Forestry Commission

Research Report

Timber, carbon and wind risk: towards an integrated model of optimal rotation length

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Contents

Summary

Forest Research (FR) has developed a prototype forest optimal rotation length model with climate change adaptation and climate change mitigation elements. The model integrates timber production with carbon sequestration and substitution, and windthrow risk considerations. The model can consider both private and societal perspectives on optimal rotation length.

In recent decades forestry policies have broadened from a traditional focus on timber production to also consider recreation, biodiversity, landscape, carbon and other benefits of forestry. An ecosystem services approach has been increasingly advocated to account for the multiple benefits of woodlands, and at the same time, forestry management has also been linked to climate change adaptation and mitigation strategies. However, the development of decision support tools to research and manage these benefits and strategies has lagged behind. Given the priority policymakers accord to ecosystem services and climate change agendas, it is important that any optimal rotation length model takes an integrated approach to accounting for these issues. The prototype developed in this study is a first step in developing such a model.

A number of scenarios considered by the Intergovernmental Panel on Climate Change indicate increasing frequency of severe storms in the future. In addition to climate change, increasing average stand age and height is also likely to create greater risks of wind damage to forests in the UK. Wind risk is one of the most significant threats to UK forests, with storms currently responsible for more than 50% of all primary damage by volume to European forests from catastrophic events.

In developing the prototype model, the approach adopted progresses from simple to complex. The starting point is the classic biological model of maximum sustainable yield which maximises the volume of timber produced. This is then extended to incorporate costs and revenues associated with timber production (the classic Faustmann economic model), which maximises the value produced. The model is further extended to include wind risk, and carbon sequestration and substitution. The optimum rotation length is based upon maximising the associated value over multiple rotations.

The prototype model provides a research tool to aid the selection of options to focus on in wider comparisons, such as in estimating marginal abatement cost curves and the cost-effectiveness of forestry for climate change mitigation and adaptation. However, the prototype also has potential to serve as a foundation for the development of tools for practical decision support in a range of wider forestry management and woodland creation applications, including informing strategic thinking about how to address climate change mitigation.

The model integrates existing models developed by FR. In particular, it uses ForestGALES – the principal wind risk evaluation tool – the growth model (M1) and the latest FR model for carbon accounting in forestry (CSORT).

The prototype model yields results in agreement with standard economic theory as applied in forestry, including the classic Faustmann model. In particular, the impact on optimum rotation length of increasing major input variables is shown in the table below:

Impact of increase in:	Effect on optimal rotation length:
Price (timber)	Fall
Costs (planting)	Increase
Discount rate	Fall
Wind risk	Fall
Carbon price (sequestration)	Increase
Carbon price (substitution)	Fall

As can be seen from the table, an increase in the price of timber would lead to a reduction in the optimal rotation length. Currently the model has been tested primarily for the case of unthinned Sitka spruce (*Picea sitchensis*) yield class (YC) 14, planted at 2 m spacing (i.e. a stocking density of 2 500 trees per hectare).

Initial results from the model for sites with low windthrow risk indicate that the optimal rotation length is very sensitive to the price placed on the carbon sequestered, but not to the price placed on carbon substitution benefits associated with use of wood products after a stand is harvested. This is mainly due to the way these effects are included and discounted in the model. In the case of carbon sequestration, the range of market prices of £3/tCO_2e to £10/tCO_2e is assumed to apply at the start of the project (i.e. before the carbon has been sequestered). This is thought to reflect prices and current practice for UK forest carbon from projects certified under the Woodland Carbon Code. At prices towards the middle to higher end of this range, the model suggests that the optimal harvesting decision, given

the other input parameters used, is not to harvest at such sites (i.e. there is no finite optimum rotation length in this case). This result is strengthened if Department of Energy and Climate Change (DECC) recommended social values of carbon, which are much higher than £10/tCO_2e, are used instead. The impact of changing the price placed upon carbon substitution benefits is less marked in the current version of the model due to these benefits being assumed to be paid subsequently (i.e. after wood has been harvested).

At sites with low windthrow risk, the preliminary results suggest that changing the discount rate has a moderate impact on the optimal rotation length: for a reduction in the rate from 3.5% to 1% the optimal rotation length increases by about five years.

At windy sites, the preliminary results suggest that including carbon benefits, or changing the discount rate, has a negligible effect on the optimal rotation length in cases where windthrown timber is of no value due to the salvage costs and lower commercial value of snapped and snagged trees.

Discussions with Forest Enterprise Scotland (FES) planners highlight the multiple factors considered when allocating felling years to individual coupes as part of a forest design plan. This process is largely subjective, which is partly a consequence of the multifunctional nature of public sector forestry and the lack of operational appraisal tools. One of the main factors is the need to meet the district's contribution to national timber volume targets. Planners are well aware that management is financially sub-optimal and tools based on the optimal rotation model that provide better economic information were seen as potentially useful. There appears to be no demand at present from FES district planners for a model that also optimises for carbon sequestration, since climate change mitigation is currently not an explicit management objective. However, the model could inform strategic thinking about how to address this issue in the future. The findings underline the need to ensure that further development of the model involves potential end users to enhance its usefulness and impact on policy and practice.

The following recommendations are made for future research and model development:

Recommendation 1: Consideration is needed to potential further development of the current prototype by:

- adding a salvage value for windthrown timber to the model (the current prototype implicitly assumes a zero salvage value);
- extending the model to include adaptation to wider climate change-related risks (e.g. drought, pests and diseases), and to other ecosystem services and biodiversity;
- using Treasury Green Book declining discount rates and DECC social values of carbon;
- allowing a fallow period between rotations as a Hylobius management strategy;
- allowing for any expected changes in timber prices over time, and for timber prices to vary with rotation length to account for changing assortments and proportions of wood harvested going to different end uses;
- linking to GIS and forestry spatial data to develop the model into a useful decision support system (DSS) tool able to produce maps of optimal rotation length and associated net present values for a forest. Maps of differences with biological or simple Faustmann models could also be produced to facilitate visualisation and discussion of different management options in cost – benefit, monetary terms.

Recommendation 2: Conduct additional social research and knowledge exchange, in particular with forest planners at district and national level, and with the private sector, to explore the demand for the developments outlined above, and, where appropriate, facilitate the co-production of model outputs and DSSs.

Recommendation 3: If climate change uncertainty is to be adequately accounted for, alternative methodologies to the approach used for the prototype model should be considered as the basis for further model development.

Recommendation 4: Additional experimental research on yields, carbon benefits and windthrow risks associated with mixed species stands is needed if the model is to be extended to cover such cases.

Recommendation 5: Research is needed on impacts of continuous cover forestry (CCF) and natural regeneration on timber yields, costs and revenues to provide a more solid foundation for extending the model to CCF. This should include research on the effects of age structure and density of typical CCF on yields, costs and revenues.

Introduction

Optimisation is a common approach in economics that is applied to a multitude of problems. Optimal rotation models of forestry are one of the oldest applications.

Ensuring the efficient use of scarce resources by embedding decision-making in an optimising framework is the overarching reason for the development and use of optimal rotation length models. The scarce resources in this case are the land used for forestry activities and the net investment required for forest operations.

The interplay of costs and benefits of a project over time (often considered over an infinite series of identical rotations) is assumed to determine the value of the land under forestry. Optimal rotation length models identify the rotation length that maximises this land value (or, equivalently, the sum of net present values (NPVs) over an infinite series of rotations).

Although considerable work has been done on optimal rotation length models since early applications at the end of the nineteenth century, concerns about climate change mitigation and adaptation, and interests in broadening forestry management goals consistent with an ecosystem services approach, have recently revived interest in this topic. For example, the adaptation of management of existing forests to climate change requires making allowance for the expected increase in the future frequency of extreme events such as storms and associated increases in windthrow risks when making rotation length decisions. Similarly, comparisons of afforestation options for climate change mitigation require consideration of forest management regime, including rotation length.

Aims and objectives

This study aims to:

- develop a prototype optimal rotation length model that accounts for timber production, climate change risks and carbon sequestration, providing private woodland owner and societal perspectives, covering both afforestation and forest management choices, and focusing initially upon wind risks;
- develop a model that links directly to existing Forest Research (FR) models of carbon balances in forestry and those associated with production and use of harvested wood products (HWP), e.g. CSORT and BSORT (Morison *et al*., 2012), and of wind risk, ForestGALES, (Gardiner and Quine, 2000; Gardiner *et al*., 2006);
- explore possibilities for drawing upon models developed by others (e.g. the CARBMOD carbon/timber optimal rotation length model developed by Professor Colin Price (formerly Bangor University), and the new Forestry Commission Forest Investment Appraisal Tool (FIAP II)), and the feasibility of using these as a base for further development;
- explore approaches to extending the model to other ecosystem services such as biodiversity;
- explore critical factors affecting rotation length decisions with land owners and forest managers as a means to inform model design.

Methodology

This study draws upon an initial literature review of the current state of the optimal rotation modelling field. The review focused on studies published since 2000, although significant earlier papers are also considered. Largely complete, though not yet finalised, the draft review is available on request (from V. Saraev) as a supporting document for this report.

Free open source software was chosen for the coding and development of a prototype optimal rotation length model. Details of the software used (which includes the Python programming language with SciPy and NumPy libraries for calculations, and Eclipse-PyDev as an integrated development environment) and approach, can be found in Appendix 3. Development of the initial prototype model proceeded from simple to complex in the following steps:

1 Developing a maximum sustainable yield (MSY) model for the maximum mean annual increment (MMAI) solution without accounting for costs or prices. (For this, data from FR's Forest Yield model needed to be interpolated in order to apply standard numerical optimisation methods developed for continuous functions.)
2 Adding economic information on prices, costs and discount rate to reproduce a classic Faustmann model (Amacher, Ollikainen, and Koskela, 2009; Faustmann, 1849).
3 Adding carbon sequestration benefits. A very simple carbon module after Hartman[1] and van Kooten (Hartman, 1976; van Kooten, Binkley, and Delcourt, 1995) was initially used based on assuming a fixed fraction of the carbon sequestered being locked up long term to represent the average amount of carbon locked up over the rotations (a standard simplifying assumption in much of the economics literature).
4 Adding wind risk to the Faustmann model.
5 Adding wind risk to the Faustmann model with carbon sequestration. This step completed the initial proof of concept integrated optimal rotation length prototype. The initial development of the model to integrate timber, carbon sequestration and wind risk follows along the lines presented in Amacher, Ollikainen, and Koskela, (2009: Chapters 2, 3 and 10).
6 Substituting the simplified carbon module used initially with outputs from FR's CSORT model, allowing greater precision and incorporation of carbon substitution effects. CSORT covers various carbon fluxes and pools,including (HWP) and estimates of associated carbon substitution benefits.

1 In a seminal contribution in 1976 Richard Hartman considered how the optimal rotation length changes if one also considers other ecosystem services provided by forests that depend on the age of the forest.

For a number of reasons, it was decided to initially focus upon unthinned stands. First, it was simpler than using options with thinnings, as saw-tooth like curves are much harder to model. Second, a major focus of the study is the influence of wind risk. This is only influential in constraining rotation length at windy sites. However, the management recommendation at such sites is generally to avoid thinnings because they open the forest structure and significantly increase the risk of windthrow. Only early interventions are recommended, and these do not yield significant amounts of timber. Third, the timing of the last thinning often depends on the year in which it is expected to harvest the stand, which is an unknown in the model. Hence, including thinnings would add an extra dimension to the optimisation problem since the time of the last thinning would need to be estimated in addition to the optimal harvest time. Nevertheless, a model including thinnings could be developed in the future.

For simplicity, only two cases of windthrow risk are considered currently: i) no windthrow and ii) catastrophic windthrow with complete destruction of a stand. No intermediate cases are considered at present, although cases of partial destruction of a stand in the event of a storm and inclusion of a positive salvage value for windthrown timber could be included in further development of the model. A positive salvage value would be expected to lessen the impact of wind risk.

Estimates from FR's M1 Yield Model are used initially in running the model. These are an input to FR's BSORT, CSORT and ForestGALES models. It proved feasible to fully integrate the wind risk module (which uses output from ForestGALES) into the prototype model. However, CSORT is still in a very active development phase requiring a number of other tools to run in a chain-like manner, and is currently far from being fully automated or integrated. The carbon and timber estimates used in the model are derived from three programmes, manually linked together:

- M1: produces timber yield data;
- BSort: uses the M1-Yield table file to produce biomass estimates for the main crop and thinnings/mortality data;
- CSort: uses the B-Sort outputs to estimate carbon balances.

M1 and BSort produce a particular file format as an output. This is then incorporated directly as an input into the next programme in the chain.

Initially, the model only produced output for a fixed rotation length. Development of the CSORT model was needed to output data automatically for a series of rotation lengths required for further use in the optimisation routine.

To assist with development of the model and its application to climate change adaptation issues, qualitative social research was conducted with forest planners to understand the factors influencing felling and restocking decisions, and the processes through which those factors are taken into account. The findings provide insights into how the model could be developed further if it were to be used in the field to add value to existing decision-making procedures, rather than being limited to its original purpose as a research tool to aid strategic–level issues such as the choice of woodland creation options to focus on in developing forestry marginal abatement cost curves

It is currently unclear what all the factors are that affect felling decisions and how important they are. For example, are decisions based primarily on MMAI adjusted for wind risk (either with expert judgement or using ForestGALES)? How and to what extent are economic factors taken into account? To what extent are the stand-level decisions determined by wider forest management issues in which multiple spatial factors and overall targets are considered, e.g. relating to landscape, biodiversity and the need to provide a target annual supply of timber?

To explore these questions, interviews, which were subsequently transcribed and analysed, were conducted with FES district forest planners in June 2014, and discussions were held with the FES national planning team in March 2015. (Further interviews are envisaged, in particular with one or more private sector forest planners, to provide a contrast with FES.) A starting point for discussion during the interviews was a list of around 20 factors (see Table 1), which in principle were seen to inform felling decisions. The factors

Table 1 Potential factors influencing felling decisions.

Objective (or ecosystem service)	Factor influencing decisions	Spatial scale determined
Timber	Biological optimum age for felling (MMAI).	Stand
Timber	Timber prices: harvest more when prices are high or harvest more when prices are low to ensure a stable supply to processing sector?	Stand
Timber	Operational costs: only harvest stands for which timber prices exceed extraction costs (i.e. where there is a positive standing sales price)?	Stand
Timber	Discount rates: fell sooner if discount rates are high?	Stand
Timber	Restructure the forest to generate a 'sustainable timber supply': how is this defined? Is early felling used to speed up the process?	Forest
Timber	Extraction: road construction and maintenance – to what extent are these barriers to felling or just cost considerations?	Forest
Timber	Wider targets: for example district timber production targets.	District
Wind risk	Site factors: for example soils, altitude, age of stand affecting risk (is ForestGALES used?)	Stand
Wind risk	Spatial factors: exposure due to neighbouring stands – is there a need to harvest towards the wind?	Forest
Tree health	Site factors: is felling employed on stands infected with pests or pathogens?	Stand
Tree health	Spatial factors: for example is sanitation felling used to prevent the spread of pests and diseases to neighbouring stands?	Forest
Climate change adaptation	Species diversification for climate change adaptation: is early felling used to speed up the process?	Stand
Climate change mitigation	Are longer rotation periods employed to increase carbon sequestration or shorter rotation periods employed to increase carbon substitution?	Stand
Water quality	Is long-term retention employed in riparian zones?	Forest
Biodiversity	Sites of special scientific interest and other designations: is long-term retention employed because of the ecological value of the site?	Stand
Biodiversity	Forest habitat networks (functional connectivity for key species): is long-term retention employed because of the ecological value of the site?	Forest
Biodiversity	Plantations on ancient woodland sites (PAWS): is early felling used to speed up conversion to natives?	Stand
Landscape/ biodiversity	Conversion to continuous cover forestry (CCF): is selective felling used to increase stand age diversity?	Stand
Landscape	Visual amenity of long-term retention near roads, trails, car parks, viewpoints: is long-term retention employed to reduce clearfell 'eye sore' at key locations? Is early felling used to maintain views adjacent to existing viewpoints?	Forest
Landscape	Conversion to natives near roads, trails, car parks: is early felling used to speed up this process?	Forest
All objectives	Site suitability for different tree species.	Stand

were assembled on the basis of the authors' knowledge of the planning process from previous research projects, OGB 36 (Forest Design Planning) and relevant policy guidance. A key question asked of forest planners was to what extent they explicitly take each of these factors into account when deciding the optimum age to fell a given forest coupe. (See the section on 'Exploring critical factors affecting rotation length decisions with land owners and forest managers'.)

The prototype integrated optimal rotation length model was initially tested on a small number of cases in terms of tree species and yield classes (YCs). The choice was influenced by analysis of Craik Forest in southern Scotland. It is envisaged that the prototype model will be fully calibrated and tested using data from Craik Forest. Some initial results of this exercise are presented in Appendix 1.

It is also envisaged that the initial documentation for the prototype model (included in Appendix 3) will be updated after further model development and testing in conjunction with work on the Craik Forest case study.

Data

Data used in developing the prototype model include:

- Yield data from growth models. Initial model development drew upon estimates from Forest Yield (Matthews, 2008), based on the previously published Forestry Commission Yield Models for Forest Management (Edwards and Christie, 1981). Subsequently the extended version of the Forest Yield growth model called M1, which accounts for a longer time horizon and competition mortality among trees, was then used in developing the prototype model.
- Probability of windthrow: estimates from ForestGALES (Gardiner *et al*., 2006; Gardiner and Quine, 2000).
- Carbon fluxes associated with the standing stock and other carbon pools and greenhouse gas emissions estimated from the CSORT model described in Chapter 5 of Morison *et al*. (2012).

Unless stated otherwise, initial model estimates are presented for the case of unthinned Sitka spruce YC 14, planted at 2 m spacing (i.e. 2 500 trees per hectare density). Other parameter values used in developing the prototype model are listed in Table 2. Sources upon which these values are based generally comprise long time-series or ranges based on typical scenarios. The values chosen are always within the sources' ranges and are considered to represent typical (rounded) values.

Models with wind risk have a second cost parameter – the cost of clearing and preparing the site after a catastrophic event (windthrow). This is currently set at 20% of the standard planting cost and could be varied. In effect, it is assumed that windthrow is a costly event that results in higher than standard replanting costs. Currently there is no salvage value assumed in cases of windthrow, making it an especially significant adverse event in economic terms – an issue that could be revisited in future refinement of the model, possibly including an option instead to include insurance premiums in cases where cover for windthrow is available.

Carbon

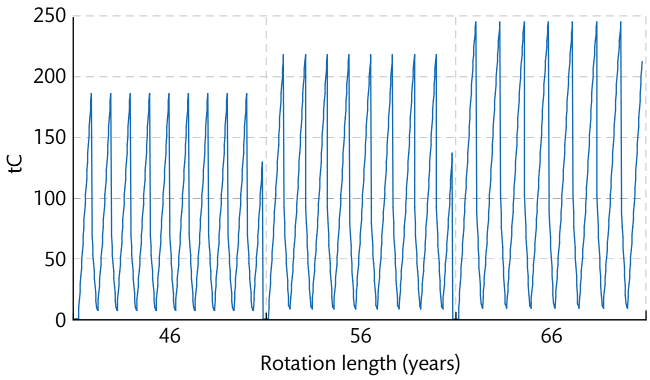
This sub-section provides background on the CSORT estimates that are used in developing the prototype optimal rotation length model. In Figure 1 we present typical output from CSORT for above ground carbon dynamics over a series of rotations of various lengths: 46-, 56- and 66-year-long rotations. Each rotation length is explored over a 400-year horizon, which is the current recommended time horizon limit in CSORT. The graph illustrates how longer rotations give rise to increased above ground carbon,[2] with higher long-term average carbon sequestration. (Note that use of the fixed time horizon gives rise to an incomplete rotation at the end of each rotation length series. Data are given in tonnes of carbon – to convert from tC to tCO_2 multiply by 3.667 or 44/12).

2 Soil carbon and below ground biomass are not currently accounted for in estimating the long-term average carbon sequestration under the Woodland Carbon Code (Tim Randle pers.com.).

Table 2 Values for the prototype model's inputs.

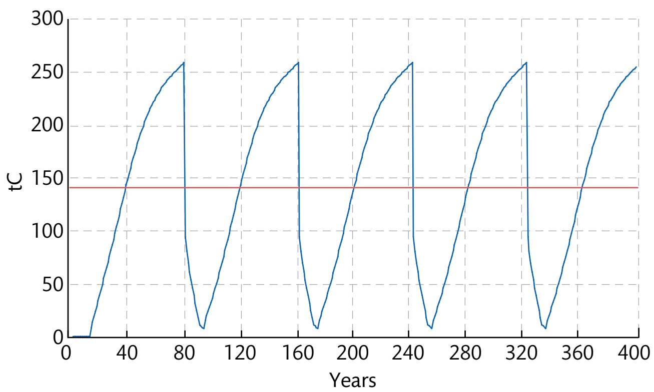
Parameter	Value	Source
Planting costs	£2 000 per ha	From the range of standard costs for productive conifers by Forestry Commission Scotland (pers. com. in 2014)
Timber price, coniferous standing sales price	£15 per m^3 overbark	Coniferous Standing Sales Price Index for Great Britain (Forestry Commission, 2013)
Discount rate	3.50%	Initial rate from HM Treasury's Green Book (HM Treasury, 2003)
Carbon prices (societal)	£200/tCO_2e	Department of Energy and Climate Change (DECC), (www.gov.uk/government/collections/carbon-valuation--2)
Carbon prices (market)	£3/tCO_2e	(CJC Consulting, 2012)

Figure 1 Above ground carbon dynamics for various rotation lengths.



The long-term average carbon sequestered is computed by summing and averaging over a time frame comprising a series of full rotations. For example, Figure 2 shows the 80-year rotations, for which the long-term average carbon sequestered is about 141 tC/ha (red constant line on the graph).

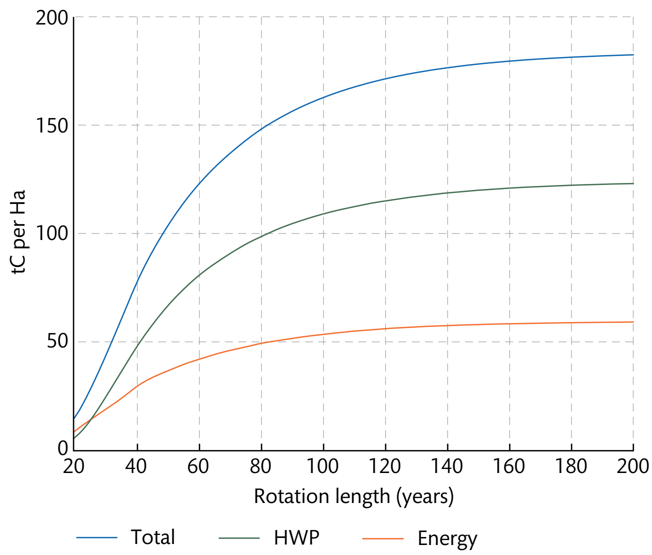
Figure 2 Long-term average carbon sequestered in a series of 80-year-long rotations.



In developing the prototype model, it was assumed that the forest owner receives a payment that corresponds to the long-term average carbon sequestered. This is in line with current practice under the Woodland Carbon Code (Forestry Commission, 2014) organisations and individuals wishing to reduce their carbon footprint while also delivering a range of other environmental and social benefits. It should be noted that although CSORT could be used to estimate the volume of carbon sequestered at an existing forest site for a given management regime, additional benefits accrue only where new woodlands are planted or, in some cases, where silviculture changes.

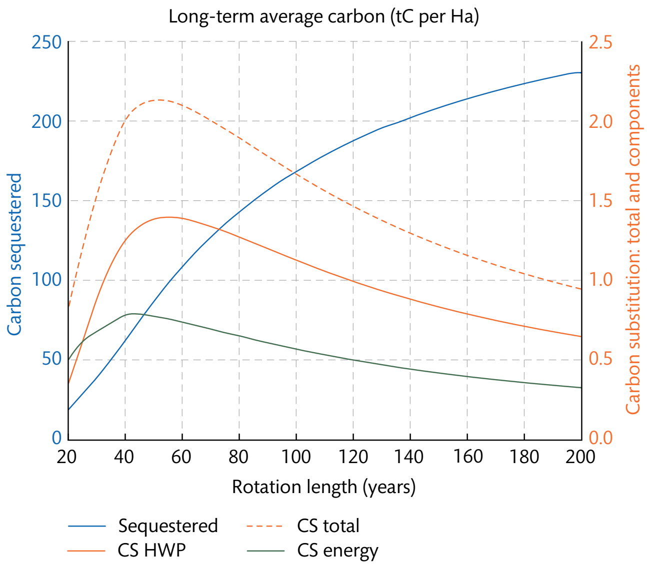
Figure 3 shows the maximum level of carbon substitution (Morison *et al.*, 2012: Chapters 3 and 5) that occurs after harvesting at the end of the rotation, which subsequently declines as products decay. It shows how the level of carbon substitution increases as rotation length rises. Carbon substitution occurs when, for example in construction, more carbon intensive materials like steel and/or cement are displaced by wood. Counterfactuals as the business as usual case would include things like: continuing to build houses from bricks only, as opposed to substituting bricks for wooden cladding; and using the current mix of resources for electricity generation, as opposed to replacing 100% coal power by 90% coal and 10% woodfuel in co-firing, etc. Carbon substitution benefits are separated into two groups: substitution due to energy generation (in electricity generation and sectors covered by the EU emission trading scheme) and other substitution due to the use of harvested wood products (HWP). The distinction between the two categories is important in policy appraisals as the DECC traded sector social value of carbon is used for the first group and the non-traded sector one for the second group. Note, however, that the two values are assumed to be the same from 2030 onwards, so the distinction between different types of carbon substitution becomes unimportant over longer time horizons.

Figure 3 Maximum level of carbon substitution that occurs after harvesting at the end of the rotation.



Next, Figure 4 shows how the long-term average carbon sequestration (blue line – measured on the left-hand axis) increases as rotation length rises, converging towards the level where the stand is left indefinitely without being felled. It also illustrates how the average level of carbon substitution benefits per year of a rotation (total – dashed red line; energy and HWP components – solid green and red lines, all measured on the right-hand axis) varies with rotation length, essentially following the timber yield curve relationship, reaching a peak at the MMAI. The average carbon substitution shown is much smaller than long-term average carbon sequestration due to the different approach adopted. Whereas carbon sequestration occurs continuously

Figure 4 Long-term average carbon sequestration and average carbon substitution rate (tC per Ha).



throughout the rotation with the mean based upon the average cumulative level of carbon sequestered in each year, carbon substitution only occurs once the wood is harvested. It is zero in other years. Thus for carbon substitution, the average cumulative level is computed simply as the total over the rotation divided by the rotation length. On this basis, total carbon substitution of just under 200tC over a 200-year rotation (Figure 3) translates into an average of just under 1tC (Figure 4). For example, if Sitka spruce of YC 14 is grown on a 50-year rotation cycle (plant – grow – clearfell) then by year 50 it sequesters about 185 tC per ha (standing carbon above ground, equivalent to about 678 tCO_2 per ha). The long-run average carbon sequestration for this rotation is about 83 tC per ha (this includes debris, about 304 tCO_2 per ha). When clearfelled and used for HWP and energy, about 2.1 tC per ha are substituted (average value over the 50 years is approximately 7.7 tCO_2 per ha).

The prototype optimal rotation length model does not take account of persistence and subsequent decay of wood products in years beyond the end of the rotation. Thus, the graphs in Figure 4 only correspond to the long-run average carbon substitution in restricted circumstances – namely where all the wood harvested is used immediately as woodfuel. Inclusion of decay functions for HWP, or HWP associated with thinning would further complicate the model significantly, often implying that carbon substitution rises as the number of rotations increases and making calculation of a long-term average less meaningful (in contrast to that for carbon sequestration).

Figure 4 shows that the average carbon substitution initially increases with rotation length until a maximum is reached around 55 years, and thereafter decreases with rotation length. The point of inflection corresponds to the MMAI.

Model dependence on timber and carbon prices was explored. As discussed below, this was found to be in agreement with economic theory.

Results

Based upon the incremental approach to developing the full model, this section is structured as follows. First, we present results for a version of the model that integrates carbon considerations with the Faustmann model (that only considers timber). Here we explored:

- dependence on timber prices (with carbon prices fixed and no windthrow risk)
- dependence on carbon prices (with timber prices fixed and no windthrow risk)

Second, we present results for the fully integrated model including windthrow risk.

Carbon and timber model

Dependence on timber prices

As the timber price increases from £10/m^3 overbark (ob) to £20/m^3 ob the optimal rotation length decreases in all the versions of the model. This is a typical range of coniferous standing sales over the last couple of decades (all conifers on the Forestry Commission estate are currently sold standing).

Figure 5 shows the land expectation value (LEV) at different timber prices for the Faustmann version of the model (LEVf) prior to including carbon sequestration or substitution. LEV is the landowner's NPV from starting with bare land and harvesting the rotations after T years since planting over an infinite series of rotations. The horizontal axis represents the rotation length in years, with values on the vertical axis for the LEV. Peaks in LEVs for different prices are the maximum values that correspond to optimal rotation lengths. As predicted by the classic Faustmann theory, when the timber price increases the optimal rotation length falls (as shown by the earlier peak in LEVf at £20/m^3 ob than at lower timber prices). A negative LEV (shown by the blue and green lines in the graph being invariably below zero) indicates that without carbon payments forestry is unprofitable at the two lowest timber prices.

Figure 6 similarly shows the land expectation value at different timber prices for the Faustmann model augmented with long-term average carbon sequestration (LEVc). In line with standard economic theory, when the value of the carbon sequestration is added, the optimal rotation length increases (as shown by the earlier peak in LEVf compared to LEVc). With the carbon price fixed at £3/tCO_2e, the graph shows that at the lowest timber price (the blue line corresponding to £10/m^3 ob) it is optimal not to harvest at all.

For the purpose of developing the prototype model, the carbon price was assumed to be paid at the outset for the long-run carbon sequestration (consistent with what is currently the most common practice generally for projects certified under the Woodland Carbon Code). By contrast, payments for the carbon substitution benefits were assumed to be delayed until the wood is harvested.

Figure 5 Faustmann model (LEVf) for three timber prices: 10, 15 and 20 (£/m^3 ob).

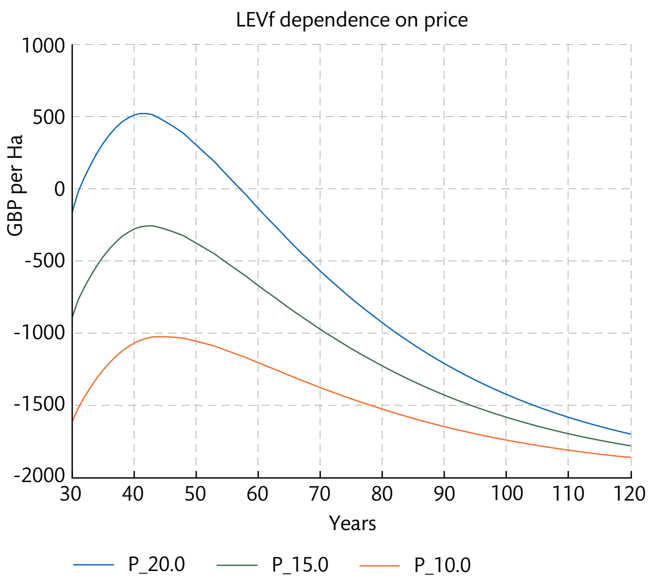


Figure 6 Faustmann model augmented with long-term average carbon sequestration (LEVc) for three timber prices: 10, 15 and 20 (£/m^3 ob).

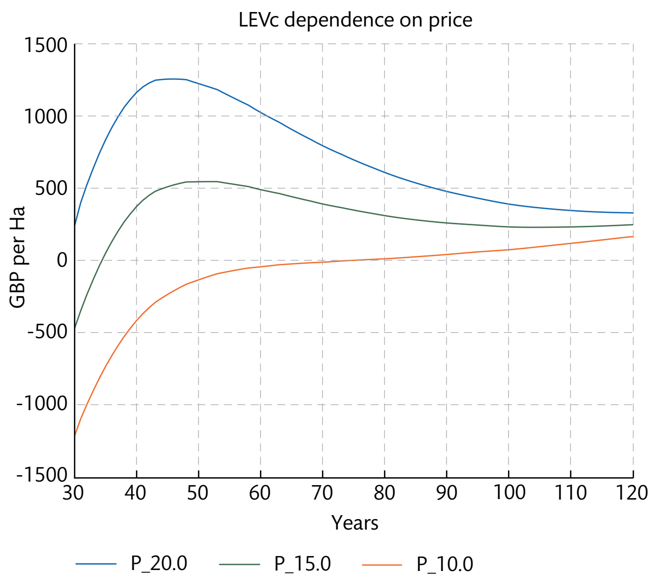


Figure 7 Faustmann model augmented with both carbon sequestration and substitution (LEVcs) for three timber prices: 10, 15 and 20 (£/m^3 ob).

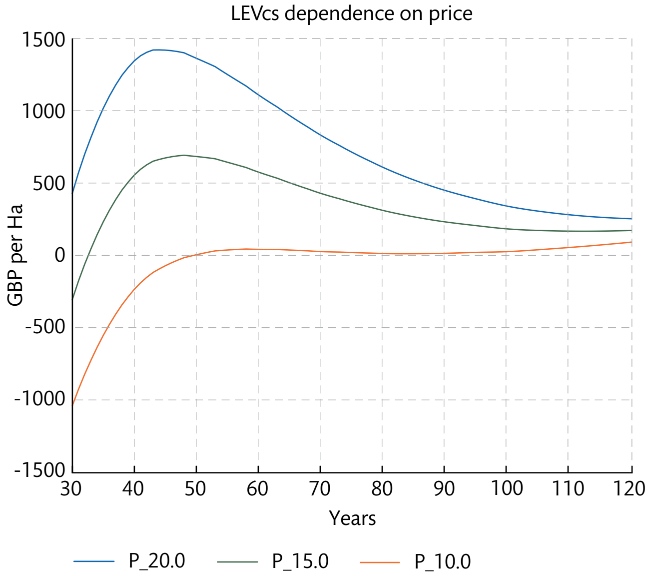


Figure 7 shows the LEV at different timber prices for the model including both carbon sequestration and substitution (LEVcs). The results are similar to the version with carbon sequestration only (Figure 6) in that it is optimal not to harvest at all at the lowest timber price (the red line corresponding to £10/m^3 ob) and a carbon price (for sequestration and substitution) of £3/tCO_2e – especially if one considers rotation lengths beyond 80 years.

While LEV values are invariably negative at £15/m^3 ob in the version of the model without carbon they become positive once carbon is added. At £10/m^3 ob, however, LEV values for the models with carbon are generally negative and turn positive only after rotations exceeding 100 years.

Dependence on carbon prices

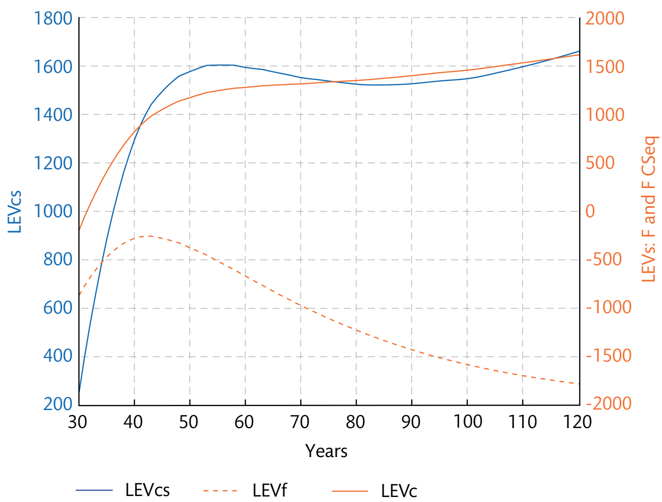
For the model with the 'long-term average carbon sequestration only' added, optimal rotation length rises as the carbon price increases. This is to be expected given that the long-run average carbon sequestration rises with rotation length.

In order to isolate the influence of prices for carbon substitution on the optimal rotation length, a version of the model with carbon substitution only was also run. As expected, this showed that optimal rotation length decreases as this carbon price is increased. This is because carbon substitution in this model formulation behaves exactly like timber with its own growth curve and price. Therefore, like the timber only model, higher prices shorten optimal rotation length.

For the model with both carbon sequestration and carbon substitution, in theory, a carbon price increase could increase, or decrease, (or have no effect on) the optimal rotation length – depending on the relative size of the effects associated with carbon sequestration and substitution. However, in the current setup including upfront payments for carbon sequestration with payments for substitution benefits at the end of the rotation, a carbon price increase leads to the optimal rotation length also increasing relative to that for the Faustmann model.

Figure 8 shows the case for a timber price set at £15/m^3 ob and a carbon price of £5/tCO_2e. To aid comparison of the relationships between LEV and rotation length, the scale for the model with both carbon sequestration and carbon substitution is shown on the left-hand axis, while that for the other two graphs is shown on the right-hand axis. (Note that, had the same scale been used, the graph for the model with both carbon sequestration and carbon substitution would lie above the other two graphs over the entire range).

Figure 8 Land expectation values by rotation length (£5/tCO_2e carbon price).



Comparing Figures 7 and 8 shows that increasing the carbon price from £3/tCO_2e to £5/tCO_2e increases the optimal rotation length from 48 years (green line in Figure 7) to 55 years for the model with both carbon sequestration and carbon substitution (LEVcs, blue line in Figure 8) if the time horizon considered is limited to around 100 years. However, if time horizons above around 106 years are considered, the increased carbon price leads to the optimal rotation length becoming undefined (i.e. longer than 100 years and probably indefinite) due to a higher and increasing LEVcs. The optimal rotation length is similarly undefined (i.e. it is best to leave the stand unfelled) for the model with only the carbon sequestration benefits added.

As illustrated above, if the carbon price is increased beyond some threshold (which also depends on other model parameters such as the timber price and discount rate assumed) or the maximum rotation length is left unconstrained (e.g. in the case of a carbon price of £5/tCO_2e shown in Figure 8), it becomes optimal not to harvest at all. DECC-recommended social values of carbon for the non-traded sector (not part of the EU Emissions Trading Scheme) are far higher than £5/tCO_2e used above. The model results imply that for coniferous standing sales prices at historically typical levels, the optimal harvesting decision from a societal perspective is not to harvest because the value of the additional carbon sequestered is greater than the value to the landowner of the timber that would be felled. Including wider factors (like wind risk), which are not accounted for in this version of the model, can alter this conclusion (see discussion of the fully integrated model below).

Prices for carbon sequestration and carbon substitution have been assumed to be the same in developing the prototype model, but could instead differ for a variety of reasons. Differences could arise, for example, as a consequence of the different social values of carbon applying to benefits arising in traded and non-traded sectors (depending on whether benefits arise in industries covered by the EU Emissions Trading Scheme or not).

The model of optimal rotation length with carbon substitution is quite insensitive to the price of carbon substitution. This is illustrated in Appendix 2. In a scenario with the carbon price for carbon sequestration fixed at £5/tCO_2 and a timber price at £20/m^3 ob, the optimal rotation length decreases by just over four years if the price of carbon substitution is increased from £5/tCO_2e to £200/tCO_2e – quite a small change in forestry timescales. This low sensitivity to prices suggests that incorporating time-dependent carbon substitution values such as those recommended by the UK Government (BEIS, formerly DECC)[3] would have minimal impact on the estimates of optimal rotation length. This might change, however, if payments for carbon substitution were made upfront (as is the case for carbon sequestration benefits), rather than after the stand is harvested as is currently assumed in the model.

Fully integrated model

This sub-section presents the initial results of the fully integrated prototype model. This is the model which uses carbon estimates from CSORT and wind risk estimates from ForestGALES.

The main determinants of windthrow probability are tree height and how windy a site is. The latter is measured using detailed aspect method scoring (DAMS). DAMS is an index of wind exposure which is used in ForestGALES to estimate the frequency and strength of winds; it is calculated from assessment of regional location, elevation, topographic shelter and aspect (Quine and White, 1993). DAMS scores for Britain range from 3 at the least windy sites to 36 at the most windy ones (Gardiner *et al*., 2006), with productive forestry rarely practised at the sites with DAMS greater than 21.

At sites with DAMS scores of 16 or below, windthrow risks were considered unlikely to be a significant consideration for Sitka spruce stands. An initial focus on a wind risk DAMS of 18 (which represents a moderately exposed site for commercial forestry) was selected as a starting point to investigate the results of wind risk in the prototype model. Statistics for Craik Forest (see Appendix 1), for example, show that a DAMS score of about 18 corresponds to the point that separates the windiest 25% of sites in this forest (in terms of DAMS scores) from the least windy 75% of sites.

To understand the influence of the discount rate on optimum rotation length, the model was run using various discount rates. Although time-dependent discount rates (which decline over time from 3.5% for the first 30 years) are recommended by HM Treasury's Green Book, this was considered too complex at the outset, so constant discount rates were used.

Table 3 presents results for each of the optimal rotation length models developed, estimated at different discount rates (and based upon the input parameters shown in Table 2). Table 3 consists of three major blocks, with results given for a specific discount rate: 3.5%, 2.5% and 1.5%. Each block contains a hierarchy of models starting with the classic Faustmann model (which does not account for carbon or wind risk), and ending with the full model that accounts for both carbon and wind risk. Models with timber where only carbon benefits (sequestration, or sequestration with substitution) or only wind risk are present, are presented in intermediate rows. Within each block the optimal rotation length (in years), LEV and timber volume are given. The biophysical maximum sustainable yield (MSY, MMAI) model is presented for reference in the top line of results. As the latter model of MSY is aimed at maximising timber volume over time and does not depend on economic information, the rotation length in this case is independent of the discount rate assumed.

3 See: https://www.gov.uk/government/collections/carbon-valuation--2 (accessed 03/10/2014)

Table 3 Optimal rotation length (T), Land Expectation Value (LEV) and timber output per rotation at various discount rates, timber price is set at £15 per m^3 ob.

Model	T (years)	LEV (£/ha)	Timber Output (m^3/ha)
MSY(MMAI)	52.9	-456	686
Discount rate 3.50%			
Faustmann	42.7	-283	528
F_CSeq	50.5	420	653
F_Carbon	48.3	667	621
F_WindRisk	39.2	-369	463
F_Carbon_WR	40.8	455	495
Discount rate 2.50%			
Faustmann	45.7	1 141	580
F_CSeq	50.6	1 892	654
F_Carbon	49.1	2 358	633
F_WindRisk	40.1	894	482
F_Carbon_WR	40.9	1 946	497
Discount rate 1.50%			
Faustmann	49.6	4 889	640
F_CSeq	52.5	5 715	680
F_Carbon	51.4	6 734	666
F_WindRisk	41.0	4 076	497
F_Carbon_WR	41.3	5 677	503

Notes: MSY (MMAI) – maximum sustainable yield (mean maximum annual increment); Faustmann (F) – the classic economic model; F_CSeq – Faustmann model augmented with long-term average carbon sequestration benefits only; F_Carbon – Faustmann model augmented with carbon sequestration and carbon substitution benefits; F_WindRisk – Faustmann model with wind risk only; F_Carbon_WR – the full model (i.e. the Faustmann model augmented with all carbon benefits and wind risk).

The results in Table 3 show that, within each block, the longest rotation length is associated with the models with carbon sequestration benefits included. It is shorter in each case than that of the timber volume maximisation MSY (MMAI) model. The results also show the shortest rotation length is associated with the models with wind risk but no carbon included. The rotation length under the Faustmann model is in between these.

Comparing the results across the blocks, the results illustrate how the optimal rotation length rises as the discount rate falls, also leading to higher values for LEV and timber volumes. The optimal rotation length increases most for the Faustmann model as the discount rate is reduced (increasing by about seven years when the discount rate falls from 3.5% to 1.5%) and least for the models with wind risk (which is seen to be the main determining factor of rotation length for this relatively windy case with a DAMS score of 18).

As might be expected, at relatively windy sites such as this, the results illustrate how wind risk largely determines the rotation length with the optimum insensitive to the level of discount rate. The optimal rotation length increases by about three years if carbon benefits, but not wind risk, are included.

As also expected, lower discount rates increase the LEV, i.e. make forestry more profitable.

Exploring critical factors affecting rotation length decisions with land owners and forest managers

To date, discussions with FES planners suggest that around half a dozen major factors (or groups of factors) are taken into account when allocating felling years to individual coupes as part of a forest design plan: MMAI, wind risk, site conditions, timber volume, restructuring, landscape, tourism and biodiversity. The weighting given to each of these by a forest planner depends upon the particular forest and zone within that forest. The process is carried out in a largely subjective manner, with little quantitative or economic evidence available to support or justify decisions. This is in no way a criticism: it reflects the multifunctional nature of public sector forestry and the state of available evidence.

Currently, carbon sequestration is not considered in forest design planning, and there appears to be no expectation for district teams to take this into account. However, it was mentioned that this might partly reflect the current lack of operationalised tools to assess carbon effectively.

It appears that decisions for most forest zones are driven largely by the need for restructuring the forest in a way that enhances the landscape, given that they were typically planted in a short period of time, within a decade, and hence most mature stands are still on their first rotation. As well as providing landscape benefits, restructuring is seen as a means to increase resilience to storms. More recently, other agendas have piggy backed on this, including resilience to pests and diseases and climate change adaptation. It is very difficult, if not impossible, to separate out the different factors within this restructuring agenda and provide weightings for their relative importance.

Restructuring decisions are shaped substantially by adjacency rules, whereby coupes neighbouring a clearfell cannot be felled until the restocked trees have reached a certain height, which often has the effect of staggering the age structure by around five years between bordering coupes. This requirement responds to multiple objectives, i.e. restructuring and diversification for resilience, landscape and biodiversity benefits.

Judgements on how best to restructure the forest are also made with a view to meeting the district's contribution to national timber volume targets. Timber production is taken into account on the basis of MMAI, adjusted for wind risk, possibly by interpreting DAMS scores, but more likely through the use of rules of thumb adjusted with expert knowledge that takes into account site conditions. Currently little economic knowledge about the costs and benefits of felling in different years is considered in forest design planning (and similarly this knowledge is not available to support species selection for restocking plans).

The way in which forest planning teams are set up, with separate specialist planners for forest management, environment, recreation and civil engineering, etc. may have had the effect of shaping (and institutionalising) the choice of factors and the ways in which they are considered in decision-making (e.g. how they are defined and grouped together and how their case is made). Better integration of planners and key stakeholders is one of the objectives of the current FES Land Management Planning project. A related point concerns the fact that the final decisions about what should be felled are not made by planners but by the district harvesting and marketing teams who then select those stands that are most suited for marketing from all the coupes identified for felling in a given year across the entire district. The factors and processes they employ to make these judgements are unclear to us, but it appears that this may result in substantial deviations in the felling year of any given stand and would be worth exploring as part of future interviews.

Felling decisions in a given forest design plan are conditioned (and sometimes determined) by the information already given in the previous forest design plan from 10 years ago. This plan will already have mapped out the zones considered suitable for felling in different five-year time intervals. The default option would be to follow the previous plan, although felling periods in the previous plan might then be adjusted to suit the current plan on the basis of more recent information or changes in circumstances.

Around 20 years ago, before the previous round of forest design plans was being prepared, a corporate planning system (FIAP) was available that provided an economic basis for optimal felling years by forecasting timber production according to standard costs and price information, and price size curves. It was also able to run forecasts on restocking plans. However, the system was eventually seen to be insufficiently accurate and perhaps for this reason it fell from use. The increasing emphasis on multipurpose forestry may also have contributed to its decline. A discussion with Forest Research's Inventory, Forecasting and Operational Support (IFOS) team should help clarify this history along with the reasons why the production forecast no longer offers this functionality. Perhaps if users were able to input more precise information on costs, prices, discount rates, etc. then it would provide more useful outputs. (While not currently geared to optimal rotation length decisions, a new investment appraisal tool – FIAP II – has recently been developed that appraises a single rotation for woodland creation projects, accounting for both timber and carbon revenues.)

Given the uniform age structure of many productive forests, a model that provides the economic optimum felling age might suggest that much of the public forest estate should be felled around the same time. Even in zones earmarked for production, some of the factors that planners need to consider in felling decisions lead to very substantial deviations away from optimal felling age. Planners are well aware that management is economically sub-optimal; the question is how to minimise the financial cost of continuing to meet multiple objectives.

An optimal rotation model that provides better economic information was seen by informants as potentially useful, even if its likely impact on decisions is perhaps lower than we might have originally expected. It could fill the gap that was addressed in part by the previous corporate system two decades ago. However, there appears to be no demand at present from FES district planners for a model that also optimises for carbon sequestration since this factor is currently not taken into account in felling decisions.

Interviewees agreed that, at this stage, the focus on felling rather than restocking decisions is most useful (although there was broader interest in improved tools for long-term economic appraisal of restocking options at particular sites, which go beyond the scope of this project).

Discussion and recommendations

The prototype model yields results which behave in agreement with standard economic theory (Amacher, Ollikainen, and Koskela, 2009), including the classic Faustmann model. For example, the impact on optimum rotation length of increasing major input variables is shown in Table 4.

Table 4 Impact of increasing input variables on optimal rotation length.

Input variable	Optimal rotation length
Price (timber)	Fall
Costs (planting)	Increase
Discount rate	Fall
Wind risk	Fall
Carbon price (sequestration)	Increase
Carbon price (substitution)	Fall

As can be seen from Table 4, for example, the response to an increase in a price of timber is negative. This implies that the optimal rotation length shortens as the timber price rises.

The results produced by the prototype model are reasonable in terms of magnitudes. This also helps verify the internal consistency of the model.

However, it is also important to run the model for a greater range of cases, varying input parameters, such as discount rates, costs, DAMS scores and prices, as well as species and yield classes. This will yield a fuller picture of model working and predictions, including sensitivities to various input parameters.

It is worth noting that the model is currently based upon maximising economic returns to the woodland owner. This implicitly assumes that there is no value to the processing sector (or users of wood) of having a higher level of timber production. (This assumption may be justified where imports are a perfect substitute for UK-grown wood, prices purely reflect those for imported timber and the latter are unaffected by the amount imported).

As part of climate change adaptation research, further testing of the full model, including sensitivity analysis with timber, wind risk and carbon components, is planned on the field data from Craik Forest in southern Scotland and possibly in Queen Elizabeth Forest Park in central Scotland. This will help to further verify that the model is working properly and eliminate major errors and software bugs if any are present in the current version.

There are a number of ways in which the model could potentially be extended. For example, more detailed CSORT estimates could be used for different categories of HWP: brush, roundwood and sawlogs. In principle, the harvesting revenue side could be refined by breaking down the harvest into these categories and applying separate prices to these products if information on the latter were available. Alternatively, account could be taken of changing size assortments as rotation length changes and corresponding price information (or price size curves).

Ideally, future developments of the model will include adding in a salvage value for windthrown timber (reminder: for the current version complete destruction of a stand is assumed), and potentially extending it to other ecosystem services, e.g. biodiversity and amenity, if found feasible. Adding a salvage value would make the model more flexible as it would be able to deal with less destructive windthrow events.

Incorporating more detailed costs for establishing and maintaining stands is expected to be quite straightforward, to the extent that the present value of these over a rotation could simply be incorporated as an element of the initial cost assumed. Allowing for a fallow period between rotations as a Hylobius management strategy could be a useful step and a relatively simple one in developing the prototype model further.

Including time-dependent variables, such as declining discount rates (as recommended by HM Treasury's Green Book) and changing carbon values (as recommended by DECC), could potentially be tackled in three different ways. In order of increasing complexity these are:

1. comparing the results of the model for various discount rates (e.g. 3.5%, 3%, 2.5%, etc.) and price projections complemented by a sensitivity analysis – the approach adopted to date in developing the prototype model;
2. moving to a different modelling paradigm: from a classical Faustmann approach, which relies on an infinite series for a typical rotation NPV, to an approach that would require the use of methods of dynamic programming;
3. adopting a stochastic dynamic programming approach to also allow for uncertainty (although it may then only be possible to calculate the optimal rotation length for the first rotation – see more explanations in Appendix 4).

In some cases, refinements such as incorporating declining discount rates and carbon values over time may have little effect on the results. For example, initial results indicate that the model is quite insensitive to carbon substitution prices and, hence, the use of a fixed carbon price for this may have minimal impact on the results. The dependence of optimal rotation length on discount rates for sheltered sites, where wind risk is not the determining factor, is noticeable but still not very significant – for example, for the optimal rotation length model with carbon benefits, as the discount rate falls from 3.5% to 1% the rotation length rises only by about five years, which is not a very large increase in forestry timescales.

Recommendation 1: Consideration is needed to potential further development of the current prototype by:

- adding a salvage value for windthrown timber to the model (the current prototype implicitly assumes a zero salvage value);
- extending the model to include adaptation to wider climate change-related risks (e.g. drought, pests and diseases), and to other ecosystem services and biodiversity;
- using Treasury Green Book declining discount rates and DECC social values of carbon;
- allowing a fallow period between rotations as a Hylobius management strategy;
- allowing for any expected changes in timber prices over time, and for timber prices to vary with rotation length to account for changing assortments and proportions of wood harvested going to different end uses;
- linking to GIS and forestry spatial data to develop the model into a useful decision support system (DSS) tool able to produce maps of optimal rotation length and associated NPVs for a forest. Maps of differences with biological or simple Faustmann models could also be produced to facilitate visualisation and discussion of different management options in cost – benefit, monetary terms.

Following a number of discussions with various people within FR it became clear that if the prototype optimal rotation length model were to be developed into a really useful DSS tool, the model should be linked to GIS and forestry spatial data. This would make it possible to produce maps of optimal rotation length values for a forest. Maps of differences with biological or simple Faustmann models could also be produced to facilitate visualisation and discussion of different management options in cost – benefit, monetary terms. This could allow greater appreciation of the opportunity costs of existing management practices and could aid greater consideration of economics in practical forest management and planning decision-making.

Recommendation 2: Conduct additional research and knowledge exchange, in particular with forest planners at district and national level, and with the private sector, to explore the demand for the development of an optimum rotation length tool, and, where appropriate, facilitate the co-production of model outputs and DSSs.

The Woodland Carbon Code look-up tables are also based on CSORT. Potentially the prototype model could be used to determine the optimal rotation length for projects under the Code – although currently there are very few traditional timber production forestry projects that have been certified under the Code. Various payment mechanisms (such as a lump-sum payment at the beginning of a project or at some fixed time into a project's lifetime, or a stream of payments) could also be assessed using the model. Model outputs, especially LEV values, could give potential carbon investors an insight into indicative NPVs for particular projects.

Adapting the management of existing forests to climate change requires allowance to be made not only for future risks such as increased frequency of catastrophic windthrow, but also for a range of other risks such as drought and outbreaks of diseases. Extending the prototype model to cover other types of risks is anticipated to be relatively straightforward in cases where probabilistic estimates of their expected incidence are available, and also if estimates are available of their impacts on timber yields, carbon and other ecosystem services. Furthermore, if one were to focus on modelling uncertainty effects, it is worth considering other approaches which are better suited for the task: the option approach and stochastic dynamic programming.

Recommendation 3: If climate change uncertainty is to be adequately accounted for, alternative methodologies to the approach used for the prototype model should be considered as the basis for further model development.

The prototype optimal rotation length model is based on existing FR models for growth, wind risk and carbon balances. Thus, caveats applicable to existing versions of these models also apply to the prototype model.

The prototype optimal rotation length model currently uses an even-aged stand approach. However, the model potentially could be developed to deal with different types of forestry apart from homogeneous stands, including CCF and mixtures, i.e. forests heterogeneous in terms of age and/or species composition. For example, to the extent that a non-homogeneous forest stand can be considered to comprise a number of homogeneous patches, one could just focus at the level of a homogeneous patch of woodland and apply the model as before.

In cases of true mixtures when species interact and affect yields, the model could still be applied using a new aggregated yield curve as for a single species stand.

Recommendation 4: Additional experimental research on yields, carbon benefits and windthrow risks associated with mixed species stands is needed if the model is to be extended to cover such cases.

In cases of CCF comprising a single tree species, the issue of optimal timing of harvesting could be addressed by considering a forest as being composed of several homogeneous stands of different ages, probably with lower density of planting than in traditional productive forestry. In a stand with natural regeneration, planting costs could be assumed to be zero in the model. On the other hand, working CCF stands for harvesting may be more involved and hence more costly. The implied costs variations could be reflected in the model parameterisation where empirical data are available. The model could then be applied to each age class within stand with the results aggregated, unless harvesting one age class affects windthrow risks and growth rates of others.

Recommendation 5: Research is needed on impacts of CCF and natural regeneration on timber yields, costs and revenues to provide a more solid foundation for extending the model to CCF. This should include research on the effects of age structure and density of typical CCF on yields, costs and revenues.

The combination of the approaches proposed above could be used to tackle CCF with mixtures (i.e. comprised of more than one tree species).

New models may need to be developed to address the problem of optimal management of non-homogeneous forest stands in a more direct way. In modelling multi-age stands the following references (Tahvonen, 2007; Tahvonen *et al*., 2010; Tahvonen *et al*., 2009; Tahvonen, 2004a; Tahvonen, 2004b; Salo and Tahvonen, 2003; Uusivuori and Kuuluvainen, 2005) may provide a good starting point, although it must be noted that these models are much more complex than the prototype currently developed.

Research gap: Further research is needed on how ecosystem services (e.g. recreation and amenity) and biodiversity vary with stand age and structure.

Given the prominence of other ecosystem services and biodiversity on policy agendas, it is important to consider how these might also be incorporated into the optimal rotation length in cases where the level or quality of provision is thought to depend on stand age. However, experimental data are often lacking in quality and coverage in trying to link these to stand age.

A related study by FR (Barsoum *et al*., 2016) examines links between biodiversity and rotation length with a view to exploring how biodiversity could be incorporated into an optimal rotation length model.

Concluding remarks

The above discussion suggests important potential avenues for future research. There are a range of areas of forest policy and practice that could benefit from the development of a tool that utilises state of the art carbon accounting in forestry to evaluate various scenarios such as changing wind risk (through changes to DAMS scores), planting different species, effects of prices, costs and discount rates on optimal rotation length.

However, as is often the case, economic analysis can only progress as far as there are empirical data available to underpin the analysis and show, for example, how various stand structures and mixtures affect timber yields, wind risk and carbon. Conducting field studies to address the research gaps identified above will be important to realise the potential to extend the model further.

In considering potential practical application of the model it is important to get better understanding of how the optimal rotation model would fit into the decision-making process of a range of forest managers. Afterwards, a useful step would be for planners to identify a forest that could be the subject of a forest design plan in the next 12–18 months and then for planners and researchers to work together to test and refine the model for that particular forest. This should reveal how, and to what extent, the outputs could inform forest design planning.

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Appendix 1: Craik Forest case study

This appendix presents initial results of the early version of the prototype model using a simplified carbon module before CSORT integration took place. As these are based upon the initial, very simple approach to incorporating carbon sequestration (assuming a fixed fraction to be stored long term), the results are not directly comparable to those in the main text of this report. Nevertheless, initial results of the model testing for the latest version, which includes outputs from CSORT, indicate that changes are small and major tendencies are not reversed.

Craik Forest in southern Scotland covers an area of 4 729 ha across an altitudinal range of 175–425 m. The main forest species is Sitka spruce (Picea sitchensis) comprising 65% of the forest. Other conifers represent 9%, broadleaves 7%, and the remainder 19% comprising felled areas, permanent open space and other ground. At present, over 85% of the forest is managed on a patch clear-felling system, and can be classed as intensive even-aged forestry.

Tables A1.1 to A1.3 show some basic statistics about Craik Forest: main species, yield class (YC) and detailed aspect method scoring (DAMS) score (one of the main determinants, together with tree height, of wind risk).

Table A1.1 Tree species growing in Craik forest.

Species	Hectares	Share %	Cumulative %
Sitka spruce (SS)	3 077	65.1	65.1
NA	899	19.0	84.1
Norway spruce (NS)	273	5.8	89.8
Downy birch (PBI)	189	4.0	93.8
Japanese larch (JL)	89	1.9	95.7
Lodgepole pine (LP)	86	1.8	97.5
Hybrid larch (HL)	49	1.0	98.6
Scots pine (SP)	42	0.9	99.5
Mixture SS/LP	10	0.2	99.7
Douglas fir (DF)	6	0.1	99.8
Grand fir (GF)	5	0.1	99.9
Beech (BE)	3	0.1	99.9
Noble fir (NF)	2	0.0	100.0
Common alder (CAR)	1	0.0	100.0
Total	**4 729**	**100.0**	

As can be seen from the Table A1.1 Sitka and Norway (Picea abies) spruces represent about 90% of tree species in Craik. Hence we currently focus on these two species.

Tables A1.2 and A1.3 show that typical values of YC and DAMS score for Sitka spruce in Craik Forest are 15 and 17 respectively.

Results from running the initial prototype model for optimal rotation and land expectation value (LEV) for various typical YC and DAMS values for SS and NS in Craik are presented in tables A1.4 and A1.5. Results are presented for five different optimal rotation length models: biological model of a maximum sustainable yield (MSY), an economic Faustmann model (F) and its variants that include wind risk (F+Wind Risk) and carbon sequestration (F+Carbon Benefits), and the prototype model that accounts for both wind risk and carbon sequestration (Integrated).

In agreement with standard economic theory as applied in forestry, including the classic Faustmann model, the optimal rotation length falls as YC and/or DAMS increases.

Table A1.2 Yield class statistics for Sika Spruce (SS) and Norway Spruce (NS).

	Minimum	1st Quartile	Median	Mean	3rd Quartile	Maximum
SS	0	13.3	14.7	15.8	17.8	24.8
NS	0.1	4.4	8.2	8.4	13.1	15.4

Table A1.3 DAMS summary statistics for Sika Spruce (SS) and Norway Spruce (NS).

	Minimum	1st Quartile	Median	Mean	3rd Quartile	Maximum
SS	11.7	15.7	17.1	16.7	17.9	20.2
NS	11.4	12.8	15.1	14.9	16.6	18.8

A1.4 Optimal rotation length (years).

	Sitka spruce				Norway spruce
Model	YC14	YC16	YC18	YC24	NS_YC10
MSY	53	52	46	43	71
Faustmann	42	41	36	33	48
F+Carbon Benefits	78	67	61	53	148
DAMS 16					
F+Wind Risk	41	37	36	31	46
Integrated	53	47	46	35	113
DAMS 18					
F+Wind Risk	40	35	33	27	
Integrated	46	40	36	30	
DAMS 20					
F+Wind Risk	35	33	30	19	
Integrated	39	34	32	25	

Note: ForestGALES output for Norway spruce (NS) of DAMS 16 is the same up to YC 12.

Estimates for LEV, which is equal to the net present value over an infinite series of rotations, are shown in Table A1.5.

Table A1.5 illustrates how LEV increases with YC and falls with the windiness of the site.

The prototype optimal rotation length model is based on existing Forest Research (FR) models for growth, wind risk and carbon balances. Thus, caveats applicable to existing versions of these models also apply to the prototype model.

In particular, there are significant knowledge gaps concerning mature stands retained substantially longer than normal rotation lengths (about 40 to 60 years for conifers). The growth model (M1) used in all simulations relies on field data collected for stands which are mainly under 100 years old (for coniferous woodland). After that age the data are constructed as a smooth extrapolation taking into account competition mortality (i.e. that some trees do not survive because of the ground area imposed carrying capacity limitations related to trees' size and number), but not biological mortality (i.e. age-dependent probabilities of a tree dying). Therefore, most of the results for coniferous stands extending beyond 100 years must be treated with caution.

Potential future research may also investigate incorporating climate change-dependent yield models. One possibility would be to draw upon the CDYsim model (developed by John Fonweban at FR) which uses climatic and biophysical (elevation, slope and age) variables to predict yield for five major conifer species (Sitka spruce, Scots pine (*Pinus sylvestris*), Douglas fir (*Pseudotsuga menziesii*), Norway spruce and Japanese larch (*Larix kaempferi*)) grown in Britain. An alternative approach that currently looks more promising is that a version of the CARBINE model (Robertson *et al*. 2003) could be used. CARBINE is currently used for national-scale carbon scenario analysis, accounting and reporting, and is being developed to incorporate climate change-dependent yield models (although the state of documentation is currently poor, with the model mostly still in the development phase). Climate change issues may also potentially be incorporated through other impacts on the suitability of different species.

A1.5 Land expectation value (LEV, £).

Model	YC14	YC16	YC18	YC24	NS_YC10
MSY	183	580	1 276	2 777	71
Faustmann	518	1 036	1 629	3 457	-767
F+Carbon Benefits	7 607	10 142	11 956	17 351	4 059
DAMS 16					
F+Wind Risk	-129	376	907	2 553	-1 283
Integrated	6 030	8 513	10 313	14 930	2 957
DAMS 16					
F+Wind Risk	-146	274	747	2 084	
Integrated	5 578	7 738	9 055	13 312	
DAMS 16					
F+Wind Risk	-440	13	321	1 509	
Integrated	4 375	6 782	8 002	11 119	

Appendix 2: Carbon substitution prices

As the price placed upon carbon substitution increases, the optimal rotation time decreases. The results in Table A2.1 illustrate the lack of sensitivity of optimal rotation length to the assumed carbon substitution price. The results are from the fully integrated prototype model (i.e. based on CSORT estimates for carbon sequestration and substitution), with a timber price of £20/m^3 ob assumed.

In the case explored above, the MSY(MMAI) model is associated with the longest rotation length and has the lowest land expectation value, indicating that it is economically sub-optimal. However, including wider factors in the model (e.g. biodiversity), could potentially alter this result.

A2.1 Prototype model and carbon substitution.

Model	T (years)	LEV (£/ha)	Output per rotation (m^3/ha)	Carbon substitution price (£/tCO_2e)	Maximum
MSY(MMAI)	52.88	183	686	NA	24.8
Faustmann	41.24	489	503	NA	
F_CSeq	43.21	763	538	NA	
F_Carbon	42.19	1 226	520	**5**	
F_Carbon	39.38	5 502	468	**50**	
F_Carbon	38.67	10 296	454	**100**	
F_Carbon	38.16	19 902	444	**200**	15.4

Notes: MSY(MMAI) stands for the MSY model which is based on maximising the volume of timber produced over time, results in the maximum mean annual increment (MMAI). F_CSeq is the Faustmann model augmented with long-term average carbon sequestration. F_Carbon is the Faustmann model augmented with long-term average carbon sequestration and carbon substitution benefits.

Appendix 3: Prototype model (2015 version) documentation

General model overview

Model inputs include:

1 Yield model data showing dependence of volume (m^3) on time (stand age).
2 Wind risk data from ForestGALES.
3 Economic data: prices for timber and carbon, discount rate and costs of replanting and windthrow damage.

The model outputs the optimal rotation length that maximises the landowner's land expectation value (*LEV*). This represents the sum of net present values (NPVs) of forest operations and the final sale, over an infinite series of rotations. The NPV is defined as revenues less costs (both appropriately discounted). It is optimal in the sense of maximising the *LEV*.

Software and licensing

The model has been built using free open source software. This has the advantage that in principle the model could be altered, distributed and used freely. The model is programmed using the following software:

- Python programming language, version 2.7.2, website: http://www.python.org/. The Python implementation is under an open source licence that makes it freely usable and distributable, even for commercial use. The Python licence is administered by the Python Software Foundation (PSF) (http://www.python.org/psf/).
- NumPy is the fundamental package for scientific computing with Python, version 1.6.1, website: http://www.numpy.org/. NumPy is licensed under the BSD licence (first used in 1980 for the Berkeley Source Distribution (BSD): http://www.linfo.org/bsdlicense.html), enabling both commercial and non-commercial reuse with few restrictions.
- SciPy is open source software for mathematics, science and engineering, version 0.10.1, website: http://www.scipy.org/SciPy. The SciPy library depends on NumPy, which provides convenient and fast N-dimensional array manipulation. SciPy's licence is free for both commercial and non-commercial use, under the BSD terms (http://www.scipy.org/FAQ#head-22f0cc18e232f57520678cd55ef7e904113fa304).
- Matplotlib is a Python 2D plotting library, version 1.1.0 or higher, website: http://matplotlib.org/. Matplotlib was written and maintained primarily by John Hunter (Hunter, 2007), and is distributed under a BSD-style licence, i.e. it only uses BSD compatible code, and its licence is based on the PSF licence (http://matplotlib.org/users/license.html).
- Eclipse SDK, version 3.7.2 or higher is used as programming IDE (integrated development environment) for this project, website: http://www.eclipse.org/. All Eclipse projects are licensed under the Eclipse Public License (EPL), a commercial friendly Open Source Initiative (OSI, http://opensource.org/licenses) approved licence (http://www.eclipse.org/org/). It is used with a PyDev plug-in that enables Eclipse to be used as a Python IDE, version 2.7.1 or higher, website: http://pydev.org/index.html. PyDev is using the same licence as Eclipse: EPL (Eclipse Public License, http://www.eclipse.org/legal/epl-v10.html).
- ALGLIB is a cross-platform numerical analysis and data processing library, version 3.7.0 (cpython), website: http://www.alglib.net/. It can be used under GPL 2+ licence (http://www.alglib.net/download.php).
- Py4J (http://py4j.sourceforge.net/) enables Python programs running in a Python interpreter to dynamically access Java objects in a Java Virtual Machine. Py4J also enables Java programs to call back Python objects. Py4J is distributed under the BSD licence (http://en.wikipedia.org/wiki/BSD_licenses). Py4J is used to integrate ForestGALES.

Major programming steps

1 The programme reads output from the Yield Model (Forest Yield or M1) into an array with two columns: Age and Volume (m^3/ha), with (t0,y0) being the first data point. Then the following data transformations are performed:
   a Exponential extrapolation of the Yield Model from time\age zero (0,0) to (t0,y0).
   b Appending this exponential extrapolation to the data read from the Yield Model to form a full time-series running from t=0 to 200 years (in case of M1 output).
   c Cubic spline interpolation (agnostic, local function approximation) on the full time-series of yield. The SciPy interpolate.UnivariateSpline function is used in current implementation. Possible alternatives are: monotone cubic Hermite (implemented in ALGLIB) or Akima (stable to the outliers) interpolations.

2 The programme reads output from ForestGALES for a location The following data transformations are applied:
   a The minimum return period (*R*) between Breakage and Overturn is selected for each data point.
   b The probability of a storm in year i is calculated as $p_i = 1/R_i$. The probability of no storm occurring until time T is then given by: $\prod_{i=1}^{T}(1-\boldsymbol{p}_i)$, and hence the probability of a storm occurring by time *T*, $\Pr(X<T)=1-e^{-m(T)}$, is: $1-\prod_{i=1}^{T}(1-\boldsymbol{p}_i)$. Therefore: $e^{-m(T)}=\prod_{i=1}^{T}(1-\boldsymbol{p}_i)$, where m(T) is the mean value of the non-homogeneous Poisson process. Function $e^{-m(T)}$ is the function that needs to be estimated from ForestGALES output.
   c Probabilities (pi) are fitted into a logistic function: $\boldsymbol{p}_i(t) = c_0 * \exp(c_1*(t-c_2))/(1 + \exp(c_1*(t-c_2)))$, with the three parameters (c_0, c_1, c_2) estimated using least squares. The estimated function makes the probabilities a continuous function of time.
   d $e^{-m(T)}=\prod_{i=1}^{T}(1-\boldsymbol{p}_i)$ is then approximated using spline interpolation.
3 Given smooth approximations of the yield and wind risk, one can construct the *LEV*. This can be maximised either using direct function maximisation or by solving derived first order conditions.
4 Except for the Faustmann model, where it is easy to derive first order conditions, other models currently use methods for direct *LEV* function maximisation from the SciPy optimize module (see SciPy documentation for details on these methods).
5 Where wind risk is involved, the *LEV* function uses an integration routine from the SciPy integrate module (see SciPy documentation for details on these methods).

Finally, to run estimations for the full model, i.e. the model with timber, wind risk and carbon benefits, one needs first to run CSORT for the particular species and management regime. Please, refer to the CSORT developers on how to run the model.

The version of CSORT used as the basis of the estimates presented in this report is identifiable from its main executive file, which is named *duration15d.exe*.

CSORT produces output in the form of a set of files in CSV format. These must be further processed to derive estimates of the long-term average carbon sequestration and carbon substitution over a series of rotation lengths.

Maximisation of the *LEV* with respect to rotation length (*T*) yields the optimal solution, with expressions for the *LEV* in the different models presented in Table A3.1.

The main Python programs created to run the suite of models developed for the project are listed in Table A3.2.

Table A3.1 Land expectation values for various optimal rotation models.

Model of optimal rotation	LEV
Faustmann	$LEV_f=(1-e^{-rT})^{-1}[\,pf(T)e^{-rT}-c\,]$
Faustmann with a non-homogeneous risk of catastrophic event, wind risk	$LEV_{WR}=\dfrac{[\,pf(T)-c_1e^{rT}\,]e^{-(rT+m(T))}-\int_0^T(e^{-m(x)})'e^{-rx}(-c_2-c_1e^{rx})dx}{r\int_0^T e^{-[m(x)+rx]}dx}$
Faustmann with carbon benefits	$LEV_C=\dfrac{[\,pf(T)+P_{C\_SUB}\cdot C\_SUB(T)]\cdot e^{-rT}-c}{(1-e^{-rT})}-P_{C\_LTA}\cdot C\_LTA(T)$
Full model, Faustmann model with carbon and wind risk	$LEV_{CWR}=\dfrac{[\,pf(T)+P_{C\_SUB}\cdot C\_SUB(T)-c_1e^{rT}]\cdot e^{-(rT+m(T))}-\int_0^T(e^{-m(x)})'e^{-rx}(-c_2-c_1e^{rx})dx}{r\int_0^T e^{-[m(x)+rx]}dx}-P_{C\_LTA}\cdot C\_LTA(T)$

Note: *r* – discount rate, *c* (or c_1) – replanting cost, *p* – the stumpage price received for harvesting, *T* – rotation age. Production function *f(t)* also called growth/yield function, denotes the volume of the forest in cubic meters (m^3) of wood at time *t*. Prices for carbon substitution and long-term average sequestration are given by P_{C_SUB} and P_{C_LTA} correspondingly. In the case of windthrow there is an extra cost incurred (c_2) currently set at 20% of c_1.

Table A3.2 Major Python programs.

Python script	Purpose	Input/output
Growth_Approx.py	Growth model approximation	Input: growth data, e.g. M1 Output: cubic spline approximation
FWindRisk_FG1.py	Solving Faustmann model with wind risk	Input: economic data and (same as required to run ForestGALES) species, yield class, spacing, DAMS and soil data Output: solution of Faustmann model with wind risk
vFormatCSV.py	Process CSORT output to obtain cubic spline approximations of carbon sequestration and substitution data depending on a rotation length	Input: AllCumsumm.csv data from CSORT Output: cubic spline approximations of carbon sequestration and substitution data depending on a rotation length
FCarbon_V31.py	Solving Faustmann model with carbon	Input: economic data and growth and carbon data approximations from CSORT output Output: solution of Faustmann model with carbon
fullORL_V4.py	Solving full model, i.e. Faustmann model with carbon and wind risk	Input: economic data and (same as required to run ForestGALES amended with CSORT output) species, yield class, spacing, DAMS, soil and carbon data Output: solution of the full Faustmann model with carbon and wind risk

Note: economic data include prices for timber and carbon, costs of replanting, costs of replanting in case of windthrow, and discount rate.

Appendix 4: Option approach and stochastic dynamic programming

Dynamic programming (DP) is particularly well suited to deal with sequential problems (of which forest management is one) and allows for incorporation of uncertainty, for example concerning future prices, interest rates and yields (due to climate), arising from the long-term nature of forestry. DP is well developed for applications in both discrete and continuous time settings. It was developed in the 1950s by Bellman and others (Bellman, 1954; Bellman, 1957) and applied initially in engineering (http://en.wikipedia.org/wiki/Bellman_equation). The DP approach has become much more widely accepted and its application in economics has increased since the late 1980s when a number of examples demonstrated how to employ DP to economic issues (Stokey, Lucas and Prescott, 1989).

Another method used in forest management and investment appraisal is the real option approach (or option approach for short). It is based on the theory of financial options valuation and is relatively new for forestry, with the majority of early applications dating to the 1990s (Hildebrandt and Knoke, 2011). Its relevance stems from the nature of investment decisions in forestry. Most investment decisions in forestry have three important characteristics:

1 The investment is partially or completely irreversible, i.e. the initial cost is at least partially irrecoverable.
2 There is uncertainty over the future return.
3 There is some flexibility in timing of significant investment decisions (planting, thinning, harvesting). One can postpone action to get more information about the future.

The ability to delay some irreversible investment actions is akin to a financial call option that gives the right to buy an underlying asset at a certain price in a certain period and offers managerial flexibility (e.g. options to wait/delay, to abandon, to change the amount invested), which has a value that can be evaluated. In general, the option value increases with the size of the sunk cost and with the level of uncertainty over the future (Dixit and Pindyck, 1994).

Option pricing yields a new and useful view of uncertainty. In particular, it demonstrates the economic value of flexibility in the decision-making process in an uncertain environment (Hildebrandt and Knoke, 2011).

Modern forest management practice increasingly adopts an ecosystem services approach to account for the multiple benefits and objectives of forestry. It is also increasingly linked to climate change adaptation and mitigation strategies. In view of the priority given to these policy agendas, it is important that new models take an integrated approach to accounting for these issues.

This Research Report focuses on the development of a prototype integrated optimal rotation length model. The model is integrated in the sense that it accounts for timber production, climate change mitigation in terms of carbon sequestration and substitution benefits, and climate change adaptation in relation to windthrow risks. Extending traditional models (which focus upon timber production only), to cover the wider benefits of woodlands in the presence of climate change risks, will contribute to more comprehensive comparisons between management alternatives in terms of net present values, rotation lengths and production volumes.

The research illustrates how several models developed by Forest Research (ForestGALES – wind risk evaluation tool, and CSORT – carbon accounting in forestry) can be linked together to produce answers to complex queries. In this case: what is the economically optimal harvesting time when timber and carbon benefits are included in the presence of wind risk?

Currently the model has been tested using Sitka spruce yield class 14. A key result of this test shows that in some cases, optimality involves leaving stands unfelled for the carbon sequestration benefits, while at some windier sites windthrow risk can be the main factor determining optimum rotation length.

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