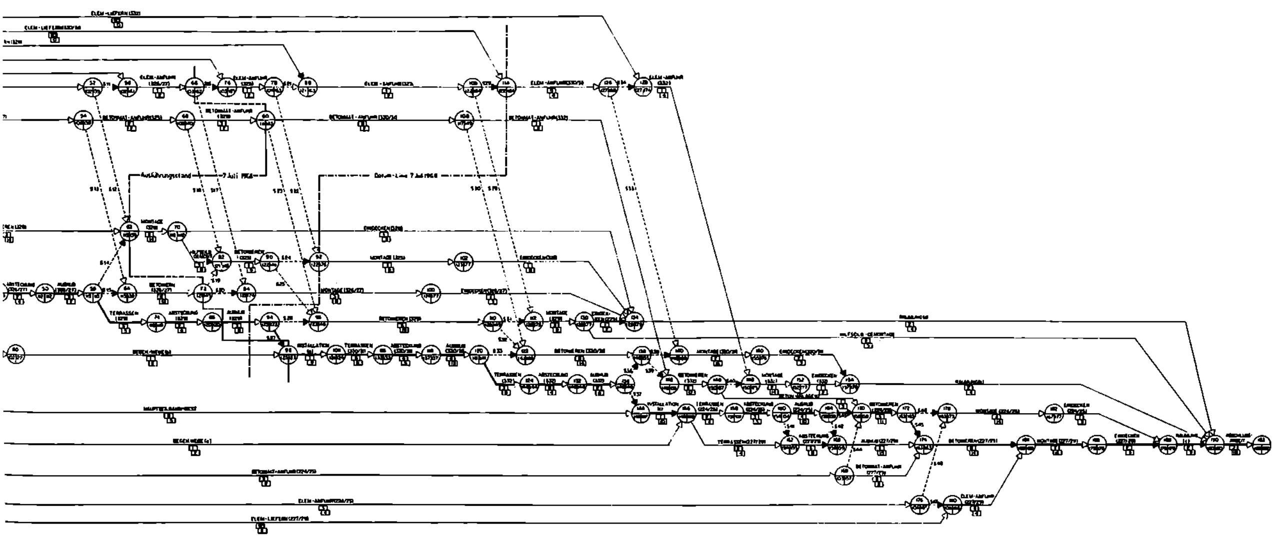


Figure 18. Network diagram for the avalanche control project—"Munt", construction programme for 1966.



of buildings in zones of avalanches. The common methods of planning had to be revised and improved. When searching for new ways and possibilities with less costs, Network Technique was adopted in 1966.

Procedure with Network Technique

Experience in the use of the Network Technique on which the following remarks are based, was gained in connection with the construction of avalanche control above the mountain called "Munt" in the Municipality of Trun (Canton of Grison), Switzerland in 1966 (Pfister, 1969).

When planning with Network Technique for avalanche controls it is recommended to work out a complete network covering all functions of the entire project, each project usually taking years to complete. In studying one year's project work all activities of the general project must be specified in detail. This makes it possible to give information and orders to contractors, subsidy authorities, construction managements and enterprises.

The CMP-method has to be based upon relatively accurate timing. In order to fix such timing, the experience of previous projects can be applied. The approximate netplans of the above construction, which were calculated manually, made it possible to reduce the total duration of the project (see Figure 18 and Table 29).

We have proof that it pays off to make analyses of structure and timing in a serious manner when planning such construction. More and more heavy machines and other technical equipments are being used for avalanche controls. Therefore, it is absolutely necessary to study very seriously also the problem of the capacity of machines and manpower, particularly those on the critical path. To optimise the netplan, an analyses of the capacity also has been made.

The first use of Network Technique in avalanche control has shown the following results:

—Construction was possible within the estimated timing due to time control.

—Differences in timing had been checked weekly. Especially knowledge about the critical activities were most valuable.

—The netplan and the resulting disposition papers were a useful instrument. They were a valuable help also for foremen not familiar with scientific

TABLE 29

EXAMPLES OF THE ACTIVITY LIST OF THE NETWORK DIAGRAM IN FIGURE 18

Tätigkeit/activity	German	English
12 — 14	Ausschreiben (der Arbeiten)	Advertisement of the works for contract
14 — 16	Arbeiter—Beschaffung	procurement of workmen
4 — 10	Elemente—Beschaffung	procurement of the components
6 — 56	Elemente—Liefern	delivery of the components
56 — 66	Elemente—Anfuhr	transport of the components
18 — 36	Betonmaterial—Anfuhr	transport of the material for concrete
22 — 34	Begehungswege	pathmaking
22 — 32	Abstecken (der Werke)	marking out the avalanche controls
34 — 40	Terrassen (–Aushub)	excavation of the terrace
44 — 46	Aushub (–Fundamente)	excavation of the foundation
42 — 62	Betonieren (–Fundamente)	concreting the foundation
62 — 70	Montage (–Fertigelemente)	mounting the components
70 — 134	Eindecken (–Fundamente)	covering the foundation
134 — 190	Räumung (der Baustelle)	clearing the site
28 — 42	Betonieranlage (aufbauen)	mounting the concrete machine
22 — 60	Vorbereiten (durch Sprengung von losem Fels)	preparing (by blowing up loose rock)
22 — 144	Hauptseilbahn (Instandstellen)	mounting the main cable way
30 — 42	Hilfsseilbahn (montieren)	mounting the auxiliary cable way
(326/27)	Nummer des erstellten Einzelwerkes, bzw. der Werkreihe	number of one constructed avalanche control, respectively of the line
1	Angaben unter dem Tätigkeitspfeil:	information below the activity-arrow:
6	Dauer in Tagen/benötigte Arbeitskräfte	duration of this activity/workmen wanted

planning. Co-ordination was also possible for remote construction sites.

—No discrepancies arose with regard to the planning thus revision of the estimated figures or a new study of other alternatives was not necessary.

—The whole program was finished with a delay of only 3 days. It was therefore possible to erect 466m of avalanche controls with 26 workmen within 86 working days in comparatively difficult terrain. The results achieved when working with Netplan Technique surpassed all positive expectations by far.

Outlook

Netplan Technique guarantees an enormous accuracy in organisation when constructing avalanche controls. Precautionary measures can be undertaken. Difficulties arising from weather conditions, problems when transporting material, falling rocks, difficult paths, limited facilities for storing material etc. can be dealt with. Researches revealed that it is possible to get an optimum of work done within the available time.

Furthermore, netplan technique assures a more profitable planning and preparatory work. Thanks to electronic evaluations, planning of projects can be prepared and completed with less people and within shorter periods than heretofore. A further advantage by applying this method lies in the fact that planning is possible "made to measure", particularly for smaller projects. For these it is sufficient to work out limited analyses regarding structure and timing. It can be stated that all these advantages verify the investments put into Network Technique.

Forced by bigger and more complex projects it will be necessary in future to develop integral systems of project organisation. We hope to be able to reach this goal.

If more interested parties will study such integral conceptions it should be possible to find a satisfying system of integral planning.

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DISCUSSION OF BITTIG'S PAPER

Wardle. Most of the work discussed at this meeting is concerned with the long or middle term and broad in relation to planning. It seems to me we have a role to play in relation to the detailed design of operations. Both Mr. Bittig's and Mr. Novotny's papers deal with short-term management problems. Bittig's is also the only paper in the collection dealing with a subject other than some aspect of timber production. I should like to ask Mr. Bittig about the possibility of developing general solutions which can act as a guide to people doing operations of the same kind and, secondly, about the economy in management possible through the use of networks. *Bittig.* It is not easy to hand network plans over to small firms not skilled in the science of planning. We find that the forestry staff have to instruct the firm in the progress of the work. The planning can be carried out in winter at a time when other operations are not possible. *Wardle.* Would this approach not be valuable in the case of forestry operations, although not so desperately critical in their timing. *von Malmborg.* The technique seems to be directly applicable to logging operations and perhaps to other operations where one is short of labour. I wonder why we haven't taken it up. *Wardle.* I often wonder why we don't use it more in relation to planning our own work. *Bittig.* You never get the people who understand the operational problem to make the network and you have to bring the manager and the technical-net planner together. *Bjora.* In practice managers are using it but not writing it down. *Morgan.* Where a project is big enough it probably needs the use of a computer to up-date the network. This in itself is an obstacle to its use. *Wardle.* What about the possibility of getting generalised solutions to logging operations. *Höfle.* I don't agree that logging operations are entirely repetitive. *Bittig.* If you are embarking on the use of new methods network technique is very valuable. *Höfle.* The technique has application to research projects showing all the details that have to be carried in order to get through the project. It is particularly valuable if parts of a project have to be assigned to others. *Wardle.* Management is a scarce resource. If one is going to run low output enterprises one is going to have to fight for economy and management. *Duerr.* Whether the manager is a limiting resource varies with circumstances. Some of the models, such as linear programming, are designed for seeing which resource is limiting. In the United States for example the ratio of Chiefs to Indians is sometimes quite high.

Paper 12

APPLICATION OF MATHEMATICAL METHODS IN OPERATIONAL PLANNING OF LOGGING OPERATIONS

By M. NOVOTNY

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In testing the suitability of linear programming application in the short-term management of timber transport carried out by enterprise, we have acquired experience that one of the factors hampering the success of linear programming in this field is a lack of reliable information on probable time order of logging operations over short periods (week, month, quarter of year).

The logging process itself is not too complicated. In analysing it separately, e.g. by stands, it would be possible no doubt to apply the results without resorting to more complicated instruments of management.

However, from the viewpoint of forest districts and forest enterprises, the whole problem becomes complicated, if we take into account that this relatively simple process

—takes place at several tens of working sites either simultaneously or in succession,

—is performed by a range of means of production with different technical parameters,

—is scattered on the area of several hundreds of hectares,

—is carried out by a range of workers belonging to a number of organisational levels, who are in mutual relationships of superiority or subordination or without any relationship.

These factors make management of the logging process difficult. Therefore, the production process is managed most empirically on the basis of information coming from the forest districts, frequently in isolation and accompanied by many improvisations. The mutual links of logging phases often result in the gradual development of situations quite different from that intended. The importance of logging work from the viewpoint of forestry is so great that it is necessary to use every effort for increasing the level of its management. In that respect several theoretical works have already been compiled. In this paper I describe one of them. It concerns the method of critical path, the application of which I suggested in a modified form in 1966. After finishing several tests,

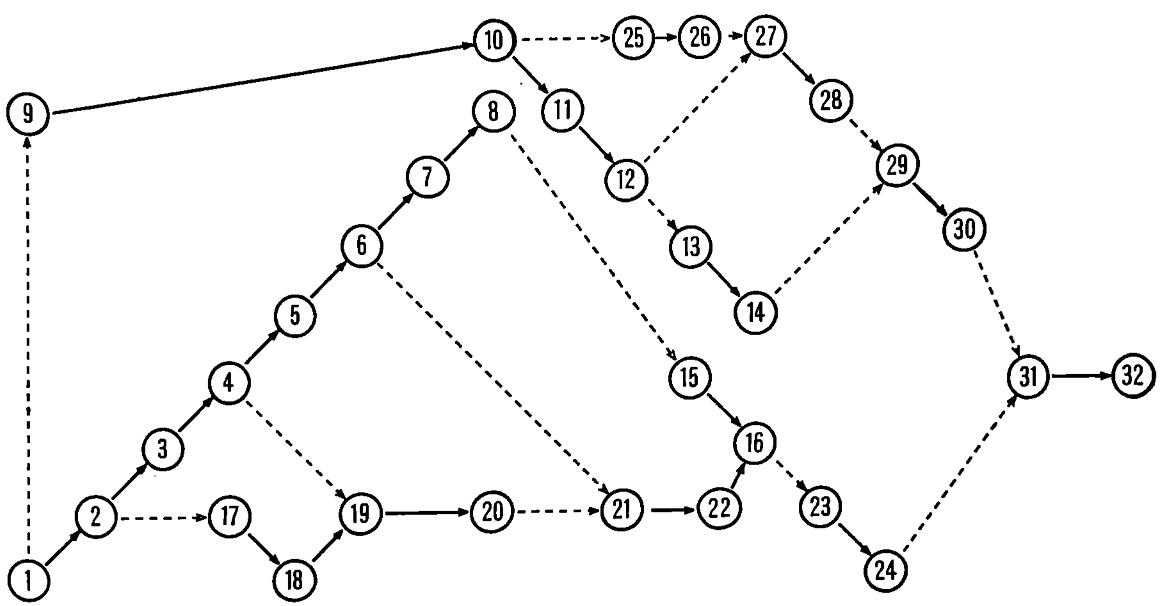

Figure 19. Network for logging.

I simplified this procedure and supplemented it with the decision model. In my opinion, the described solution is one of other possibilities for the introduction of exact management methods into a further field of productive activity in forestry.

Theoretical solution

The critical path method (CPM) is based on the graphical illustration of the analysed action by means of so-called network diagram. The principles of its construction have been sufficiently described in the literature and I shall not deal with them in this paper.

In applying CPM for logging management, I recommend the construction of the network diagrams for logging activity by individual stands and only then to link them up at the level of forest section, forest district and forest enterprise. In this way, we shall obtain a perfect knowledge and basis for necessary operations. A simple example of such network diagrams is given in Figure 19. A list of appropriate activities is given in Table 30.

The critical path in this case consists of 41 shifts. It is, at the same time, the shortest time within which the realisation of the whole operation may be achieved.

However, this classical procedure includes a basic disadvantage. That is to say it does not take into account the size of resources that forest district or forest enterprise has at its disposal for securing the given logging tasks. If this fact is not respected in the practice, it occurs very often that the terms calculated on the basis of this network cannot be

TABLE 30

LIST OF ACTIVITIES IN THE NETWORK FOR LOGGING

Node denomination		Duration of activity	Description of activity
i	j		
1	2	7	Logging in stand 1 by working group 1
1	9	0	Fictive activity characterising linkage
2	3	1	Technical pause
2	17	0	Fictive activity
3	4	6	Skidding in stand by working means 1
4	5	0	Fictive activity
4	19	0	Fictive activity
5	6	5	Wood haulage from stand 1 by work means 1
6	7	1	Necessary technical pause
6	21	0	Fictive activity
7	8	2	Processing of wood from stand 1 in depot
8	15	0	Fictive activity
9	10	10	Logging in stand 2 by working group 2
10	11	1	Technological pause
10	25	0	Fictive activity
11	12	3	Skidding in stand 2 by working means 2
12	13	0	Fictive activity
12	27	0	Fictive activity
13	14	4	Wood haulage from stand 2 by working means 2
14	15	9	Necessary pause
14	29	0	Fictive activity
15	16	2	Processing of wood from stand 2 in depot
16	23	0	Fictive activity
17	18	10	Logging in stand 3 by working group 1
18	19	1	Technical pause
19	20	6	Skidding in stand 3 by working means 1
20	21	0	Fictive activity
21	22	7	Wood haulage from stand 3 by working means 1
22	23	1	Technical pause
23	24	4	Processing of wood from stand 3 in depot
24	31	0	Fictive activity
25	26	17	Logging in stand 4 by working group 2
26	27	0	Fictive activity
27	28	4	Skidding in stand 4 by working means 2
28	29	0	Fictive activity
29	30	7	Wood haulage from stand 4 by working means 2
30	31	0	Fictive activity
31	32	3	Processing of wood from stand 4 in depot

kept if the given means are used. Of course, the theory of network planning developed some methods, by means of which the results of classical procedure may be additionally modified and the shortage of resources in a given situation settled to a maximum degree. Nevertheless, the trials made for the solution of our problem showed that the procedure necessary for making such modifications is relatively complicated and can hardly be applied in the forestry practice.

For this reason, I suggested a procedure for network diagram construction, which respects from the very beginning the limitation of resources for individual activities in a given organisation level. In general, this procedure consists in the use of the combination of network diagram with Gantt progress chart and with linear programming methods. It is a well-known fact that the Gantt progress chart expresses the relationship between the activities and the time. On the other hand the network diagram points out the relationships among the activities. If combined, the two diagrams suitably complete each other. The methods of linear programming then make possible a certain optimisation in the use of existing resources.

The suggested procedure utilises the fact that, before starting the proper logging operations, the means of production and manpower which will be at disposal for a given level, and their capacities expressed by a number of shifts or hours which will be at disposal (estimation may be done more accurately by the Monte Carlo method) and the outputs in the stands to be logged are known for a certain period. Basing on these data, it will be possible to construct

—the optimum plan for assignment of individual resources for the fulfilment of given tasks and

—the Gantt progress chart of working process for each means of production in real time.

Because various resources are needed for the realisation of individual partial activities in timber production, it is very advisable to construct the optimum assignment plans and the Gantt progress charts gradually. The results of the preceding activity form the input data for the following activity (e.g. the stumpage volume at the beginning of a certain period and the logging results obtained within this period determine the skidding tasks). Thus, we have gained a very important information which, otherwise, could be obtained only with great difficulty. This is one advantage of this procedure.

The other advantage consists in the possibility of constructing the Gantt progress charts for individual activities separately, in this way gaining maximum knowledge of these activities. It is very easy to construct from individual and appropriately numbered abscissae of Gantt progress charts a whole model of the operation in the form of incident matrix or network diagram and to obtain, in this way, basic information for the necessary analysis. The confrontations of working capacities of individual means with the tasks through optimisation calculations shows in advance whether the planned task is realistic from the viewpoint of the resources available. A carefully constructed Gantt progress chart informs also

—on the shortest time within which the planned action may be done (in this way we obtain one of the basic informations from CPM method without any laborious calculations) and

—on the utilisation of individual means from the viewpoint of time.

The method of Gantt progress chart construction is demonstrated through a practical example in Figures 20, 21 and 22 and is not further described in detail.

For ensuring the minimum total costs of an operation and for testing the feasibility of planned times, I suggested the use of linear programming. In making this suggestion I suppose that we may always find the following information

—the list of means available for a given period,

—the available capacity of these means, expressed by a number of hours or shifts,

—the output of these means in individual stands, expressed in cubic metres either per hour or per shift,

—the costs of their work in individual stands expressed in kcs either per hour or per shift, and

—the tasks to be fulfilled in each stand.

This information is sufficient for making decisions on the allocation of individual means, based on objective calculations. In the main, it is a case of a generalised transport problem which may be solved by finding

—either the approximate

—or the optimum result.

For finding the optimum result, a special algorithm was elaborated. Of course, the simplex method may be applied too. The practical cases, coming into consideration in forestry, are too extensive to allow the optimum solution to be found manually. However, in this regard, it is not necessary to be afraid of any complications, because there the appropriate programmes for electronic computers are at our disposal.

The algorithm for finding approximate solution is not too complicated and it may be quite well mastered manually for all extents of tasks under our consideration. Because in many cases even a suboptimal solution is sufficient for improving present practice, we recommend its application.

Methodology

The general procedure in the application of the CPM method combined with the optimisation calculations may be divided in several steps.

Step 1. In the first step it is necessary to select the stands, in which the logging operations have to be carried out within a chosen period. In this selection it is necessary to take into consideration the demands of the sale plan for this period, the technical possibilities of logging and its technological provisions. The stand selection is carried out by the chief of the forest district on the basis of information obtained from the headquarters of the forest enterprise. We take it for granted that the stands to be logged have been determined for the whole year in harmony with the provisions of forest management, that the trees to be logged have been marked and the industrial inventory performed.

Simultaneously with stand selection it is necessary to carry out inventory of the means available for logging, skidding and transport within individual forest districts and the forest enterprise. It is recommended that the means for wood processing are included among logging means. The entire situation may be simplified by regarding the wood processing and transport depots as consumer outlets.

Step 2. After finishing the stand selection and inventory of means, it is necessary to complete the appropriate technical-economic parameters for both data groups. The data on assortments produced from individual stands and on the duration of partial activities are entered in an auxiliary summarising table.

Step 3. After filling up the prescribed tables, it is possible to proceed to the first phase of decision making, in which it is determined what kind of means will be applied in individual stands. This phase may be accomplished either by a simple consideration or on the basis of optimalisation calculations. Where the application of particular means is confined to a certain operational area, it is necessary, in the eventual mathematical model, to use the prohibitive rates for all stands situated outside this accessible area.

Step 4. In this step the first partial Gantt progress chart is constructed, i.e. for the logging phase including processing at the felling site. The respective form and procedure may be seen in a practical example. After entering all stands to be logged, the so-called fictive stands will be posted separately in our example (figure 20) marked as 1T, 2T and 3T. The number of such fictive stands corresponds with the number of logging means. They are used to make it possible to record in the Gantt progress chart the fact that some means of production are not available for the entire planned period. In such a case, the interruption in the work of a means is marked by a corresponding abscissa in the respective fictive stand.

The individual abscissae of the Gantt progress chart are gradually plotted by the stands with regard to the applied means of production. The beginning and the end of each abscissa is marked with respective numbers.

Step 5. After finishing the fourth step, we are able to construct a table informing us on the possible starting times for skidding in individual stands.

The steps 1 to 5 may be done at different organisational levels, e.g. in forest section, forest district and forest enterprise. In my opinion, the best way is to choose the forest district level, because it is

—so small that all described steps may be done manually without great difficulty on the one hand, and

—a sufficiently self-dependent unit for making decisions required by these steps, on the other hand.

Step 6. In this step decisions are made on the allocation of skidding means for the operations in individual stands. The starting requirements for these decisions are as follows: a list of skidding means, their available capacity recorded in step 1 and a list of tasks to be fulfilled in the stands according to the tabulated data in step 5. In this second decision phase either subjective considerations or objective mathematical methods may be applied.

It is advisable to solve the eventual decision model within the level of forest enterprise. The information obtained from individual forest districts and from processing and transport centres form the input parameters. The output information is formed by the decision on which means are to be applied.

Step 7. After finishing the decision phase mentioned in step 6, we may proceed to the construction of the second partial Gantt progress chart for the skidding phase. The construction of this chart is governed by the same principles as those for logging.

Step 8. On the basis of the step 7 it is possible to construct a table informing us on the volume available in individual weeks and forests both for the consumers and the handling yards.

Step 9. The timber stock to be prepared at despatch points within the planned period will be summarised by the individual organisation units (forest section,

TABLE 31

STARTING DATA ON FOREST STANDS

Stand Nr.		Planned assortments		Duration of working phase of logging activity			The means planned for task fulfilment—Nr.		
by forest management	Serial Nr.	raw logs	roundwood	logging	skidding	transport	logging	skidding	transport
		tree species/cu.m		shifts					
7 b_5	1	SM/11*	SM/70	6	5	2	1	1	1
20 b_6	2	SM/10	SM/128	7	12	3	1	2	1
15 f_3	3	SM/30	SM/46	5	7	2	2	2	1
17 f_2	4	SM/20	SM/142	9	17	4	2	1	1
18 b_3	5	SM/30	SM/41	6	10	2	1	3	1
6 b_6	6	SM/20	SM/28	3	2	1	3	2	1
7 b_{16}	7	SM/20	SM/143	10	8	4	3	3	1

*SM=Norway spruce.

forest district, etc.) for the whole forest enterprise. In this way, one part of the input information for a further decision model will be obtained.

The second part of input information for this model is represented by the consumers' demands and the third by a matrix of transport rates. On the basis of these three categories of data it will be possible to decide by means of the algorithm of classical transport problem from which organisational units timber will be delivered to individual consumers.

Step 10. On the basis of the preceding step it is possible to construct the third partial Gantt progress chart. It is again constructed in individual forest districts by the same principles as described in step 7, whereby regard is paid to the terms involved in the Gantt progress chart for skidding.

Step 11. From the Gantt progress chart of transport it is again possible to construct a table which gives information on the chronological order of deliveries and on the use of transport means under given conditions.

Step 12. In this step, which is not always necessary for the concrete needs of management, we unite all the three partial diagrams by the principles of network analysis in a single complete diagram. In this way, we shall obtain all data necessary for the calculation of the course of critical path and individual kinds of reserves. It will be sufficient to record this union in a table comprising a list of activities.

Step 13. After constructing a complete table of activities, we can proceed to the calculation of critical path and appurtenant reserves. The length of critical path is already known from the Gantt progress charts, so that we have a possibility to check up on the rightness of all previous considerations.

Step 14. In this step we shall confront the results of our calculations with the planned tasks and deduce from this analysis the appropriate organisational measures.

Practical example

I shall demonstrate the procedure described in the preceding paragraphs on an example illustrating the elaboration of the monthly operational plan of logging activities in a forest district. To save space, I shall not mention all steps of methodological procedure. Table 31 comprises the starting data on the forest stands.

The following means are available:

- —for logging 3 working groups equipped with JPM (one-man power saw).
- —for skidding 1 wheel-tractor and 2 teams of horses with necessary crew.
- —for transport 1 lorry with usual crew.

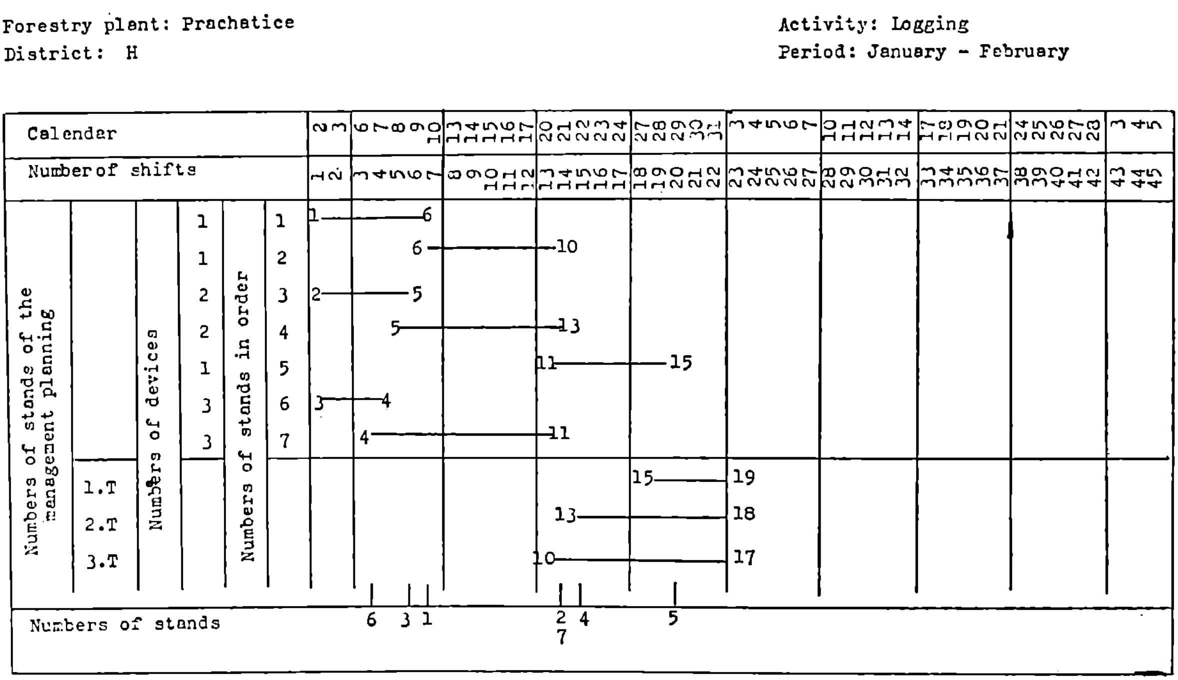

Figure 20. Partial Gantt progress chart for logging.

Forestry plant: Prachatice
District: H

Activity: Skidding
Period: January - February

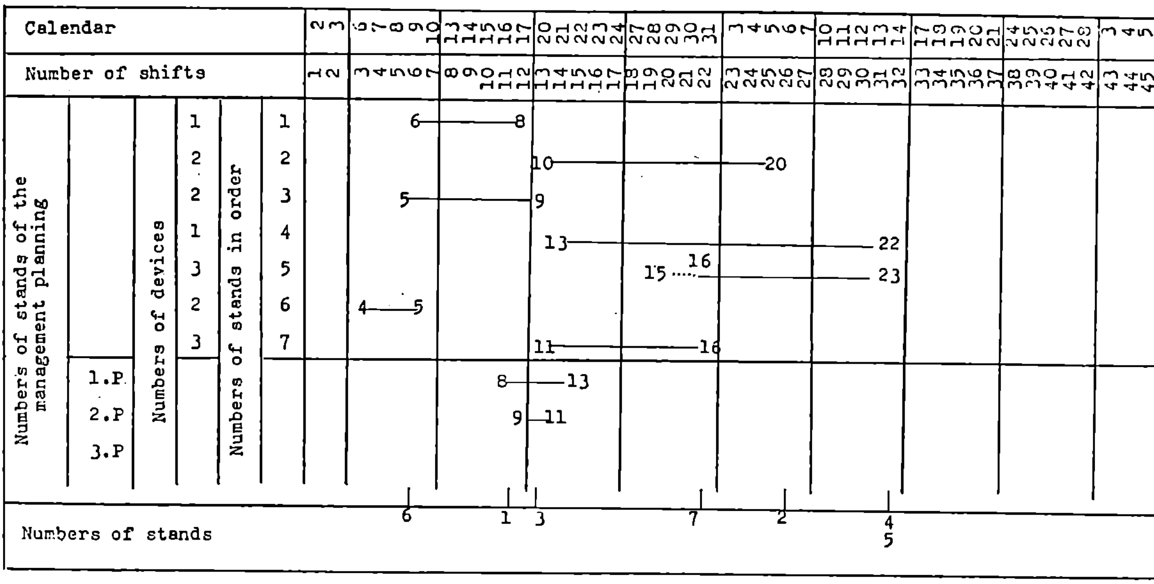

Figure 21. Partial Gantt progress chart for skidding.

Forestry plant: Prachatice
District: H

Activity: Transport
Period: January - February

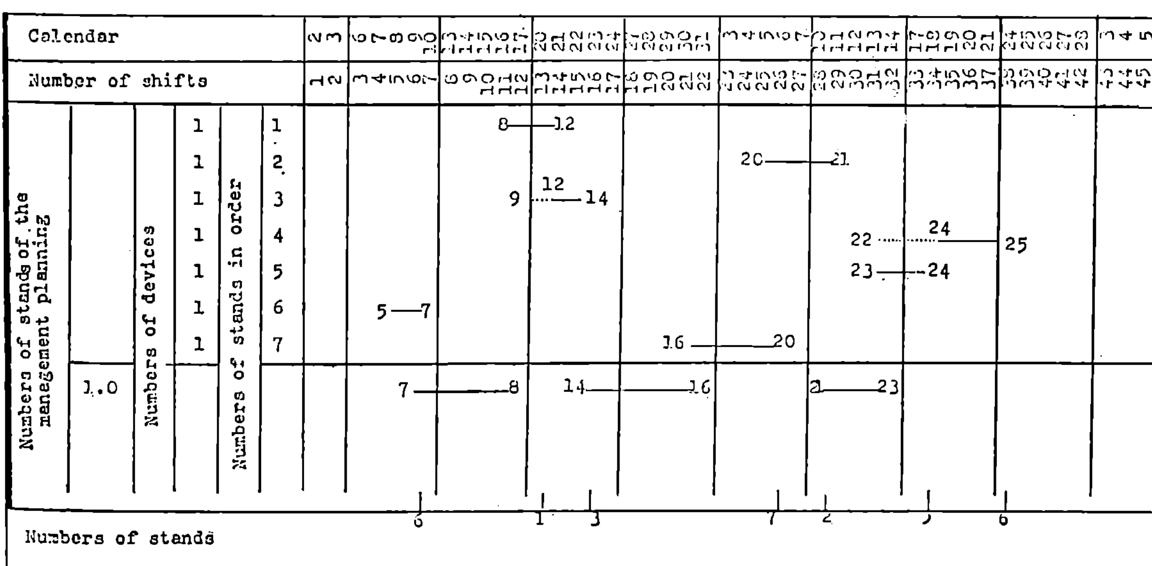

Figure 22. Partial Gantt progress chart for transport.

The figures 20, 21 and 22 include the respective partial Gantt progress charts. Information which may be read from them are given in a summarising table 32.

Problems of application in practice

For the time being, I suggest applying the described modification of CPM method and the decision models only as an instrument for a single analysis. This allows the forest enterprise to make a general analysis of the situation in the field of wood production to such an extent that after the confrontation of planned tasks with existing production sources it may obtain a knowledge of the probable chronological order of the whole operation. On the basis of this knowledge it is possible to divide its means in such a way as to secure timely fulfilment of tasks with minimum production costs. This suggestion makes the whole application to some degree easier. This easiness consists, first of all, in the fact that:

—there exists a sufficiently long period for the collection of necessary data, for the construction of network diagrams and for appropriate calculations and analyses on the one hand, and

—it is not necessary to elaborate and organise an information system, to provide regular information, e.g. for checking up weekly on the deviations of the actual performance from the calculated programme.

Many problems arise in the practical application of this method. I shall briefly mention only two groups of them, in my opinion the most important ones. They are the group of psychological and the group of organisation problems.

The psychological problems, which may arise in connection with this new method are essentially the same as those which accompany every change in an accustomed working procedure. They are commonly known, and it is not necessary to discuss them in detail. We met several times with their existence in seeking the fellow-workers for our experiments in this field. This situation is rendered yet more difficult in the case of CPM application, because use of this method brings about the improvement of management activity, but it does not result in saving of labour. On the contrary, if compared with traditional procedure, the laboriousness of management work increases even where modern data processing techniques is applied. However, this increase of laboriousness if fully justified by the results obtained.

The success of the application of the new method for the complex operational planning of logging activity is in practice dependent, on its organisation. If this organisation is not perfectly secured, the success is doubtful.

The application is dependent on a detailed elaboration of work procedure and on a full ensurance of its organisation. It is necessary, first of all, to realise the right inter-relationships between the managing and executive staff in securing the logging operations. The managing staff must determine the basic links, relationships and technology, obtain manpower and means of production, set up general organisation of work and formulate a basic variant of solution. On the other hand, the executive staff must carry out the project in prescribed terms and according to suggested procedure.

With regard to the peculiarities of logging process in forestry, caused mainly by the extent of the territory in which this process is realised, it is necessary to solve in its management first of all the problem of inter-relationships between forest enterprise and forest district.

TABLE 32

SUMMARY OF DATA FROM CHARTS

Serial number of a week	Timber volume prepared at the end of a given week for further phase of production			Deliveries
	Logging	Skidding	Transport	
	m^3	m^3	m^3	
1	0	0	0	0
2	205	48	20	28
3	0	157	0	0
4	463	0	41	116
5	71	163	0	0
6	—	138	20	143
7	—	233	10	128
8	—	—	50	60

Whereas the headquarters of forest enterprise represents in the logging process the decision unit and the forest section the executive one, the leading officers of forest district or transport centre perform both activities. Therefore, the chosen procedure must correspond to this fact.

In my opinion, the decision competency may be divided between the two mentioned organisational levels in such a way that the leaders of forest district or transport centre submit the basic variant of solution for such means, which they possess and administrate. Their decisions are then confronted with the general tasks and possibilities by the headquarters of forest enterprise, which co-ordinates the activities of all organisational levels linking them up with each other, without their being in a position of mutual superiority or subordination. The procedure suggested in this paper corresponds to this principle, which essentially presumes the enlargement of the competency of lower organisational levels.

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Paper 13

A SIMULATION MODEL FOR ENTERPRISE PLANNING

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Introduction

This paper is an account of a model which describes certain activities of the Forestry Commission over a period of time extending into the future. The model is deterministic and does not seek an optimal solution. It can be regarded as falling into the class known as "simulation" models.

The techniques and the general approach reflected in this paper cannot be viewed, in 1970, as being new but the detailed approach and areas of emphasis should be of interest to others involved in similar studies.

Purpose of the model

The main use of the model in its present state is in the field of corporate planning though its use is not confined to this field. A particular concern of corporate planning is the process of identifying alternative medium or long term strategies, evaluating them in respect of one or more criteria and presenting them to the managers of an enterprise so that they may decide on the most suitable course for the enterprise to attempt to follow.

The identification of alternative strategies may be a matter of discussion and inspiration but their evaluation is a rather more mechanical matter of calculating the costs and revenues which arise as a result of physical actions. It is in the carrying out of the arithmetic for this latter aspect of corporate planning that the model can be particularly useful.

Though the model is specifically designed to assess the results of different cutting and new planting plans other factors can also be tested. The output variables include net discounted revenue (NDR) and projections of net annual income and employment.

An important use of the model is to test the sensitivity of the results of particular plans to changes in the assumptions.

Description of the model

The model takes the enterprise in its initial state, as described by the input variables (which include cutting and new planting plans), and forecasts, year by year into the future, the physical programme and financial and other results which follow from the initial state.

Figure 23 outlines the main relationships which are included in the model. A more detailed account will be found in Appendix I. (See page 117). The relationships described by the model are those which follow from the primary activities of thinning and felling the existing and future growing stock, and from the planting of newly acquired areas. These primary activities are dependent on the specified cutting and new planting plans. Thus a given new planting plan generates each year a land acquisition programme, a crop establishment programme and a road construction programme. At some time in the future it will also generate a further road programme and a harvesting programme. A given cutting plan generates each year a harvesting programme, a road construction programme (in areas entering the production phase), an establishment programme on newly felled areas and also a timber sales programme. The existing estate generates a maintenance and protection programme. Each programme gives rise to a demand for labour, machines and materials. The demand for these resources will give rise to costs which when subtracted from revenue will result in an annual net income. The discounted net income for any year contributes to the total net discounted revenue associated with the future management of the enterprise.

Though the results of the simulation are of interest for, at most, 20 years ahead, the simulation is run for a period of sixty to eighty years in order to provide an adequate estimate of the net discounted revenue in perpetuity. New planting plans can be specified for up to twenty years ahead. Cutting policies are specified for the duration of simulation. Volume production, restocking programmes and roading programmes are normally projected for the duration of the simulation. Projections of employment and net annual cash flow are made for up to twenty years ahead.

Exogenous Variables

The exogenous (or input) variables required to evaluate the relationships in the system can be usefully classified into four types:

(*a*) Variables which describe the new planting and cutting plans.

(*b*) Variables which describe the demand for the resources of labour, machines and materials per unit of crop establishment, harvesting, etc. and the wage rate per unit of labour used in different operations.

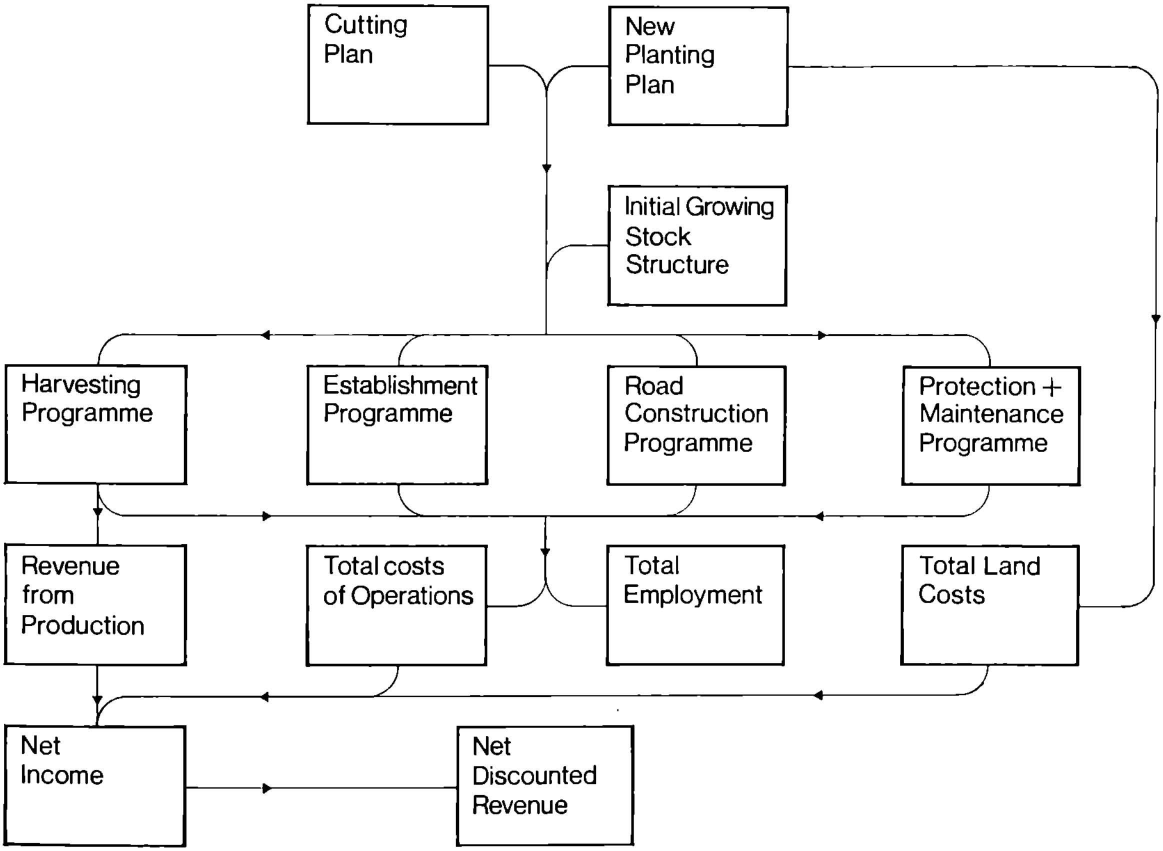

Figure 23. Generalised outline of a forest enterprise model.

(*c*) Variables which describe the annual rate of change in the demand for the resources in (b) for different operations.

(*d*) Yield models which describe the development of timber production in different yield classes for different cutting treatments.

The variables included in (a) and (d) are described in some detail in the sections that follow. The variables included in (b) and (c) are tabulated in Appendix II. (See page 121). The variables representing the age class structure within different yield classes can be regarded as "status" variables and the values of these in year "0" must also be input at the start.

Controllable variables. Most if not all of the variables in (a), (b) and (c) above are under the control of management to some extent or other and a manager may wish to regard any of them as "controllable variables." The model can of course describe the results of changes in the level of any of these but the extent to which possible interactions are considered varies with the variables being considered. For example the intensity of pre-planting road construction can easily be changed but since a change interacts with establishment costs, changes in the level of these costs will take place and these changes must be studied outside the model. Similarly different levels of changes in labour productivity for an operation may be stipulated but the interaction with changes in the level of machine and other costs must be handled outside the model. The construction of the model is such that most of the interactions arising from different cutting and new planting plans can be handled within the model and it is therefore particularly suitable for studying the effects arising from changes in these plans.

New planting plan. The new planting plan is specified as shown in Table 33. New planting should not be confused with the "restocking" activity which takes place after clear felling. New planting extends the total forest area and land must usually be acquired by purchase before new planting can take place. New planting can be specified to be in any yield class and in up to six site treatment types, each treatment type representing a different establishment cost. For each yield class and site type a total area of new planting can be specified for any of four five year periods extending up to twenty years from the start of the simulation.

Cutting plan. The cutting plan specification is shown in Table 34. A cutting plan must be specified for each yield class and each "growing stock category" within the yield class as indicated in the table. The growing stock is subdivided in the way shown because the manager may wish to test different policies for different parts of his growing stock within a yield class and it is between the defined categories in particular that cutting plan differences are most likely to occur.

Within a cutting plan two alternative felling ages may be specified (Table 34 column (6) and column (7)). The number of years which must elapse before the alternative felling age is used is specified in column (8). A clarification of column (4) of the felling policy is delayed until the next sub-section which describes the yield model specifications.

The specification of the cutting plan is very flexible since, apart from the growing stock categories recognised within a yield class, each yield class may be split up in any arbitrary way to represent, for example, the growing stock in that class over a geographical region and to which a particular cutting plan is to be applied. The splitting up of a yield class in this way rather than by the growing stock categories involves penalties owing to the greater time needed for preparation of input data and to the increase in computing time.

TABLE 33

SPECIFICATION OF THE PLAN FOR NEW PLANTING

Species	Yield class	Site treatment type	Total Area of New Planting ('000 acres)			
			1–5 years	6–10 years	11–15 years	16–20 years
Sitka spruce	120	1				
		2	5·0			
		3	5·0	5·0	2·5	
		4	5·0	10·0	15·0	20·0
		5	5·0	5·0	2·5	
		6				

TABLE 34

SPECIFICATION OF THE CUTTING PLAN FOR A YIELD CLASS

	Column (1) Species	Column (2) Yield class	Column (3) Growing stock category	Column (4) Yield table index no.	Column (5) Age of first thinning	Column (6) Felling age	Column (7) Alternative felling age	Column (8) Number of years before alternative felling age is used
Row (1)	Sitka spruce	120	Existing growing stock at or beyond age of first thinning stated in row (1) col (5)	1	25	55	50	5
Row (2)			Existing growing stock younger than first thinning age in row (1) col (5) and growing stock derived from new planting	2	20	55	50	5
Row (3)			Growing stock derived from future restocking	3	20	50	45	5

Yield models. Ideally an enterprise model should incorporate a subroutine representing a yield model which would describe the mensurational result of any cutting policy for any species or yield class. This would greatly add to the flexibility of the model but at the time of the model's construction it was not feasible from the mensurational point of view.

In the absence of a universal model one alternative is to input a yield table to tabulate the results in terms of volume production, for each cutting plan, for each yield class. This is rather cumbersome and though it is basically what is done in the present model some attempt has been made to achieve flexibility in meeting different cutting plans from the same yield table.

The form of yield table input for each yield class is described in Table 35. All the parameters shown in the table depend upon age. For a given yield class the total volume production and the mean breast height quarter girth (BHQG) of fellings and thinnings depend on the specified thinning yield.

Cutting plans which differ only in their felling ages can of course be easily accommodated by the yield table as it stands since felling volumes for any year are merely total volume production less total thinning yield to date. The stated mean BHQG for thinnings and fellings are appropriate in these circumstances.

The effects of changes in the age of first thinning can also to some extent be derived from an existing yield table. This is possible since the changes usually being considered do not result in changes in total volume production and felling volumes can therefore be calculated as before. However there will also be changes in the mean BHQG for fellings and thinnings. These can to some extent be accommodated by additional cutting plan specifications which are not shown in Table 34. They are of the form m_1, c_1, for thinning and m_2, c_2 for fellings. During the running of the model the original yield table BHQG's are amended by the factor y, where $y = m_n x + c_n$, x being the age of the crop and "n" takes values 1, 2. Estimates of m_1, m_2, c_1, c_2 are intended to be made from short mensurational studies.

Other than the fairly minor changes involved when altering the age of first thinning a new yield table must be introduced in order to describe the changes in all parameters consequent on a change in the specified thinning yield. The computer programme representing the model can store up to four yield tables for each yield class. Thus column (2) of the cutting plan specifications in Table 34 specifies the index number of the yield table to be used in conjunction with the remainder of the cutting plan. The different yield tables which can be stored may represent different thinning intensities but they need not be confined to this. They can be used to represent the results of different initial planting distances, different stockings, or they can be used to represent different yield classes from that originally considered. In this way the result of allowing for an increased growth rate for restocked areas can be represented by inserting an appropriate yield table index number in row (3), column (2) of the cutting plan specifications (Table 34). This facility adds considerably to the flexibility of the model.

Endogenous Variables

The endogenous variables are the dependent or output variables of the system which is modelled. Those variables at present regarded as output variables by the programme which represents the model are shown in Table 36.

Though the financial objective of the Forestry Commission is to maximise net discounted revenue (NDR) at a given rate of interest the relationship between this and other objectives such as provision of employment and the provision of amenity are by no means clear cut. It is possible that maximising NDR might be regarded as the only objective whilst other objectives are regarded as constraints. Even so,

TABLE 35

FORM OF YIELD TABLE INPUT

Species	Yield class	Year	Total volume production (h ft per acre)	Thinning yield (h ft per acre per annum)	Mean breast height quarter girth of felling (ins)	Mean breast height quarter girth of thinning (ins)
Sitka spruce	120	20	935	28	4·0	3·5
		25	1,730	98	4·25	4·0
		30	2,715	98	4·5	4·25
		.	.	.	.	.
		.	.	.	.	.
		.	.	.	.	.
		.	.	.	.	.

the main problem still remains in that the constraints are usually only vaguely defined. The intention at the present time is that the output variables of each solution are inspected and subjectively evaluated, taking into account any definite constraints which may exist. Different solutions can then be ranked in order of preference.

Computing Aspects

The computing capacity available on site at the Forest Research Station is limited. This has led to the model being divided into two programmes each running on a different computer.

The major programme is written in Fortran IV. It takes the cutting and new planting plan for each individual yield class together with a description of the initial growing stock and calculates forecasts of timber production, restocking areas and areas entering the production stage. The programme is run on an external computer and provides input in the correct format for the second programme which is run on the ICL Sirius computer at the Research

TABLE 36

FORM OF OUTPUT OF THE MODEL

		Years									
		1–5	6–10	11–15	16–20	21–30	31–40	41–50	51–60	61–70	71–80
Total Production from fellings (h ft)	BHQG Size class 2^{2}—6^{0} 6^{1}—9^{0} 9^{1}—12^{0} 12^{1}—15^{0} $\geqslant 15^{1}$										
Total production from thinnings (h ft)	BHQG Size class 2^{2}—6^{0} 6^{1}—9^{0} 9^{1}—12^{0} 12^{1}—15^{0} $\geqslant 15^{1}$										
Total restocking (acres)	Treatment type 1 2 3										
Total area at first thinning stage (acres)											
Total new planting (acres)											
Mean labour demand per annum (man years)											
Mean machine and material costs per annum (£)											
Mean revenue per annum (£)											
Mean net income per annum (£)											
Net discounted revenue (£)											

Station. This programme is written in Sirius Autocode. It calculates the resource requirements and other consequences of the forecasts previously made.

The computer programmes which represent the model exist in three modes. Mode A assumes normal restocking over the whole of the period of the simulation; Mode B assumes no restocking activity at all; Mode C assumes no restocking after twenty years. Mode A or Mode C is intended for normal use, Mode B being used for an evaluation of the existing growing stock only.

A practical example of the use of the model

A typical problem is to assess the implications of changing the present Forestry Commission cutting plan to some alternative plan. The model can be used to help predict the main results of the change and is therefore of use to the manager in deciding whether such a change is desirable.

In this particular example the cutting plans which are compared are defined as follows:

Cutting Plan I:

The normal Forestry Commission cutting plan. Thinning yields are as specified in Forestry Commission management tables (Bradley *et al.*, 1966) and felling is planned for an age which is "optimal" for a 3½% discount rate. Thinning yield and felling age varies with yield class and species.

Cutting Plan II:

The same as Cutting Plan I except that felling is carried out at an age which is five years earlier than the age which is optimal at a 3½% discount rate. This earlier age corresponds approximately to an age which is optimal at a 5% discount rate.

A new planting plan of 50,000 acres per annum is assumed.

The factors of interest in making the comparison include the volume production for each cutting plan, the employment results, the annual net income and the N.D.R. generated by each plan. It is assumed in this example that the proportion of production sold in the form of standing trees to merchants remains constant. The change to the new felling age specified in Cutting Plan II would take place over the first five years.

An evaluation of the two plans is contained in the following Figures and Tables, which are based on the output from the model.

(*a*) Figure 24 displays summary production forecasts for each cutting plan. These can be

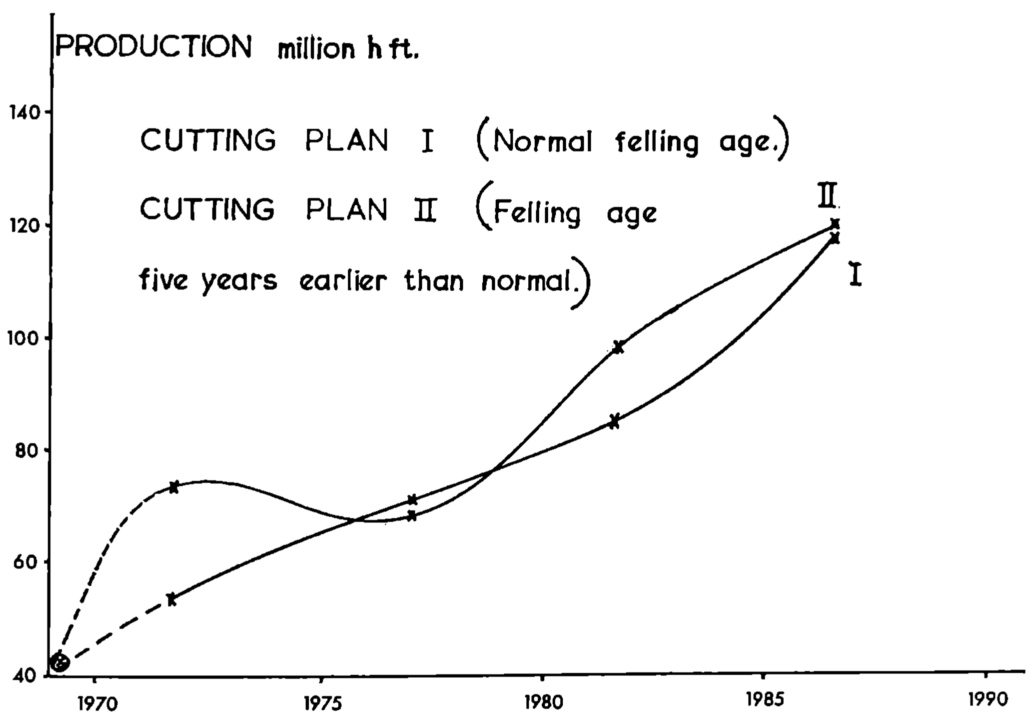

Figure 24. Production forecast for different cutting plans: Cutting plans I and II.

examined to decide whether anticipated constraints imposed by market demand are likely to be violated by either plan. These forecasts are more or less independent of the new planting plan for the first twenty years or so.

(*b*) Figures 25 and 26 indicate the employment effects of each cutting plan. Since the relative effect of the two plans on the enterprise as a whole is important, they are considered together with the new planting plan. The effect of excluding new planting is shown in Figures 27 and 28. The graphs show that the forecasts are sensitive to the labour productivity assumptions included. These employment patterns require study to determine the costs of meeting the need for extra labour or labour reductions shown in the graphs. (The model does not account for all activities and these figures underestimate total employment by about 10%.)

(*c*) Figure 29 shows the pattern of annual net income produced by the two cutting plans. Overheads at supervisory level and above are excluded. The absolute results and the absolute value of the differences between the two cutting plans is sensitive to the assumptions made (only the extreme results are shown). However Cutting Plan II produces the higher net income for the major part of the period in all cases considered.

TABLE 37

EVALUATION OF DISCOUNTED REVENUE FROM EXISTING GROWING STOCK

Discount rate	3½%	5%
	DR in £ million	
Cutting plan I	329	226
Cutting plan II	318	228

(*d*) Table 37 shows the discounted revenue (DR) from the existing growing stock, associated with each cutting plan. Table 38 is an evaluation of the N.D.R. from the existing growing stock, for each cutting plan. These tables are based on the normal Forestry Commission price/size curve. This growing stock evaluation does not include restocking or new planting activities, but includes only those roading, maintenance and harvesting activities which are necessary

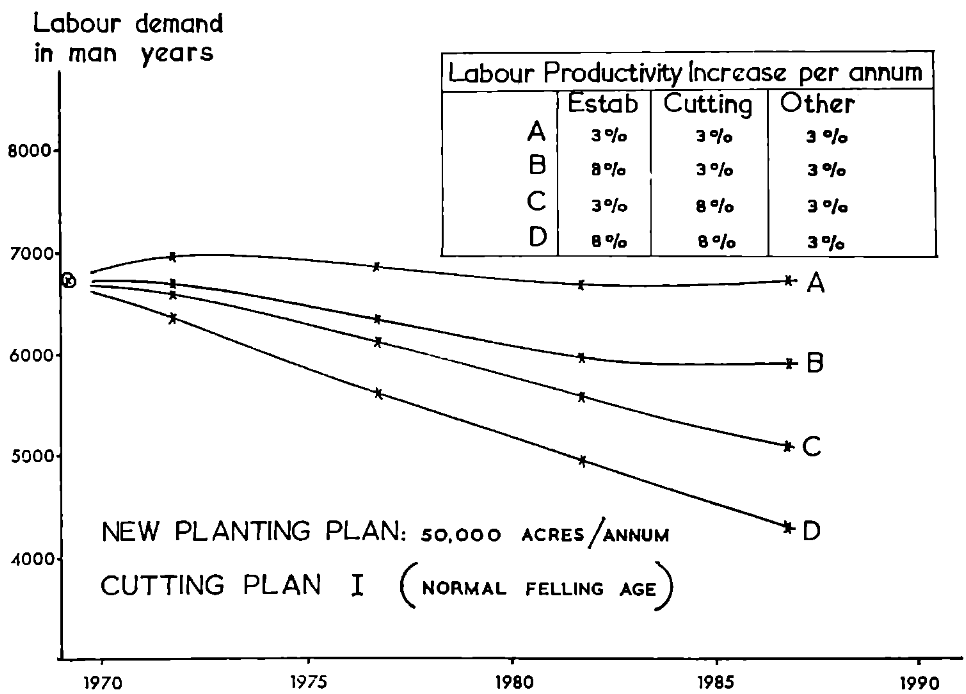

Figure 25. Labour demand for different rates of change in labour productivity: Cutting plan I, new planting 50,000 acres per annum.

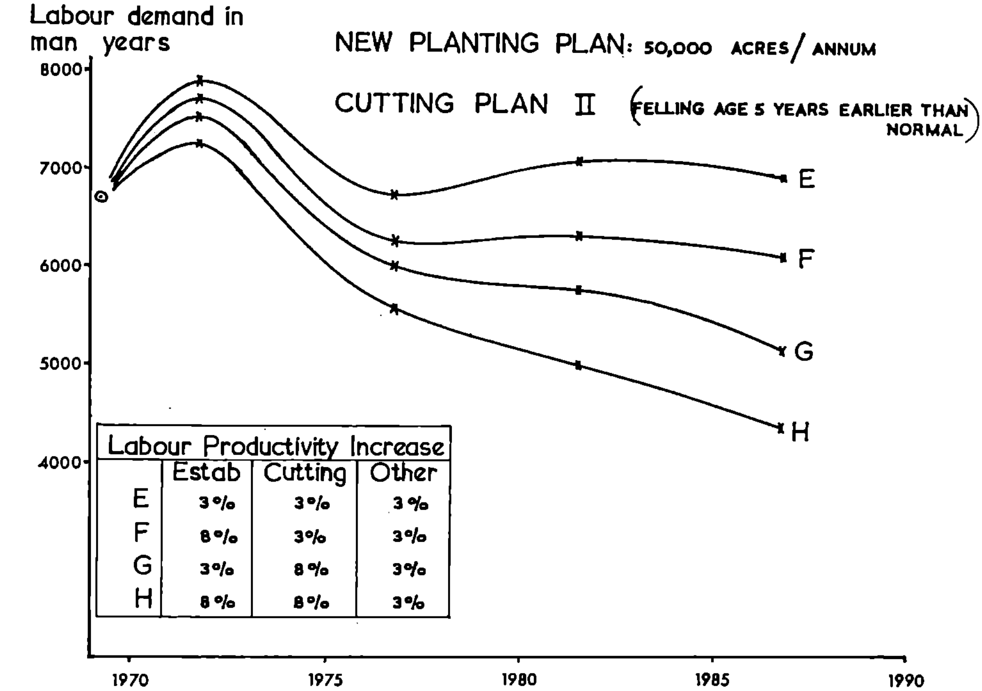

Figure 26. Labour demand for different rates of change of labour productivity: Cutting plan II, new planting 50,000 acres per annum.

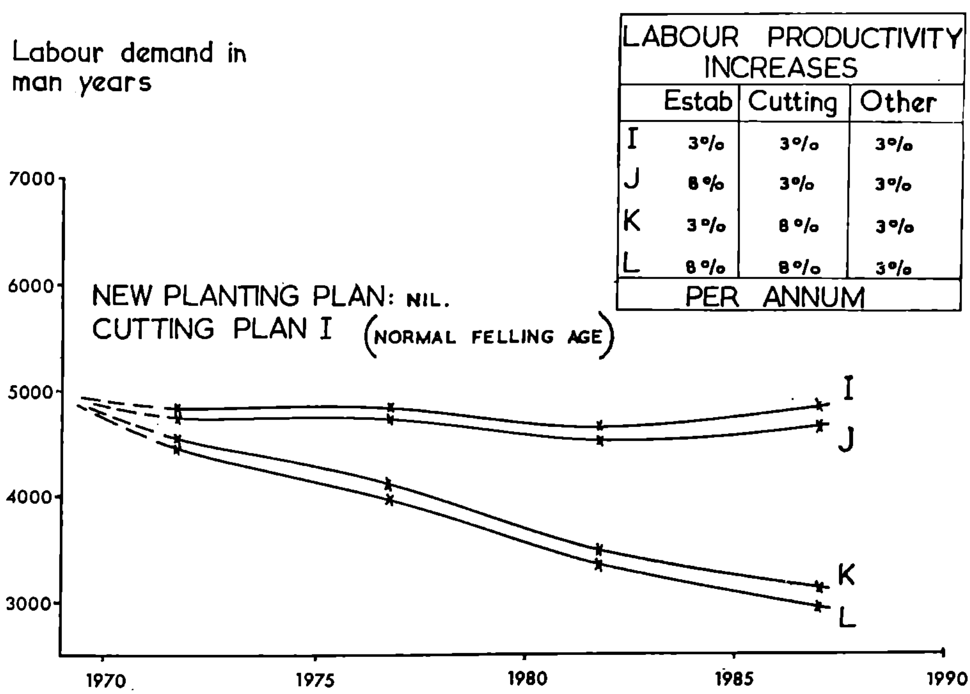

Figure 27. Labour demand for different rates of change of labour productivity: Cutting plan I, new planting nil.

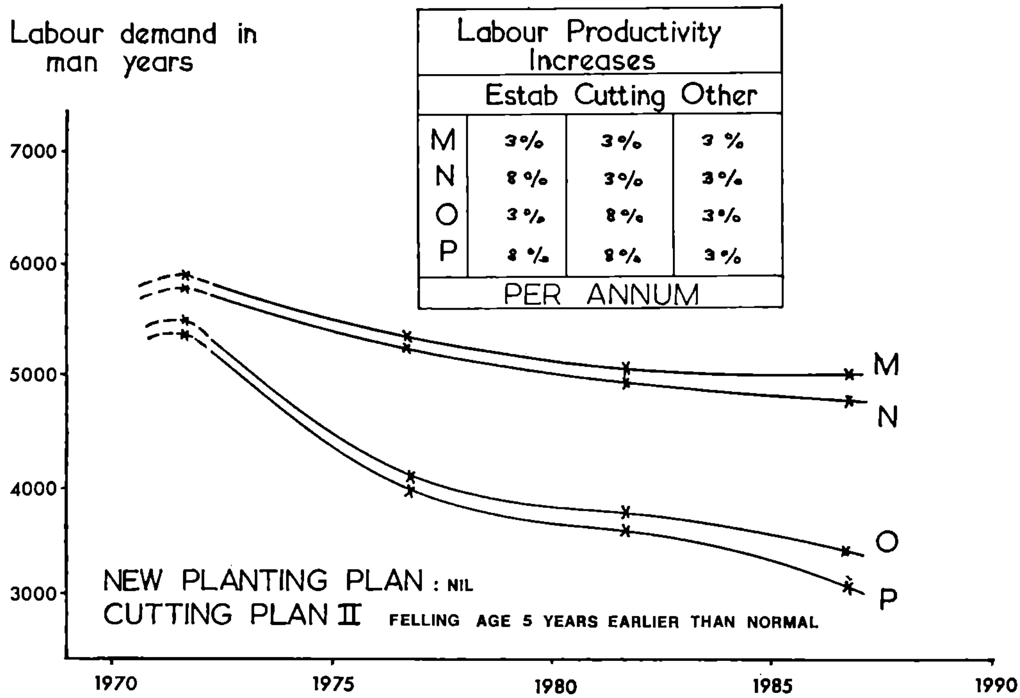

Figure 28. Labour demand for different rates of change of labour productivity: Cutting plan II, new planting nil.

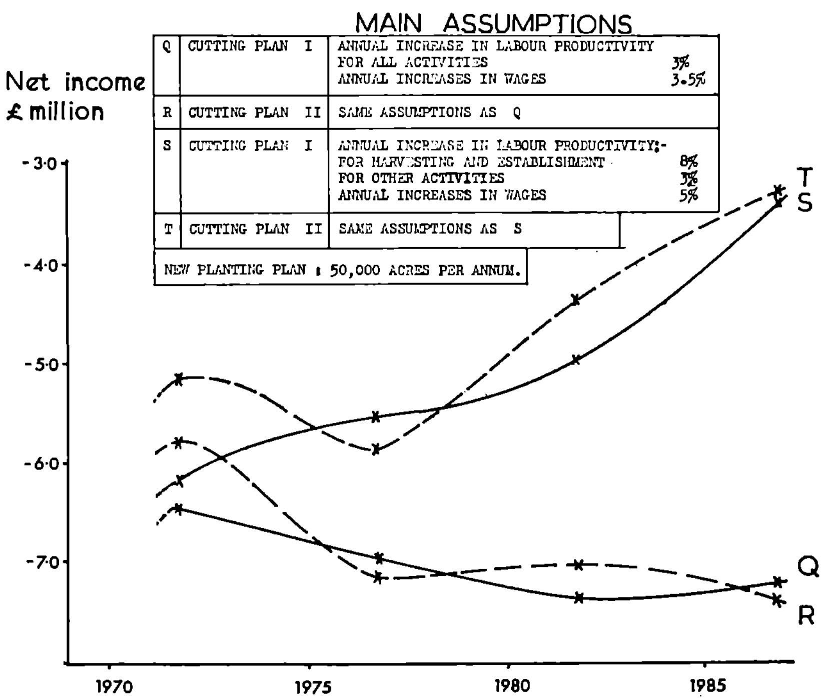

Figure 29. Annual net income: Cutting plan I and II, new planting 50,000 acres per annum for two sets of labour productivity and wage cost assumptions.

up to the time of felling of the existing growing stock. Table 39 shows the N.D.R. generated (over 60 years) by managing the existing estate, for each cutting plan taken together with the new planting plan. This table assumes that restocking is carried out throughout and a flat price/size curve based on recent Forestry Commission prices is assumed. The absolute values shown in these tables is not of particular significance. The relative differences are of far more interest and taken together the three tables will imply that there are only minor differences in the NDR generated by the two cutting plans.

TABLE 38

EVALUATION OF NET DISCOUNTED REVENUE FROM EXISTING GROWING STOCK

	Discount rate	3½%	5%
A	Main assumptions	3% annual increase in labour productivity 3·5% annual increase in wages 0·8% annual increase in machine costs	
		NDR in £ (million)	
	Cutting plan I	214	136
	Cutting plan II	208	141
B	Main assumptions	% annual increase in labour productivity in harvesting 3% annual increase in labour productivity in other activities 5% annual increase in wages 0% annual increase in machine and other costs	
		NDR in £ (million)	
	Cutting plan I	206	130
	Cutting plan II	201	136

N.B. The discounted revenues in this table are netted of direct costs only.

TABLE 39

EVALUATION OF NET DISCOUNTED REVENUE GENERATED BY FUTURE MANAGEMENT OF THE EXISTING ESTATE WITH REPLANTING AND NEW PLANTING

	Discount rate	3½%	5%
A	Main assumptions	3% annual increase in labour productivity 3·5% annual increase in wages 0·8% increase in machine and other costs, based on recent Forestry Commission timber prices	
		NDR in £ (million)	
	Cutting plan I	−112	−94
	Cutting plan II	−112	−92
B	Main assumptions	8% annual increase in labour productivity in harvesting 3% annual increase in labour productivity in other activities 5% increase in wage costs 0% increase in machine and other costs, based on recent Forestry Commission timber prices	
	Cutting plan I	−27	−44
	Cutting plan II	−26	−41

The model output as represented by the above Figures and Tables enable the manager to assess the main results of a change in plan, to identify where constraints are likely to be violated, and to indicate a preference. In the present example a manager will have to balance the disadvantages of the initial fluctuation in production and in the demand for labour against the immediate advantage in net income, since the NDR from the two plans is similar. He might well come to the conclusion that the possible net income gain would be unlikely to compensate for the initial disadvantages and either adhere to the present cutting plan or define a new alternative plan for further evaluation.

Discussion

The model which has been described is of use in evaluating cutting and planting plans and future work will concentrate on utilising the model for this purpose. The sensitivity of different plans to changes in the initial assumptions will require to be tested at different levels of the initial assumptions. It is in this respect that the model is of major use and it is this facility as much as anything else which justifies the cost of construction.

A further area of work lies in the sphere of "flexibility analysis". A cutting and planting plan will be based on assumptions which are, in part, predictions. An initial plan may be optimal with respect to existing predictions but at some time in the future these predictions may change and a new "optimal plan" may need to be specified. The aim of flexibility analysis is to specify an initial plan which maintains a wide range of planning options for the future but which has a low opportunity cost with respect to the optimal plan based on the present assumptions.

Reliability and Deficiencies

The reliability of the output of the present model is not easy to assess. It depends partly on the reliability of the available data. One of the advantages of model construction is not only that data inadequacies which prevent efficient decision making are highlighted, but also that the importance of these inadequacies can be tested by sensitivity analysis. The physical forecasts of volume productions, restocking areas and areas entering the production phase, depend on the specific planting and cutting plans, on the input yield tables and on the growing stock data.

These forecasts are considered to be of reasonable reliability which will be further increased when the results of a recent growing stock survey become available. The N.D.R., employment and net income estimates are inherently less reliable since more predictions of greater uncertainty have to be made in their estimation. Section 5 indicates that the results obtained are generally sensitive to labour productivity changes, wage cost changes and changes in other costs. The reliability of the model in absolute terms depends on further studies of these factors which will lead to better estimates for inclusion in the model. The reliability of the model will also depend on the structure of the model. While certain areas of the model need structural amendment, the effect of present inadequacies are not considered to be a particularly significant source of unreliability. Relativities are as important as absolute values and in this sense the output of the model can be used with some confidence.

The main deficiencies of the existing model lie in its lack of full comprehensiveness and in the interface between model and user.

The model purports to be an enterprise model and it does include the main activities of the Forestry Commission which accounted for 90% of industrial employment and direct expenditure in 1969–70. The model cannot adequately accommodate overhead activities other than on the assumption of them varying directly with scale of programme. Overhead activities amounted to about 30% of the total forest enterprise expenditure in 1969–70. Manpower predictions are essentially of industrial labour where labour demand varies directly with the scale of programme, and supervisory staff cannot be included other than on this basis. In reality staff numbers and overhead costs are not likely to vary directly with scale and this is a deficiency of the model.

Inasmuch as the model is primarily intended as a practical tool for the manager, an important deficiency lies in the interface between the user and the model. If the model is to be used then the means of data input must be made as simple as possible and changes in the details of the data, especially in planting and cutting plan specifications, must be capable of being easily accomplished. At the moment the model is distributed over two different computers each with its own specific input medium. Though forms (Appendix II page 121) are provided for the systematic recording of input data, a manager can only access the model through an intermediary who has special knowledge of the running of the programmes. This does make it more difficult to use the full flexibility of the model to carry out extensive sensitivity analysis.

The deficiencies of the model can be overemphasised and must not be taken out of context. The overall consideration is that despite its limitations the model is a powerful tool in comparison with what was previously available.

Future Development of the Model

The present model must be seen in the context of it being a first attempt and it can be argued that much of its benefit is in the way of technical experience for the model builders. The benefits of experience also extend to the model users as well as the model builders, and after a first attempt the two groups should be in a much better position to communicate and discuss what might be required from future versions of the model.

In any case model building should be regarded as a continual process of development. It is by the running of a working model that one learns what areas are important and need modifying in further development. For example it is becoming clear from our present model that we shall be looking more closely at the maintenance activities and perhaps describing them in more detail in future versions since absolute values and the effects of different felling policies may well be sensitive to the maintenance assumptions. We are also contemplating the addition of further controllable variables such as the proportion of our production which is sold directly to the timber trade, before processing. We may also wish to give the user more latitude in formulating a restocking plan over a given period. The deficiencies in the description of overhead activities have already been mentioned.

Model building must be a continuous process of development partly because the Forestry Commission and its environment is continually changing, usually slowly but dramatic changes cannot be ruled out. A descriptive model must be modified to meet these changes. For example, as at present defined the model takes no direct cognisance of the recreational activities of the Forestry Commission. These could become important over the next few years and future models may find it necessary to incorporate recreation as an activity in its own right. Even before this it may be necessary to make some assumptions about recreation and to incorporate them in the model so that the effect on the criterion variables can be studied before final policy is formulated. Changes in the external environment may take place in a way that demands changes in the structure of the model. The supply of labour is a case in point. The best present estimate is that the supply of labour is not likely to constrain the Forestry Commission's activities. If views on this change then it may be necessary to make structural changes and to incorporate assumptions in the model on future changes in labour supply and also to provide further controllable variables to indicate high priority activities when the assumed supply of labour does not meet the simulated demand.

The need to regard model building as a continuous process has implications for the representation of the model by a computer programme. The programme must be reasonably easy to amend. This will mean that thought must be given to the design of a modular form of programme in which individual modules can be amended or replaced without undue difficulties.

One obvious line for the further development of the model is towards the added sophistication of optimisation. Whether this line will be pursued will depend on the sensitivity analyses to be carried out with the existing model. Linear programming would be the most likely method of optimisation and if this step is taken the present model or its successor would be useful in evaluating the overall consequences of different activities, and also in making some initial selection of the activities that should be included in the linear programme. Thus if this line of development is taken, the work already carried out or envisaged in the near future will have been a valuable preliminary.

The need to input yield tables to represent certain cutting plan consequences is an important restriction. Recent work in the Forestry Commission has enables production to be characterised by mathematical functions, making it more convenient to prepare a computer programme to construct yield tables. Previously a graphical method was used. It should therefore now be possible to incorporate a subroutine within the computer programme representing the model which will itself calculate the yield for a given yield class and cutting regime. This will obviate the need to input yield tables and result in a major gain in the flexibility of the model as well as an appreciable reduction in the time and cost of input.

On a practical plane, further development could take place in the form of the input required for the model. The calculation of the input data shown in Appendix II (see page 121) is time consuming and most of it is prepared from the Forestry Commission financial accounts. The intention is that most of the input data should be in the same format as the accounts and further calculations would be carried out within the computer. Likewise the presentation of final results would be in a somewhat similar form, which is familiar to managers.

The accessibility of the model within the computer will also receive attention. The problem can be met by transferring the model to an on-line terminal, which is now available. Data would be permanently stored within the computer and only changes in this data would be input by the user. In these circumstances the manager himself should be able to test the outcome of different plans quickly and with comparatively little effort.

General Conclusion

The general conclusion that is drawn from the work carried out is that an essentially simple model such as described can be a very useful aid to decision-making. We are operating in a situation where we have little knowledge of the effects of our present long-term plans, or possible alternative plans, on the enterprise as a whole. In such circumstances even a simple model can add greatly to our state of information by helping to map out the boundaries of the possible future results of alternative plans and by highlighting those areas where lack of information affects the quality of decision-making.

REFERENCE

BRADLEY, R. T., CHRISTIE, J. M., AND JOHNSTON, D. R., 1966. *Forest management tables*. Bookl. For. Commn. 16.

DISCUSSION OF MORGAN AND BJORA'S PAPER

Bjora. This model is to do with strategic and tactical planning and has importance in the case where we don't exactly know the objectives, but we do know in which fields there are expected to be objectives, such as net discounted revenue, cash flow, employment and cutting levels. We don't know to what extent they may be relevant. We can review the short-term and long-term effects of courses of action in respect to these fields. We are working in a situation of uncertainty and can only make more or less reliable predictions about such things as labour productivity. With this model we can examine the effects of several sets of assumptions to test the robustness of a given course of action. It doesn't give optimal solutions but at this stage I think the question of optimisation is less important than enlightening the future. *Jackson*. How do you handle overheads. *Morgan*. At the moment these are being dealt with outside the model. *Paillé*. How does this relate to management games and forest management simulations being developed in the United States for example. *Morgan*. There is no essential difference. We make no pretence of doing anything new or revolutionary but wish to demonstrate things that we in the Forestry Commission have found it desirable to emphasise. *Wardle*. I think this is a valuable type of contribution. It is in contrast to micro-economic studies of the optimal way of cutting individual stands or the optimal roading densities. Instead it takes assumptions about cut and road density and calculates the volumes of wood which will flow in the economy, the labour force and profitability if the practices are adopted, which I would have thought was most appropriate.

APPENDIX I

DETAILED DESCRIPTION OF ENTERPRISE MODEL

Growing Stock Categories (See also Table 36, page 108).

B: that part of existing growing stock which is at or older than the age of first thinning defined by fB_t (see page 118 for definition of fB_t).

C: that part of existing growing stock which is younger than fB_t. All new planting (excluding restocking) is also regarded as being within this category.

D: growing stock derived from future restocking activities.

A_Y Total growing stock in year Y.

B_{t,k_Y} That part of growing stock category B which is classified as yield class t and which is in age class k in year Y.

C_{t,k_Y} That part of growing stock category C which is classified as yield class t and which is in age class k in year Y.

D_{t,k_Y} That part of growing stock category D which is classified as yield class t and which is in age class k in year Y.

$IB_{t,k}$ Component of cutting policy. $IB_{t,k}$ is the thinning intensity (volume thinned per year per unit area) assumed for growing stock category B, of yield class t, in age class k.

$IC_{t,k}$ As for $IB_{t,k}$ except that growing stock category C is involved.

$ID_{t,k}$ As for $IB_{t,k}$ except that growing stock category D is involved.

LAC_Y Total land costs in year Y.

LC_j Cost of land per unit area in site treatment type j, in year O.

LD_Y Total demand for labour in year Y.

LDE_Y Total demand for labour (in year Y) generated by establishment operations.

LDH_Y Total demand for labour (in year Y) generated by harvesting operations.

LDM_Y Total demand for labour (in year Y) generated by protection and maintenance operations.

LDR_Y Total demand for labour (in year Y) generated by road building operations.

LE_j Labour demand per unit area of establishment operations in site cost type j, in year O.

LHf_i Labour demand per unit volume for harvesting operations in clear fellings, for girth class i, in year O.

LHt_i Labour demand per unit volume for harvesting operations in thinnings, for girth class i, in year O.

LM Labour demand per unit area of forest for protection and maintenance operations, in year O.

LRE Labour demand per unit area of new planting, for road construction, in year O.

LRH Labour demand per unit area of forest at first thinning stage (X_Y), for road construction, in year O.

MCE_Y Total machine and material costs of establishment operations in year Y.

MCH_Y Total machine and material costs of harvesting operations in year Y.

MCM_Y Total machine and material costs of protection and maintenance operations in year Y.

MCR_Y Total machine and material costs of road operations in year Y.

ME_j Machine and material costs per unit area of establishment operations in site cost type j, in year O.

MHf_i Machine and material costs per unit volume for harvesting operations in clear fellings, in girth class i, in year O.

MHt_i Machine and material costs per unit volume for harvesting operations in thinnings, in girth class i, in year O.

MM Machine and material costs per unit area of protection and maintenance, in year O.

MRE Machine and material costs of road construction per unit area of new planting, in year O.

MRH Machine and material costs of road construction per unit area of forest at stage of 1st thinning in year O.

NI_Y Net income in year Y.

NDR_Y Net discounted revenue generated up to and including year Y, discounted to year O.

$P_{t,j,Y}$ New planting carried out on treatment type j, yield class t in year Y (planting policy).

R_i Revenue per unit volume in girth class i, in year O.

$SB_{t,k}$ Volume felled per unit area of growing stock category B, in yield class t, in age class k.

$SC_{t,k}$ As for $SB_{t,k}$ except that growing stock category C is involved.

$SD_{t,k}$ As for $SB_{t,k}$ except that growing stock category D is involved.

TR_Y Total revenue in year Y.

$Vf_{Y,i}$ Volume in girth class i, obtained from clear fellings in year Y.

$Vt_{Y,i}$ Volume in girth class i, obtained from thinnings in year Y.

$W_{Y,j}$ Total area of restocking in treatment type j in year Y.

WG_Y Total wage costs in year Y.

WGE Wage cost per unit of labour demand, for establishment operations in year O.

WGH Wage cost per unit of labour demand, for harvesting operations, in year O.

WGM Wage cost per unit of labour demand, for protection and maintenance operations, in year O.

WGR Wage cost per unit of labour demand, for road construction, in year O.

X_Y Total area at first thinning stage in year Y.

Y No. of years elapsed from start of simulation (takes values 1........80).

a_i Given mensurational factor indicating proportion of total volume removed in thinning or felling which falls into girth class i,

d Factor by which area at first thinning stage, in growing stock category D, is decreased.

eB_t Component of cutting policy. No. of years from start before r_2B_t is used in cutting policy.

eC_t As for eB_t except that growing stock category C is involved.

eD_t As for eB_t except that growing stock category D is involved.

fB_t Component of cutting policy. f_{Bt} is the age of first thinning for crop in growing stock category B, in yield class t.

fC_t As for fB_t except that growing stock category C is involved.

fD_t As for fB_t except that growing stock category D is involved.

i Subscript denoting girth class (takes values 1–5).

j	Subscript denoting site treatment type for establishment (takes values 1–6 for new planting, 1–3 for re-stocking).
k	Subscript denoting age class (takes values 1–80).
le_1	Rate of change of labour demand per annum per unit area in establishment operations.
lh_1	Rate of change of labour demand per annum per unit volume in harvesting operations.
lm_1	Rate of change of labour demand per annum per unit area in protection and maintenance operations.
lr_1	Rate of change of labour demand per annum per unit area in road construction.
me_1	Rate of change of machine and material costs per annum per unit area, in establishment operations.
mh_1	Rate of change of machine and material costs per annum per unit volume, in harvesting operations.
mm_1	Rate of change of machine and material costs per annum per unit area in protection and maintenance operations.
mr_1	Rate of change of machine and material costs per annum per unit area, in road construction.
n	Total number of yield classes.
p_1	Rate of change of timber price per unit volume per annum.
q_1	Rate of change of land cost per unit area per annum.
r_1B_t	Component of cutting policy. r_1B_t is the first felling age used for growing stock category B in yield class t.
r_1C_t	As for r_1B_t except that growing stock category C is involved.
r_1D_t	As for r_1B_t except that growing stock category D is involved.
r_2B_t	Component of cutting policy. r_2B_t is the felling age used for growing stock category B, in yield class t, after eB_t years of simulation have elapsed.
r_2C_t	As for r_2B_t except that growing stock category C is involved.
r_2D_t	As for r_2B_t except that growing stock category D is involved.
t	Subscript used to denote yield class ($t = 1...n$).
we_1	Rate of change of wages cost per annum in establishment operations.
wh_1	Rate of change of wages cost per annum in harvesting operations.
wm_1	Rate of change of wages cost per annum in maintenance operations.
wr_1	Rate of change of wages cost per annum in road construction operations.
z	Discount rate.

Cutting policy for yield class $t \equiv fB_t;\ lB_{t,k};\ r_1B_t;\ eB_t;\ r_2B_t;\ fC_t;\ lC_{t,k};\ r_1C_t;\ eC_t;\ r_2C_t;\ fD_t;\ lD_{t,k};$ $r_1D_t;\ eD_t;\ r_2D_t;$

Planting policy for yield class $t \equiv P_{t,j,Y}$ ($t = 1...n,\ j = 1...6,\ Y = 1...20$)

$$B_Y = \sum_{t=1}^{n} \sum_{k=1}^{80} B_{t,k_Y}$$

$$C_Y = \sum_{t=1}^{n} \sum_{k=1}^{80} C_{t,k_Y}$$

$$D_Y = \sum_{t=1}^{n} \sum_{k=1}^{80} D_{t,k_Y}$$

$$Vt_{Y,i} = \sum_{t=1}^{n} \sum_{k=fB_t}^{(rB_t-1)} B_{t,k_Y}.\ IB_{t,k}.\ a_i + \sum_{t=1}^{n} \sum_{k=fC_t}^{(rC_t-1)} C_{t,k_Y}.\ IC_{t,k}.\ a_i + \sum_{t=1}^{n} \sum_{k=fD_t}^{(rD_t-1)} D_{t,k_Y}.\ ID_{t,k}.\ ai$$

$$Vf_{Y,j} = \sum_{t=1}^{n} \left(\sum_{k=rB_t}^{80} B_{t,k_Y}.\ SB_{t,k}.\ a_i + \sum_{k=rC_t}^{80} C_{t,k_Y}.\ SC_{t,k}.\ a_i + \sum_{k=rD_t}^{80} D_{t,k_Y}.\ SD_{t,k}.\ a_i \right)$$

$$W_{Y,j} = \sum_{t=1}^{n} \left(\sum_{k=rB_t}^{80} B_{t,k_Y,j} + \sum_{k=rC_t}^{80} C_{t,k_Y,j} + \sum_{k=rD_t}^{80} D_{t,k_Y,j} \right)$$

$$X_Y = \sum_{t=1}^{n} B_{t,fBt_Y} + C_{t,fCt_Y} + D_{t,fDt_Y}\ .d$$

$$P_Y = \sum_{t=1}^{n} \sum_{j=1}^{6} P_{t,j,Y}$$

$$A_Y = B_Y + C_Y + D_Y + P_Y$$

For lh_Y, le_Y, lr_Y, lm_Y, wh_Y, we_Y, wr_Y, wm_Y, mh_Y, me_Y, mr_Y, mm_Y, p_Y, q_Y all of the form x_Y, then if $\alpha = lh_1, le_1, lr_1$...etc

$$x_Y = \left(1 + \frac{\alpha}{100}\right)^Y \quad (0 \leqslant x_1)$$

$$x_Y = \left(\frac{1}{1 + \left|\frac{\alpha}{100}\right|}\right)^Y \quad (0 > x_1)$$

$$LDH_Y = \sum_{i=1}^{5} (Vt_{Y,i} \cdot LHt_i \cdot + Vf_{Y,i} \cdot LHf_i) \cdot lh_Y$$

$$MCH_Y = \sum_{i=1}^{5} (Vt_{Y,i} \cdot MHt_i \cdot + Vf_{Y,i} \cdot MHt_i) \cdot mh_Y$$

$$LDE_Y = \sum_{j=1}^{6} P_{Y,j} \cdot LE_j \cdot le_Y + \sum_{j=1}^{3} W_{Y,j} \cdot ME_j \cdot le_Y$$

$$MCE_Y = \sum_{j=1}^{6} P_{Y,j} \cdot ME_j \cdot me_Y + \sum_{j=1}^{3} W_{Y,j} \cdot ME_j \cdot le_Y$$

$$LDR_Y = (P_Y \cdot LRE \cdot + X_Y LRH)\, lr_Y$$

$$MCR_Y = (P_Y \cdot MRE \cdot + X_Y \cdot MRH)\, mr_Y$$

$$LDM_Y = A_Y \cdot LM \cdot lm_Y$$

$$MCM_Y = A_Y \cdot MM \cdot mm_Y$$

$$TR_Y = \sum_{i=1}^{5} (Vt_{Y,i} + Vf_{Y,i}) \cdot R_i \cdot p_Y$$

$$LAC_Y = \sum_{j=1}^{6} (P_{Y,j} \cdot LC_j \cdot q_Y)$$

$$TV_Y = \sum_{i=1}^{5} (Vt_{Y,i} + Vf_{Y,i})$$

$$LD_Y = LDH_Y + LDE_Y + LDR_Y + LDM_Y$$

$$WG_Y = LDH_Y \cdot WGH \cdot wh_Y + LDE_Y \cdot WE\ we_Y + LDR_Y \cdot WGR \cdot wr_Y + LDM_Y \cdot WGM \cdot wm_Y$$

$$TMC = MCH_Y + MCE_Y + MCR_Y + MCM_Y$$

$$NI_Y = TR_Y - WG_Y - TMC_Y - LAC_Y$$

$$NDR_Y = NDR_{Y-1} + NI_Y \Big/ \left(1 + \frac{Z}{100}\right)^Y$$

TABLE 40

APPENDIX II

INPUT VARIABLES

		BHQG CLASS				
		2²—6°	6′—9°	9′—12°	12′—15°	> 15′
Present labour demand per 1,000 H ft ('000 working hours)	Felling					
	Thinning					

ESTABLISHMENT	Treatment type	1	2	3	4	5	6
Present labour demand per 1,000 acres ('000 working hours)							

WAGES	Harvesting	Establishment	Roads	Maintenance
Present weighted wages for each operation (£'000 per 1,000 working hours)				

ROADS	Roads (Establishment)	Roads (Harvesting)
Present labour demand per unit for roads ('000 working hours per 1,000 acres)		

PROTECTION AND MAINTENANCE	
Present labour demand per unit of maintenance and protection ('000 working hours per 1,000 acres)	

TABLE 40—*continued*

ESTABLISHMENT	Treatment type	1	2	3	4	5	6
Present machine costs and sundries per unit (£'000 per 1,000 acres)							

HARVESTING	BHQG CLASS				
	2²—6°	6′—9°	9′—12°	12′—15°	⩾ 15′
Present machine costs and sundries per unit	Felling				
(£'000 per 1,000 H ft)	Thinning				

ROADS	Roads (Establishment)	Roads (Harvesting)
Machine costs and sundries, (£'000 per 1,000 acres)		

MAINTENANCE AND PROTECTION		
Machine costs and sundries, (£'000 per 1,000 acres)		....

TIMBER PRICES	BHQG CLASS				
	2²—6°	6′—9°	9′—12°	12′—15°	⩾ 15′
Present timber price per unit (£'000 per 1,000 H ft)					

LAND PRICES	Treatment type	1	2	3	4	5	6
Present price of land (£'000 per 1,000 acres)							

INTEREST RATE		COEFFICIENTS INDICATING RATES OF CHANGE					
		Labour Productivity, Cutting		Labour Productivity, Establishment		....	....
Alt. 1*	Alt. 2*	Alt. 1	Alt. 2	Alt. 1	Alt. 2	....	....
						....	....

TABLE 40—*continued*

Labour Productivity Road Building		Labour Productivity Maintenance and Production		Sundry and Machine Costs, Cutting		Sundry and Machine Costs, Maintenance	
Alt. 1	Alt. 2	Alt. 1	Alt. 2	Alt. 1	Alt. 2	Alt. 1	Alt. 2

Sundry and Machine Costs, Roads		Sundry and Machine Costs, Maintenance and Protection		Annual Wages, Cutting		Annual Wages, Establishment	
Alt. 1	Alt. 2	Alt. 1	Alt. 2	Alt. 1	Alt. 2	Alt. 1	Alt. 2

Annual Wage, Roads		Annual Wage, Maintenance and Protection		Annual Change in Timber Price		Annual Change in Land Price	
Alt. 1	Alt. 2	Alt. 1	Alt. 2	Alt. 1	Alt. 2	Alt. 1	Alt. 2

TOTAL FOREST AREA ('000 acres)		....

'000 HOURS PER MAN YEAR		....

* Alt. 1, Alt. 2 refers to alternative values, only one value is included in the simulation during any single run.

Species No.	Yield class	GROWING STOCK (acres)							
		01–10 yrs	11–20 yrs	21–30 yrs	31–40 yrs	41–50 yrs	51–60 yrs	61–70 yrs	> 71 yrs

Paper 14

THE LINEAR PROGRAMMING PLANNING SYSTEM OF THE SWEDISH FOREST SERVICE

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Linear programming implemented

The linear programming technique is often thought of as a tool of operations research intended to help the analyst to find optimal solutions to allocation problems. As such it is successful insofar as it leads to optimal solutions with less computational effort than other techniques of operations research.

The central theme of this paper is that the optimal solution aspect of linear programming tends to lose importance at the stage of implementation and that other considerations such as sensitivity analysis and presentability tend to come into the foreground. It is maintained by the author that the dual prices associated with a linear programming solution can be of great help at this stage.

This in itself is nothing new. It is hoped, however, that a presentation of the experiences of the Swedish Forest Service (S.F.S.) may stimulate discussion on the problems of implementing linear programming in forestry.

The need for a system of economic planning

Since the beginning of this century an inventory of the land of the forest districts has been made approximately every ten years. The first step of the inventory is to divide the forest land into compartments. The compartments are mapped and constitute the basis for the registration of data collected in the inventory. Data for each compartment are available in a compartment description list.

At the inventory a recommendation is made as to the next operation to be performed on every compartment. (The economic planning system, generally speaking, assumes that this recommendation is valid and is only concerned with the preference ordering of the compartments and the operation that "goes with it".)

Before the introduction of the linear programming planning system the available data was condensed or stratified into fairly wide classes or categories such as age class, cutting class, etc. This condensed information was presented in the inventory report and was in this form available as a basis for policy decisions. Economic computations of revenues and costs on the basis of these wide categories could not be very detailed or accurate. On the other hand, the need for detailed and accurate revenue and cost estimates was not felt to be especially large, since the long-range forestry plans were based on biological data.

The need for an economic planning became evident when prices dropped during the 1960's. In order to show at least a nominal profit the central administration had to impose economic restrictions on administrative regions and on the forest districts (subdivisions of the administrative regions). These budget restrictions took the form of a demand for a minimum surplus* from cuttings and thinnings and set an upper limit for the expenditure on silvicultural measures. They were delivered to the administrative regions at the beginning of the budget cycle so that they could be taken into account during the budget work.

(*Note:* *In this paper the term "surplus" is defined in the following way: surplus = revenue—cutting cost—cost for lumbermen's huts, etc.—terrain transport cost—road transport cost—cost of planting a new stand in the case of clear-cuttings.)

The time element was more or less neglected when surplus demands were first used as a budget tool. The selection of compartments for treatment is in itself a decision that has to be taken about five years in advance. Road building, fertilisation and investment planning require that a preliminary selection is made that far in advance. Marginal changes can however be made at shorter notice. Experience suggested that surplus demands should be used within a budget system that included one short run and one long run horizon of 1–2 years and 5–10 years respectively.

It was soon found that considerable deviations from surplus demands had to be permitted in the final budget. In many cases it was established that the revenue and cost estimates were misleading. As a rule the error resulted from the use of stratified data for the computations.

In a few cases the difference between the estimate of the central administration and that of the responsible forest officer of an achievable surplus could not be analysed. Again, the stratified basis for the computations was at fault since the surplus estimate could not be broken down into sufficient detail.

The experience from this method of directing the budget work was nevertheless positive, since it focused attention on revenues and costs and demonstrated "at the grass-roots" that the S.F.S. is to be run according to business principles. It was however obvious that accepting it as a standard procedure would seriously endanger the fulfilment of long range objectives. There was also a danger of sub-optimising in the sense that the central administration may insist on a high surplus from one administrative region when the amount in question could be obtained in another region with much less sacrifice in long range objectives.

Summing up, then, the S.F.S. needed (1) a method of quantifying long range objectives in the budget procedure and of balancing them against short range objectives, (2) a method of grouping the relevant compartments into "temporary strata" on which computations of revenues and costs could be based, (3) a method that insured that the balance between long range and short range objectives was in reasonable agreement in different regions or districts, (4) the integration of five to ten year planning with short range planning and with the budget, information and accounting systems.

Translating long range objectives into a goal function

Ideally the objective function in forestry planning should be based on what we may call an investment theory formulation of the long range objectives. That is to say, the policy should be so strictly defined that it is possible to compare the present value (computed on the basis of the relevant social rate of discount) of a particular operation (such as planting, thinning or cutting a specified forest stand in a specified period) with the present value of every other operation and/or sequence of operations. The investment theory formulation would not exclude "non-economic" considerations such as maintaining a minimum production level in one or several future periods. It would require, however, that such policy considerations were formulated as quantified policy constraints or—in the dual formulation—as the permissible "scarcity" values of such constraints. It has been demonstrated by Wardle (1964) that a considerable number of constraints may be needed in order to formulate current policy.

Whether these policy constraints are explicitly formulated in the linear programming model or incorporated into the objective function of the model depends on the planning horizon. If, for instance, the LP model is designed for formulating a five to ten year program it seems natural to include policy constraints regarding the production level in the year 2010 in the objective function. Policy formulations regarding the regional distribution of the volume of output during the planning period could best be formulated as an explicit constraint.

An investment theory approach is no doubt an ideal to strive for. There are several reasons why we can never hope to realise it to the full. The mathematical set-up will rapidly become unwieldy if we try to maintain detail in a multi-period analysis. (It should be pointed out, however, that simulation techniques open up great possibilities when it comes to handling this type of problem.) Management will find it difficult to conceive the interconnection between to-day's operations and a set of quantitative constraints concerning the future. Since management is accustomed to "aspiration level thinking"—that is, to making gradual adjustments in policy on the basis of current results—they will be uncomfortable at the idea of committing themselves to rigid policy formulations regarding the future. Furthermore, if the objective function is to be used as a basis for budget discussions with responsible officers it had better be translated into terms that are somewhat less abstract than "weighted Lagrange multipliers of policy commitments". In the year to year budget work there will also be a need for a consistent measure of the long range effect of different operations.

The S.F.S. can be said to have used a preference approach to determine the objective function. The starting point has been tables of land values and of the value of growing forest. (These tables are commonly used in Swedish forestry not only for evaluating real estate but also for making those yield percentage calculations that are prescribed by law for determining whether it is permissible to cut growing forest.) The central idea behind the preference function as it is used in the planning system of the S.F.S. is to evaluate that part of the effect of an operation that falls beyond the planning horizon by making policy decisions about the relative desirability of different operations.

For example, management may decide that those plots on which it was recommended at the inventory that the operation of planting should be performed should have a higher priority than similar plots on which the operation of thinning was recommended. This can be achieved by assigning a preference function value to planting that is greater than the preference function value of thinning.

The priority of forest plots on which the same operation is to be performed can however be decided on by the use of land values or by the use of the value of growing stock. Thus, if we compare two forest plots on which the same operation is to be performed we may find it reasonable to say that the priorities should be proportional to their land values or to their growing stock value. (The decision to rely on the land or growing stock values is of course in itself a policy decision.) To obtain a complete objective function we only have to express the objective function value of every operation as a certain percentage of land or growing stock value.

For example, the operation of planting a plot of forest land may be estimated to have an objective function value of $x\%$ of the land value of the plot. The cost of planting, on the other hand, falls within the planting period and is deducted from the surplus of the forest district.Thinning in a stand of a certain age in a specified site class may represent an improvement in future yield that can be evaluated to $y\%$ of the value of the growing stock. Clear-cuttings typically give a surplus within the planning period, but they also add to the long range objectives since the plot is cleared for a new forest generation that is expected to grow faster and give a better economic yield than the old one. Therefore, clear-cuttings may be assigned an objective function value of $z\%$ of land value. It has been the task of the top management of the S.F.S. to determine the values of x, y, $z\%$ etc. that seemed to give a "reasonable" preference ordering of different measures. On the basis of these percentages the addition in the achievement of long range objectives—referred to as W-values—is computed for every compartment. These W-values constitute the objective function of the planning model.

The evolution of computer techniques has made it possible to have the entire area description readily available for a computer on a magnetic tape. An up-dating routine can easily change data with respect to currently undertaken cuttings and silvicultural measures, recent growth, etc. It is also possible to perform computations of revenues, costs, manpower and machinery requirements for each compartment and store this information on another magnetic tape. The capacity of the computer makes it feasible to use all the information in the compartment description list as indicators for the estimates and to handle computations of a complicated and selective nature.

To compute revenues, costs or requirements for aggregates—for example, the cost of cutting a specified volume in a district—the computer needs a specified list of those compartments that are to supply this volume. The selection of compartments is a difficult problem. There are, for example, many cutting plans that can produce the required output. Linear programming can be seen as a tool enabling the planner to find the relevant aggregates.

The use of linear programming solutions in the budget procedure

The linear programming model used by the S.F.S. for five to ten year planning can be described technically in the following way: Set a value to each of the variables x_1, x_2,...x_m corresponding to the activity of performing the forestry operation recommended at the inventory on each of the 500-1,000 compartments of a forest district. The selection should maximise the value of the objective function, w_1, w_2,...w_m, which evaluates that part of the effect of the operation that falls beyond the planning horizon of the five-year (or ten-year) period. The selection should take into consideration the constraints b_1, b_2,...b_n, for example on surplus, maximum volume to be cut, maximum or minimum amount to be cut during the winter season and the amount of timber or pulpwood from pine and spruce. Furthermore there are constraints specifying that $0 \leqslant x_j \leqslant 1$. (Since the solution in most cases is bounded by a small number of restrictions, most of the x_j:s will automatically take on integer values.)

The solution obtained from the LP model is of course optimal only in a very technical sense. The optimal solution of the LP model is subject to errors in input data and to the simplifying assumptions of the model formulation. At the S.F.S. the problems of implementation have to a large extent been concerned with these two types of error or deviation.

The most striking simplifying assumption in the model is the neglect of "concentration". The model regards the benefits and costs of a compartment as being independent of the operations performed on nearby compartments. In reality considerable gains can be obtained by performing "heavy operations" (i.e. cuttings or thinnings in old stands) on compartments that are close to each other.

To obtain a "true" optimal solution it is necessary to substitute some of the isolated compartments chosen by the LP model for some non-selected compartments located in the vicinity of other compartments. Substitutions are however profitable only if the gain from greater concentration compensates for the loss in optimality of the LP solution.

In the IBM Mathematical Programming System/360 (described in the IBM manual with the code number H20—0476—1) the loss in optimality can be read off immediately from the data list. To every integer constraint $0 \leqslant x_j \leqslant 1$ there corresponds a dual value ("reduced cost" in the IBM terminology) which gives the marginal increase or decrease in the value of the objective function that would be obtained if a marginal change were made in that variable. That is to say, if we contemplate a substitution we can register (*a*) the loss in optimality resulting from the elimination of the variable included in the optimal solution (*b*) the loss in optimality resulting from the insertion of a variable not represented in the optimal solution. The sum of (*a*) and (*b*) represents a loss (measured in terms of the values of the objective function) from the point of view of the LP model. It has to be compensated for by better concentration.

The errors of the input data can be divided into systematic errors and stochastic errors. Systematic errors in the inventory or in the benefit, cost or goal functions are eliminated as far as it is feasible. Some of them, for instance local differences in benefits and costs, are regarded as "experience factors" and are corrected for by means of consulting and "experience file".

(*Note:* *So far a considerable amount of extra information (with no direct relevance for the planning or budgeting procedure) such as the profitability of marginal quantities delivered to various buyers have been obtained by means of manual calculations on the basis of output data. These manual calculations—which the staff of the planning section may be able to perform on a desk calculator in a day or two—will no doubt constitute an important by-product of the data planning routine.)

The remaining stochastic error in the estimate of the surplus from a compartment is relatively large. For a compartment with a revenue of, say 55 Sw crowns and a cutting and transportation cost of 25 Sw crowns the standard deviation of the surplus may be 5 Sw crowns. That is to say, if the computations of the planning model indicate a surplus of 30 Sw crowns this must be interpreted at the two standard deviation levels as an interval of 20-40 Sw crowns.

This uncertainty complicates implementation at the local level. The solution of the LP model will suffer from what the statistician calls type I and type II error. That is to say, we have adopted in our plans compartments that should have been left out, and left out some that should have been included. This is demonstrated in figure 30.

The solid, S-shaped curve of figure 30 illustrates the model estimate of the LP value, that is to say the "reduced cost" of the different compartments. (The compartments are ordered so that those that have the higher LP value per cubic metre of output are placed to the left.) The shaded area between the dotted line represents the uncertainty interval. Significant type I or type II errors are to be found in the compartments producing the quantity q' to q''.

We can now distinguish between those compartments that produce the quantity to the left of the q'—border, the intermediate category where we may have type I or type II errors and the category to the right where the exclusion from the program is not subject to significant error.

In the first or third category the central planner can make a fairly definite recommendation as to whether a certain compartment should be included

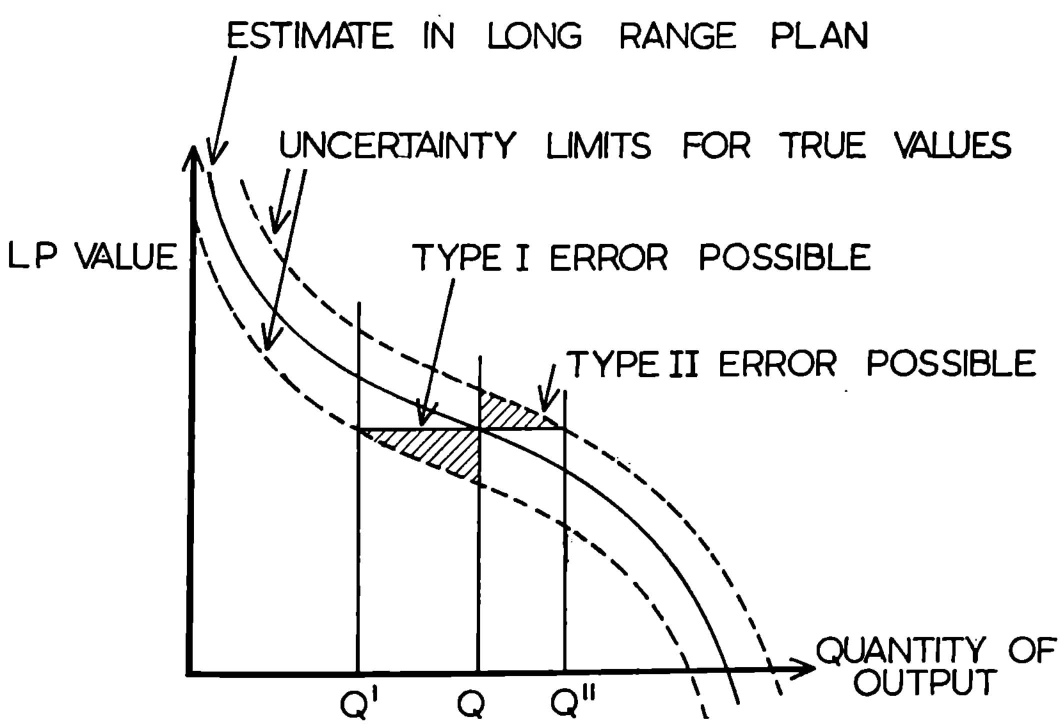

Figure 30. Stochastic errors in the estimated LP value of output.

in, or excluded from, the program. Errors of type I or type II should not affect the inclusion or exclusion of these compartments. Also, since making a substitution between the two categories would incur a considerable loss in LP optimality it is fairly unlikely that it should be profitable to make such substitutions.

As regards the in-between category, errors are large enough to affect the solution. If no better information were available the best decision under uncertainty would nevertheless be to use inventory data. Knowing that a compartment is in the uncertain area we may however look to local experience to give us further information. (This is somewhat similar to a multi-stage decision rule.) Furthermore, it is to be expected that the need for modifications on account of concentration will be comparatively great in the intermediate category.

On account of the errors in the inventory data and because of the "disregarded" need for concentration, the demand in the budget procedure for surplus (and for other aggregates such as the amount of timber or pulpwood from pine and spruce, the use of manpower and the amount produced in winter) cannot be selected on the basis of the LP solution alone. Allowance must be made for the fact that there are errors in data and that the LP solution must be modified to permit the necessary concentration.

Thus, the central planner must try to acquire systematic knowledge concerning the magnitude of the necessary deviations from the LP solution. Our experience within the S.F.S. has been limited. In a simplified pilot study the necessary reduction in surplus amounted only to 2–3 per cent of total surplus. It can be expected that additional modifications must be made under "real-life" conditions. In the official five-year plan—which is based on the figures of the linear programming planning model—this margin has been estimated to amount to 7 per cent of total surplus. This margin takes into account that the planning model will not be implemented at the forest district level until the end of the five years period. An adjustment in the magnitude of 3–5% is perhaps a reasonable estimate of the margin that will be necessary in the long run.

The errors in data and the need for local modifications of the LP solution must be kept in mind when the responsibility of the district officer is defined. The general lines along which the S.F.S. is trying to develop a standard long term budget procedure are briefly as follows:

(1) A decision on the general guide-lines for the long term budget is taken by the central authority on the basis of a modified solution to the linear programming model. (As will be shown in the following section of this paper the decision is really based on one LP solution for every district and on a master programme that co-ordinates these solutions.)

(2) Surplus demands and demands for the output of timber and pulpwood from pine and spruce, demands for maximum or minimum employment, etc. are distributed to the administrative regions. These demands are broken down by forest district and can—after readjustments at the level of the administrative region—be distributed to the forest districts.

(3) The forest district checks the feasibility of the demands by making a detailed plan. The data list of the LP solution for the forest district is distributed to the responsible officer and constitutes the starting point for his work. He is instructed to regard the selection made by the LP model as a fairly strong recommendation when it comes to the first category. In the intermediate category the recommendation of the LP model should be supplemented by local experience to a greater extent.

The detailed plan of the forest district is discussed with the management of the administrative region. In the case where it is found that it would be impossible to implement the demands of the central authority, special studies should be made to determine whether this depends on errors in revenue computations, costs computations, etc., or whether there are special reasons for making extra modifications to the LP solutions.

(4) Regular comparisons between inventory data and data obtained when trees are marked and registered (this is done up to two years before cutting or thinning) should be performed at the forest district level and reported to the regional and central administrations.

Co-ordinating the plans of different forest districts with policy decisions

The total forest area of the S.F.S. is divided into 8 administrative regions that are in turn subdivided into 68 forest districts. The boundaries of the forest districts are, generally speaking, set so as to facilitate the planning and supervision of production. The shape of the administrative regions on the other hand is mainly determined by market factors.

The LP programme used in the economic long-term planning of the S.F.S. uses a considerable amount of computer capacity. Doubling the size of the LP programme at this stage would incur processing costs that are at least four times as high. The cost of combining the forest districts in an entire administrative region or in the country as a whole into one

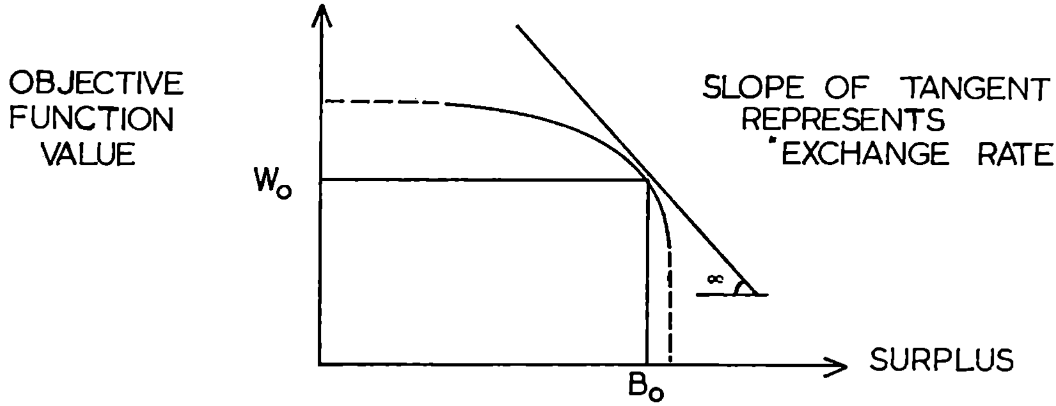

Figure 31. The convex set of surplus and objective function values.

programme would be prohibitive. The necessary co-ordination has to be secured by other means.

The need for co-ordination arises when several LP programmes have a constraint in common. The best example is the constraint on minimum surplus. Co-ordination must insure that the marginal rate of substitution of surplus for long term objectives is the same in every district. If there were discrepancies, a re-allocation of surplus demands would permit a better realisation of long range objectives.

The technique used by the S.F.S. is a simplified version of the decomposition programming described by Dantzig (1963). The decomposition principle requires that every term in the matrix row of the constraint in question is multiplied by an arbitrary scalar or constant and added to (or subtracted from) the objective function. The new objective function is inserted into all LP programmes. Solutions to the modified programmes are worked out. The solutions are fed into a master programme that gives a new and more nearly optimal scalar value. This re-iterative algorithm goes on until the master programme indicates that an optimal solution has been reached.

In the method used by the S.F.S. the scalar value is determined by judgement. The scalar value of the surplus constraint represents the marginal rate of substitution of the last surplus crown for the last "future crown"—i.e. the latest increment in the objective function. In the optimal solution the scalar value will be equal to the dual value of the surplus constraint. The scalar value is often referred to as the "exchange rate" since it determines the long run sacrifices necessary to increase current surplus by comparing "real crowns" with "objective function crowns".

The meaning of this exchange rate is demonstrated in figure 31. Surplus is measured along the abscissa and the value of the objective function along the ordinate. The slope of the curve indicates that increases in surplus will eventually lead to considerable reductions in long range objectives and vice versa. There is however an interval within which reasonable policy decisions can be made. The point where the line is tangential to the curve demonstrates one such point. This point will give a surplus of B_0 and a value of the objective function value of W_0. The slope of the tangent to the curve illustrates the "exchange rate".

Figure 32 demonstrates how the application of the same exchange rate co-ordinates the solution of several forest districts. Changes in the exchange rate

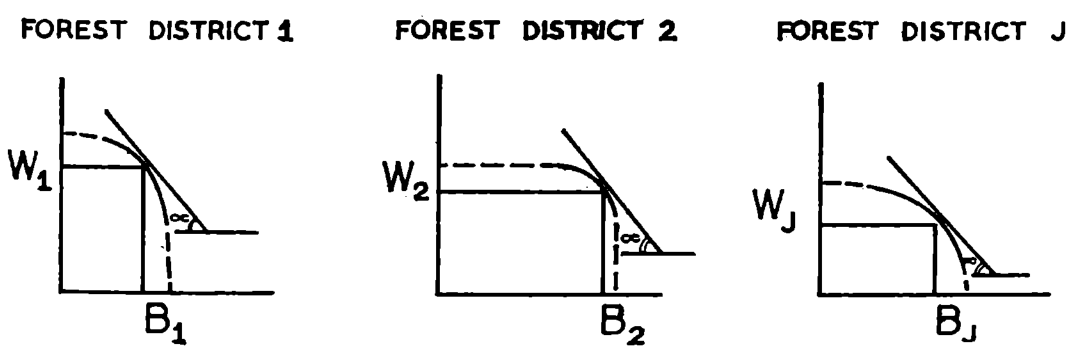

Figure 32. Co-ordination of surplus demands in different forest districts by means of the "exchange rate".

will of course increase or decrease the total amount of surplus.

To balance surplus against long range objectives a trial and error procedure will be sufficient. If there are more constraints in common for groups of forest districts, the problem will become more complicated. This is for instance the case when the districts within a region have market restrictions in common. In a number of pilot studies the S.F.S. has developed a routine by which a number of reasonable corner solutions can be combined into near-optimal solutions. This routine utilises judgement and avoids the many reiterations of the decomposition technique.

Five to ten year planning integrated with short range planning, the budget, information and accounting systems

The main purpose of the five to ten year planning is to select—five years or more in advance—those compartments in which operations (cutting, thinnings, silvicultural measures) should be effected. Five years is the necessary time lag for road construction and for the acquisition of machinery. There is also a waiting time of five years after fertilisation if the full effect is to be permitted to develop.

The system for short range planning (which incidentally also utilises data processing) aims at allocating available machinery and manpower in a way that permits operations to be performed at low cost and in a manner that makes certain that timber and pulp wood deliveries can be made in accordance with contracts. Marginal changes in the selection of compartments can however be made in the short range plans. Incremental changes in manpower and machinery are generally possible. When the short range plans are made the trees of the forest areas selected for treatment will to a large extent be marked and registered. This means that better data than the inventory data used in the five to ten year planning routine are available.

A more definite procedure for the co-ordination between five to ten year or linear programming planning and short range planning will be worked out during the fall of 1970. The procedure must, among other things, permit comparisons between data of the short term planning routine and the linear programming planning routine. This comparison is aimed at (1) correcting the estimates of the five to ten year plans, (2) establishing and analysing differences between five to ten year plans and short-range plans.

The planning systems must in turn be integrated with a budgeting, information and accounting system. It is only then that the planner can analyse the difference between the standard revenues and costs of the plans and real revenues and costs. This analysis of discrepancies between plans and reality is the very core of a management system.

The S.F.S. is presently developing a budgeting, information and accounting system that is to be integrated with the five to ten year and short range planning. The project is scheduled to be put into effect in 1972.

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Paper 15

A LINEAR PROGRAMMING MODEL OF THE U.K. FOREST PRODUCTS SECTOR

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Objectives and Criteria

The general aim of the study is to suggest an optimum strategy for the development of the wood-based sector in the economy of Great Britain. This sector is defined as wood-growing and its primary processing, plus imports. (These categories will be defined more precisely later.) At present, only about 8% of British consumption of wood products is obtained from domestically-produced wood, on a wood-raw-material-equivalent basis (Grayson, 1969, p. 5). The present study concentrates attention on this relatively small proportion of the total supply, partly because it is large in absolute terms and partly because the proportion is likely to rise in the future, as young plantations reach maturity.

The objective function to be maximised is defined as net discounted social benefit. In general the "social" evaluation is based on market prices, although if there appears to be some obvious difference between private and social valuations an adjustment is made. The use of market prices is open to many criticisms, but in a sectoral study it is necessary to take many variables as given externally that is by the rest of the national or world economy. In particular it is assumed that there is no merit *per se* in creating employment in forestry or forest industries, i.e. the existence of full employment is assumed, with a social opportunity cost of labour represented by the wage rate.

The benefits included do not allow for any gains in amenity which may result from the existence of forests. There appears to be disagreement amongst the public concerning the beauty or ugliness of tree plantations, especially conifers. The "weighted average" opinion is unknown and is taken as zero. The costs subtracted from benefits to obtain net benefits are the usual inputs labour, chemicals, water, but for wood-processing the program itself values the wood output from British forests, which has to compete with imports.

As part of the social evaluation, costs incurred in the past are ignored; this includes the establishment cost of existing plantations, and the capital investment costs of processing factories. Similarly, the discount rate used is supposed to be the *social* time preference rate, at constant prices. For those reasons, it is likely to be lower than current market interest rates (Henderson, 1968 p. 98, and Webb, 1966) and a rate of 5% will be used at first, with other rates to discover how sensitive the solution is to the rate of discount.

Resources and Present Wood-Processing

The total forest area in the United Kingdom, excluding waste areas, is estimated at about 1·2 million hectares (1969). Approximately half this is state-owned, i.e. owned by the Forestry Commission, but the species and age-classes in the state and private sectors are very different. Broadly, about 95% of state forests are coniferous, but only about 50% of private forests; also, the state forests are mostly relatively young. Table 41 indicates the differences.

TABLE 41

APPROXIMATE AGE CLASS DISTRIBUTION OF BRITAIN'S FOREST 1965

Age-class	*State*	*Private*	*Total*
	%	%	%
Below 20 years	65	28	47
20–49 years	33	20	26
50 years +	2	52*	27
	100	100	100

*42% broadleaved, 10% coniferous.

Source: Adapted from Forestry Commission data.

Probably much of the privately-owned broadleaved forests is not truly commercial woodland.

Although there is relatively little woodpulp production in Britain there is a great deal of paper manufactured, from waste paper and imported dried woodpulp. There are also relatively small amounts of sawnwood and wood panels produced. The flow chart in Figure 33 indicates the approximate sizes of the main components in 1968, in money terms (£ sterling). Waste paper consumed has been valued at the average import price, which tends to overstate the value of British-produced waste paper.

In the future, the potential supply of wood from British forests will increase, and it will be possible to increase the production of sawnwood, particle board,

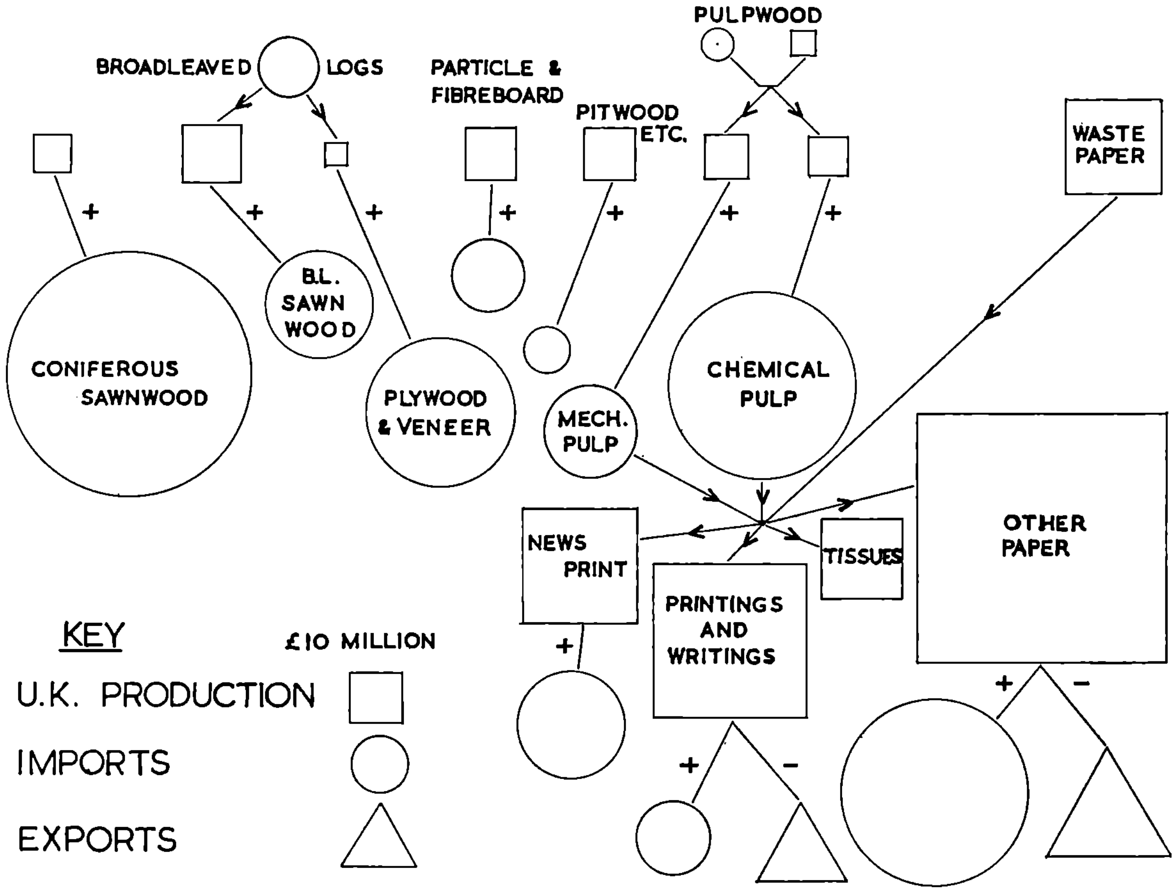


Figure 33. Summary flow chart of U.K. wood products sector 1968 (excluding dissolving pulp).

fibreboard and mechanical pulp. Production of construction-grade plywood and chemical (Stora) pulp may also be feasible; the main problems are log quality (for plywood) and water pollution (for chemical pulp). However, the approach used in this study does not assume that available wood is necessarily used. It may be more economic to leave the trees standing and import wood products instead, but this result is unlikely because existing plantations are not charged with their establishment costs.

Structure of the Model

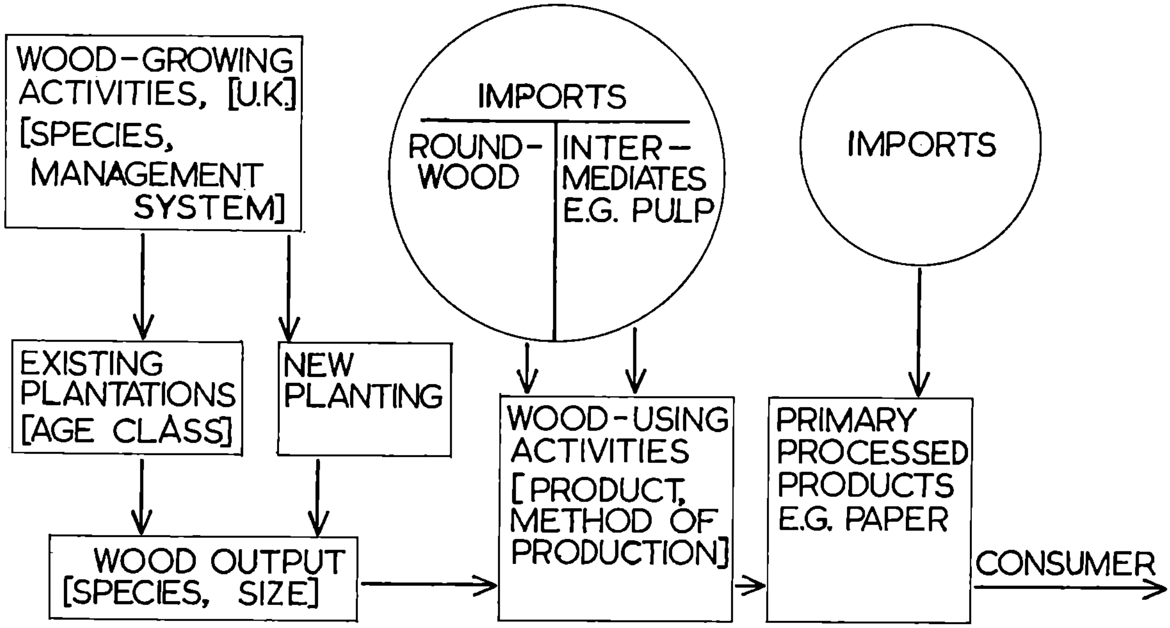
The basic structure of the model is shown in the summary flow chart (Figure 34). The linear programming method requires the definition of a set of variables or "activities", for each section of the flow chart. The time period must also be specified, and in the present model 7 periods are used, covering a total of 60 years:

Time Period	1	2	3	4	5	6	7
Length (years)	5	5	5	10	10	10	15
Years from present	0–4	5–9	10–14	15–24	25–34	35–44	45–59

Because calculations are usually made to the centre of a time period, the effective planning period is only about 50 years (actually 52·5, the mid-point of period 7). The general idea of varying the length of time periods is that the difficulty of predicting such things as prices increases with time. The model's structure is also shown in Table 42 at the end of the paper.

Wood-Producing Section

The wood-growing activities are based on a grouping into four "species", spruce, larch, other conifer and



"EXISTING PLANTATIONS" REFER TO YEAR ZERO. THE REST APPLIES TO ANY OF SEVERAL TIME PERIODS.

DESCRIPTIONS IN BRACKETS SHOW WHICH FACTORS ARE VARIED —

E.G. WOOD OUTPUT IS GIVEN FOR FOUR GROUPED SPECIES AND TWO LOG SIZES.

Figure 34. Summary flow chart of model.

Notes to Figure 34

(*a*) This applies to any of several time periods, except that "existing plantations" refer to year zero.

(*b*) The descriptions in brackets show which factors are varied, e.g. wood output is given for 4 grouped species and 2 log sizes.

broadleaved. For existing plantations there may be up to 8 age-classes of ten years each, viz:

Age-class:	1	2	3	4
Planting years:	1960+	1950–59	1940–49	1930–39
Age-class:	5	6	7	8
Planting years:	1920–29	1910–19	1900–09	Pre-1900

For example, with spruce only 5 age-classes are needed, but there are nearly 200 thousand hectares of pre-1900 broadleaved high forest. Activities in this section of the model are therefore based on species/age-class combinations, together with variations in rotation age and system of management. The latter is mainly a choice between frequent thinning and no thinning, but some intermediate thinning systems are also included, although there are few data on the increased risk of windblow when thinning intervals are increased beyond about 5 years (Hamilton, 1970). Altogether there will be about 200 activities in this section.

New planting is allowed in periods 1 to 3, and this is a main reason for choosing a planning period as long as 60 years. Even so, the maximum rotation possible is only about 50 years. One complication is the provision of roads for the extraction of thinnings or fellings. Any area felled in periods 1 to 3 is assumed to be available for replanting and must already have roads. Other things being equal, it is preferable to plant this land rather than unroaded land, and therefore it is necessary to have two sets of planting activities, of which the roaded land will presumably be chosen first, if at all. Extra columns are also needed to transfer unused plantable land from one period to the next. Planting activities are charged the cost of establishment, including ploughing, draining, fencing, planting and weeding. Their net discounted revenues are therefore considerably more negative than are those of the existing plantations (Heady and Candler, 1958 pp. 112–113).

The estimated wood volume outputs from these wood-producing activities are recorded in the matrix according to the time period and size of log, i.e. volume to 18 cm. top and to 7·5 cm. top. With 4 species and 2 sizes there are eight equations per time period, a total of 56 for the model. Wood volumes are limited by equations for the existing area of each species/age-class combination, with the restriction that the total area assumed in the solution must not exceed the area available; about 30 equations are needed. Wood volumes from new plantations are restricted by limits on the area which can be planted. In programming terms, the wood-producing activities are complete strategies in timing of output but are linked to wood-processing by material balance equations described below. (Beale, 1968 pp. 82 *seq.*)

Wood-Processing Section

Although there are only ten "final products" in the model, the number of processing activities per period is much larger, because of the different possible raw materials and methods of production. The "final products" are:

- coniferous sawnwood
- broadleaved sawnwood
- plywood
- particle board
- fibreboard
- pitwood
- newsprint
- printing and writing paper
- tissues
- other paper (i.e. packaging)

Veneers and dissolving pulp are excluded from the model because they have little relevance to the utilisation of British forests, and they are not directly competitive with the products listed above.

Particle board and fibreboard are in fact not very important in total value of output but they are interesting as a possible growth point. Pitwood is a mixture of sawn and round wood but its use is decreasing rapidly as the mining industry becomes smaller. For example, consumption in 1950 was 2·7 million m^3 (true volume) but in 1970 will probably be only 1 million m^3 (Timber Trade Federation, 1970 p. 31). Tissues (paper) are separated from the other paper because consumption is growing rapidly and there seems less likelihood of substitution by plastics. Railway sleepers are also a declining market. In the model, processing activities are defined as, e.g. produce 1 tonne newsprint from small sprucewood (i.e. size 1) in period 1. This column has a coefficient (representing the roundwood required per tonne of newsprint) in the row for small sprucewood output in period 1, and the sign of the coefficient must be different from those representing wood volume output, elsewhere in the equation. Similarly there is a 1 in the equation representing demand for the particular product per time period.

The specification of processing activities takes account of the existing types of factory in Britain but allows for their expansion or the introduction of others. The values in the objective function equation for these activities are positive (except for capacity expansion) and represent sale value of the product *less* production costs (costs exclude wood and amortisation—the repayment of fixed capital, and interest). The sale values per unit of product have to be forecast, and presumably are inversely related to the amount sold. For this reason, forecasts of demands for each product are being made, taking into account past reactions to price changes. Product prices for Britain are of course determined by world conditions, but it is hoped at least to make the demand quantities specified on the right-hand side of the demand equations consistent with the prices assumed for the objective function.

The linear assumption necessary for linear programming is not normally realistic for manufacturing industry, although average costs may be fairly constant over ranges of outputs. With the programming system being used (MPS/360) the only method of dealing with the problem of economies of scale is to try several successive values for the net discounted social benefit of processing activities, and of capacity expansions. The total number of processing activities per time period is likely to be about 70, although for later time periods the amount of detail may be reduced. Unfortunately the inclusion of particle board and fibreboard as separate panel products increases the number of activities in this section by about one quarter (as compared with including them with plywood) although in terms of output they are not very important (see Figure 33).

IMPORTS

The minimum quantities demanded, specified in the right hand side, are intended to be the total British demand and therefore most of the amounts must be met by imports. Imports can be of roundwood or of final products, the difference being that roundwood is given a negative net discounted social

TABLE 42
OUTLINE MATRIX OF THE MODEL

Description of activity (variable)		Produce wood in period 1	2	Import wood in period 1	2	Produce output in period 1	2	Import product in period 1	2	Sign ↓	Resource (RHS) ↓
Constraint no. ↓	Variable no. →	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8		
	NDSB	$-C_1$	$-C_2$	$-C_3$	$-C_4$	C_5	C_6	C_7	C_8		
1. Forest area	p.1	1								$\leqq$	b_1
2. „ „	p.2		1							$\leqq$	b_2
3. Wood vol.	p.1	$-a_{31}$		-1		a_{35}				$\leqq$	$b_3 = 0$
4. „ „	p.2		$-a_{41}$		-1		a_{46}			$\leqq$	$b_4 = 0$
5. Demand	p.1					1		1		$\geqq$	$b_5 \leqq b_7$
6. „	p.2						1		1	$\geqq$	$b_6 \leqq b_8$

benefit whereas final products are given a small but positive value. The latter represents the average net profit of wood-product-importing firms, per unit of product. With this method, the solution is unbounded unless there are upper as well as lower limits to demand quantities. However, if price has any influence on demand, it would be proper to set such limits even if all imports had a negative net discounted social benefit (NDSB).

The general structure of the model is summarised in a very simplified form in Table 42. This has only one type of wood and one product, in two time periods, and is therefore not itself a realistic problem.

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DISCUSSION OF JACKSON'S PAPER

The question of the social cost benefit function in relation to imputs was raised. *Jackson.* The importers activity is regarded as morally equally justifiable to producing wood in this country. The profit of the importer is a social benefit. The intermediate product is a means to an end not wanted for its own sake, while the import for the final products is a good thing and is what the economy wants. *Bjora.* What is the social time preference rate as distinct from the market interest rate. *Jackson.* For example, because of the mortality of individuals while society has an indefinite life society has a lower discount rate, the individual's rate is closer to the market rate. *Grayson.* You have major difficulties in identifying objectives and the number of assumptions you have to make as you go into a large model is multiplied and it becomes less manageable. There is a danger too of leaving out things which some people may regard as important. For example, how do you deal with the foreign exchange function, how do you take into account the price of imports and processed products compared with the value of import substitution. If you leave these things out, it may be regarded as unrealistic. *Jackson.* I am not very sympathetic with the obsession with balance of payments questions, but in general your suggestions are interesting and a possible basis for an alternative look. *Vornstad.* I find the idea of social costs and benefits dangerous. It would be better to handle social costs through constraints. Then one knows exactly what is going on. *Wardle.* Is the problem here that we are dealing with only a small part of the economy and one has to introduce other constraints or shadow costs or prices to bound the model. I wonder at this point if anyone would like to comment on the experience of attempting to model parts of the economy or enterprise. *Bjora.* My impression is that for every question answered several questions are raised. Model building is an on-going process you don't stop. *Vornstad.* I find the iteration between model building and policy forming very exciting. I am finding, however, that it is necessary to have a little model for policy exploration as the cost of using the big one necessary for programme formulation is prohibitive. *Paillé.* To some extent the experience is one of frustration, being unable to reproduce the system you don't necessarily get more confident in your capacity or in your models. *Wardle.* I think the important step that is demonstrated by a number of the papers we have heard is toward the use of aggregative models which show the manager what the consequences of carrying out the processes suggested by micro-analysis are and this is most meaningful. *von Malmborg.* I am interested in the use of models to study possible behaviour of the enterprise.

FINAL DISCUSSION AND SUMMING UP

As with the discussion of the papers this report on the final discussion sets out more or less what was said where the recording was at all coherent. The points raised in discussion were sometimes disconnected and incompletely dealt with. The hope is that they give a lead to what the person who raised them had in mind and provide a stimulus to further thought when the subject had not been exhaustively discussed in the time available to the original participants.

The topics raised in the final discussion were:

(i) Communication between the researcher and the potential user of his results.

(ii) Institutional restrictions.

(iii) Co-ordination between model builders.

(iv) The appropriate model.

(v) Partitioning.

(vi) Subjects which might be taken up or taken further.

Communications between the researcher and the user

Duerr. I am thinking in raising this subject about human relations aspects and specifically about communications between the research man and the potential user of his results. We can recognise several different situations;

(i) The completely detached researcher who is interested in the work for its own sake.

(ii) The semi-detached worker commissioned to help.

(iii) The man completely on the inside—for example a man working in an Operational Research (O.R.) department of a large corporation or the research branch of a government agency with an executive function.

Each of these situations presents its own problems. Let me describe my situation vis a vis the Bureau of Land Management. There are, as I mentioned, a dozen of us working on this project. We are conscious that unless we are careful our research may produce only a feeling of inner warmth. We find that the Bureau is proud of us and refers to us with a gleam in its eye. It might be sufficient for them merely to have us working for them. We are concerned, however, that our work should be effective and in order to achieve this we find we have to spend twice as much time on the problems of having our results applied as we are on getting the results. We find antagonism among certain echelons. Management decisions become more and more concentrated. It is possible for the man in Washington to make decisions which his predecessors couldn't conceive of taking. The field man is suspicious of this and sees the O.R. man taking away from his prerogative. How do we convince him that we are not depriving him but putting into his hands tools which will make him more effective. How do we get people to understand sufficiently. *von Malmborg.* It is very difficult to reach ones own board. From the beginning of my project there was a complete lack of communication, they just didn't want to know. This year this has changed since we have been able to explain our model I have been able to convince people that it was worth while. *Vornstad.* Top management who were exposed to external pressure were very willing to listen, the District Manager was more sceptical. *Bjora.* There are two main reasons for doing work for a central agency (i) they are politically forced to start the work (ii) they start the work to increase their own power. Usually, we are working more for the central administration which will tend to increase the power of the central organisation and decrease the power of the local organisation. *Jackson.* The difference between the three levels mentioned by Duerr is the type of constraints imposed on the research. The person who is commissioned has constraints imposed by the sponsors. *Wardle.* The feeling I have is that if one is to do effective work, one has to defend oneself against terms of reference. It is the job of the operational researcher to discover what is of importance and then to design his models to tackle these problems. This leads me to attach primary importance not to the techniques but to the identification of problems. A particular difficulty of the person in academic research is that of not having a sponsor but having to find his own problems. The sponsored researcher had his sponsors to react with, thus Duerr in his problem analysis was concerned to examine the position now reached in the project in order to determine what to work on next. *Vornstad.* The function of the O.R. worker is to make management formulate the problem. *Duerr.* The O.R. worker is in a peculiarly advantageous position to spot the problems whereas management often isn't or doesn't identify them in the appropriate form. The O.R. worker must put this right if he is to help the situation through research. *Vornstad.* The first objective is to get management to understand the problem so that they will believe in that which is chosen for research. Secondly, communications must be maintained, the model must not be too far removed from reality so that they can recognise the relationship to their situation. *Duerr.* This is the matter of working it out between the researcher and management.

Institutional Restrictions

Jackson. I am prompted to raise this point by Sayer's paper. Where private estates do not have a marketing organisation this places a restriction on the possible courses of action open to them. There are subsidies for planting but there is no subsidy for marketing. A useful field for investigation might be into the methods of improving this sort of situation. *Wardle.* You are referring to a situation where the normative solution is not feasible because of institutional constraints. In order to approach the optimum it would be necessary to change those constraints and the research would be directed to finding out the optimum way to change them.

Co-ordination between Model Builders

Paillé. It seems to me to be desirable to ensure the continuity of effort on the part of the researchers in the field of model building of the forestry system. At this meeting we have been working on three different floors, myself, Risvand and Kostov on the ground floor, von Malmborg and Vornstad on the first floor, Jackson, Duerr and Wardle on the top floor. They are working in different countries with different models, different variables and different techniques. To me it looks as though some of these are incompatible and I would be interested in knowing if there were any way of ensuring the additivity of effort or the continuity of effort. Is there any other means than passing round the publications after their independent researches have been done by which stage it is already too late. *Duerr.* How do we know what is going on in other words. We have tried various schemes—newsletters, different ways of keeping people informed, but the results haven't been exciting enough to warrant continuation of the schemes. *Paillé.* We have to find a way for workers to use information from the other floors. Stand modelling is the basis of the whole building. There are about 25 of these models available and they are all trying to simulate the same thing. It would seem preferable to have workers adding their efforts, not re-working the same example. *Morgan.* Are there special inter-relationships within a given enterprise which cut it off from the outside? I wonder if Jackson has found himself co-ordinating with other economy modellers. *Jackson.* I have been asked to contribute forestry quantities to the Cambridge model, but forestry in that model contributes but one row and one column. *von Malmborg.* For biology in Sweden there is a scheme whereby you write and tell them you are working on a particular subject and get information on what is going on in the field. Have you, Professor Duerr, only tried to get results or have you also tried to get information about people now working? *Duerr.* You probably know about our bibliography. We do include once a year a list of theses produced by graduate students. There is a group in North Carolina, SOFU, Southern Forestry Economics Workers, and they are trying to publish news about what people are doing. *Wardle.* Within the British Government there has been set up a Register of O.R. work going on in the departments. It is useful since it gives some indication whether it is worth while writing to a colleague about work in hand or in mind. Should we be doing something like that. *Bittig.* Would it be worth making abstracts of models? *Paillé.* One possible way of solving this would be to have a meeting such as this at which people say what they are working on, giving opportunities for an exchange of views while the work is going on. *Wardle.* I think what we are talking about is the provision of lively channels of communication. The problem is enormously rich with all sorts of dimensions. I would hesitate to recommend particular structured information flows, but the possibility of newsletters, bibliographies, abstracts should be recorded. Each has its merits so long as nobody has too much work. Perhaps the most effective means is people coming together for such meetings as this and including the loose talk over lunch.

The Appropriate Model

Vornstad. Where the problem to be solved is very complicated, it may not be possible to handle it in an optimising model, but the dead weight should, if possible, be carved away. Policy formulations should be translated into tactical plans and the simulation model is of particular value for this. When it comes to co-ordination within the enterprise it is rather important to work with optimisation models. Co-ordination within models is what capital theory is trying to do and this is why forest economists are so fond of capital theory in spite of the fact that they are working with percentages which are ridiculous from a point of view of the bank manager. It is an important means of achieving consistency between decisions if various silvicultural possibilities are evaluated at the same 3 per cent, and one can at least ensure that the chosen ones are in accord. Consistency between choices and getting management to accept O.R. models is an argument for optimisation models. *Morgan.* If there are no extra costs of using optimisation models there is no argument. If there are costs then we have seen that in the long term results are sensitive to assumptions about productivity, output and price. There is no real point in trying to aim at any sophisticated precision rather to ensure the long term fits in with the short term. *Wardle.* The point that Vornstad is making is that the rigorous, legalistic optimisation model, such as the linear programming one, is selecting a solution according to a criterion function and has a particular value in confronting a management problem. *Morgan.* The question is whether extra work involved in getting the advantages of optimisation is worth the effort, this again depends on the organisation and facilities available. *Höfle.* I see it as a question of the knowledge you have before you start to build the model in two ways—if you want to apply an optimisation model you should know your objectives before embarking on one. If on the other hand you don't know the objective, or you don't know costs, for example from Work Study of damage to roots in extraction, or the physiological strain on workers, or what I thought we saw in Morgan's model, we don't know how conflicting parameters are going to behave in aggregate. In these cases you have the merits of simulation. *von Malmborg.* We are talking about sophistication of optimising models, why should they be more sophisticated. *Morgan.* I don't know whether they are or not. In the case of the simulation model we write it according to our own format whereas if one is talking about optimising by linear programming we have to squeeze all our thinking into that format. *von Malmborg.* This is very good for you. *Morgan.* In Jackson's case he didn't really know the motivation or objective function of the private owners and I would have thought in those circumstances there was much less reason for adopting an optimising format. *Jackson.* I am going to tell them the things they are meant to be doing.

Partitioning

Wardle. People haven't been able to satisfy themselves about the perfection of their models and there have been two reasons (i) there have been data problems and (ii) there have been integration problems—oh, I have done a sub-optimisation, and it feels very painful. Somehow it seems necessary to solve the higher level problem before one can be sure one is getting an appropriate solution to the lower level one. I think one has to accept that one will have to work on little bits of the system. What one does have to do is to explore the boundaries to ensure that the outer system isn't contributing something critical to the solution for the particular bit. For example, prices in 20 to 30 years' time really have very little to do with the best way to market the decided cut over the next two or three years. *Vornstad.* This is really another formulation of one of my points. What I wanted to say was that partitioning requires an explicit formulation of the function you intend to partition. I was quite interested in your example which is quite contrary to the one I took up yesterday. That is to say, the ordering of clear cutting today must depend on the prices prevailing 20–30 years from now and you find that out if you do an objective function formulation. *Bjora.* It is important to emphasise the relationship between parts of the enterprise and the whole. *Wardle.* I think it has to be accepted that you can often find an optimal solution to a local problem which is compatible with the overall optimum without resolving the overall problem. *Bjora.* In making assumptions about the whole enterprise it is important to know how sensitive the local solutions are to change in the total. *Höfle.* We seem to be closing the circle which we started on Monday afternoon with Duerr's thought that everything depends on everything else. It is a question of which parts we try to deal with. We are looking for a criterion about which part to take, a theory for selecting the problem at that particular time. *Wardle.* I am suggesting that you can select and must and I gather Duerr is doing this in his continuing project through problem analysis. Possible criteria for choice of projects seem to be the degree of interest you can expect from management, the significance of the problem and the importance the results might have for the enterprise. Ultimately it must be the overall objective function of the enterprise that chooses the problems. *Jackson.* If we accept that everything is connected with everything else it is only logical to try to reduce the 'itsy bitsy' approach. Personally I would feel happier if my model was part of a larger model including agriculture and possibly other land using activities and I feel this is another example of an institutional constraint—human weakness really—small empires are perhaps more comfortable to live in. People giving the money should be much more strict in their requirements for making people work together, we must make it too expensive for people not to work together where this is important. *Wardle.* I don't necessarily accept the concept that everything is connected with everything else as of over-riding importance. We do have to select projects and these are considered on their particular merits in relation to the objective the project is supposed to serve. *Jackson.* I am thinking for example of land use activities such as grouse moors and forestry in Scotland. Who is going to do work on the economics of grouse moors, maybe the Countryside Commission or Nature Conservancy. It seems to me there should be some liaison so that all these pieces of work are not in isolation.

Subjects which might be taken up

In the discussion of subjects which might be taken up *Höfle* remarked that the papers we had heard during the week exploited rather few of the techniques of operational research and that it would be useful to know more about the suitability of other techniques. *Wardle* while accepting that there were numbers of techniques of which there was no experience of their application to forestry, there were also numbers of areas where foresters already had valuable experience and where we would benefit by sharing such experience. This was particularly true in the area of uncertainty and the stochastic behaviour of systems. There is also the problem of dealing with situations where there is little or no hard data and where very interesting approaches are being developed and would merit investigation. *Höfle* mentioned the subject of systems analysis. *von Malmborg* mentioned the integration of planning which could valuably be taken further. *Vornstad* suggested there was more to be done in determining the appropriate model to fit the particular planning level. *Wardle* suggested that O.R. had a contribution to make in other fields than the timber production field. Questions had been raised in our earlier discussions about non-market values, questions sometimes associated with cost benefit analysis. *Vornstad* felt one might not be altogether fond of cost benefit analysis, but that one had to get one's reasons clear. There was also the question of how do you make management accountable under a cost benefit decision system, that is if one cannot produce a viable alternative. *Wardle* mentioned that Schreuder had pointed out to the Working Group that there is still important work to be done in the area of decision theory.

Summing Up

The papers in this collection demonstrate that operational research has an important contribution to make to the management of the forestry enterprise and that it is already doing so in a number of important cases. The objective in seeking out the particular papers has been to show this contribution in relation to the central problem of enterprise management planning, namely the optimum allocation of resources. The major themes running through the discussion were the identification and structuring of the system being considered—the systems analysis and the use of either descriptive models—simulation models to show management possible states of the enterprise or the consequences of possible courses of action, or optimising models which select courses of action according to objectives, requirements and restrictions presented by management. It has been shown that the completeness, degree of perfection and precision of the models are to be decided like other investment choices according to the cost related to the value of the improvement in results.

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