Forest Research

Developing multi-stand / CCF version of optimal rotation length prototype model

Final Report

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Executive Summary

The report presents an investigation of potential ways to develop a multi-stand version of the prototype optimal rotation length model based upon a classic Faustmann approach. Development of a new model aims to help address optimal management issues arising in management of and transition to continuous cover forestry (CCF) and uneven-aged forestry.

The report consists of two parts. The first part presents results of a literature review on previous work on developing economic models of multi-stand forests and CCF. The second part presents initial results for a prototype model based upon numerical analysis of optimal harvesting decisions. The literature review showed that there are currently three major approaches to modelling optimal management of uneven-aged forests:

1. forest age-class models;
2. modified or generalised Faustmann models;
3. matrix models.

Of the three approaches, matrix models, is the most widely cited and applied. However, it is also the one requiring the most advanced growth models, harvesting costs models and detailed forest inventory data. Due to current model and data limitations, it was agreed to proceed initially with development of a prototype forest age-class model.

The review also considers terminology and the definition of different forest types, including CCF. It also identified evidence on the multiple benefits of mixed uneven-aged forests.

A prototype forest age-class model was developed, extending the classic Faustmann model, with long-run equilibria investigated for the case of Sitka spruce yield class 14, a species yield class category typical of large areas of existing British forests. A simple harvesting rule to achieve a normal forest structure (an equal area of land allocated to each age class, providing a constant harvest over time) was identified from any initial age-class distribution. While significant complexity prevented a full dynamic analysis of the optimality conditions, a numerical example of convergence to a normal forest structure is provided.

The following recommendations are made for future research and model development:

◇ **Recommendation 1:** *The best way to proceed with further research and model development is to develop the prototype forest age-class model. This could ultimately be extended to include carbon sequestration and windthrow risk as well.*

◇ **Recommendation 2:** *Further work is needed to improve our understanding of existing age-class models such as Uusivuori and Kuuluvainen (2005), especially the numerical solution procedure for the full model including the amenity value of forest stands. This would facilitate exploration of the influence of carbon sequestration (and potentially biodiversity) on an optimal harvesting schedule. The dynamics of forest transformation to a desired structure and associated harvesting strategies could also be investigated for this case.*

Developing an age-class model is only a first step towards a fully-fledged model of an uneven-aged forest. However, it is the only step that is feasible unless tree growth models that exhibit dependence on stand density variables are available.

Matrix models are capable of analysing management decisions for a much wider range of forest types than forest age-class models, and could help in making forest management decisions for forms of CCF that are more complex than those simply comprising of single species age classes. The feasibility of developing a matrix model will depend crucially on availability of growth models capable of accounting for species interactions and/or spatial structure (i.e. distance-dependence) of stands, such as MOSES GB, which is currently being calibrated for some of the major species grown in Britain.

◇ **Recommendation 3:** *Once the prototype forest age-class model has been sufficiently developed, the development of a matrix model for uneven-aged forestry management should be considered. Given the progress with MOSES GB growth model re-calibration for Britain it is recommended to proceed from forest age-class modelling to development of a matrix model based upon the MOSES GB model as soon as feasible.*

◇ **Recommendation 4:** *To further aid our understanding of optimal forest management strategies for uneven-aged forestry and CCF, future research should also account for risk and uncertainty issues due to climate change (possibly through impacts on growth and wind risk) and due to changes in the economic environment (prices, costs and interest rates). It would benefit from exploring application of stochastic programming approaches, including an option approach, and /or Monte-Carlo simulations.*

Major research and data gaps identified include the current lack of appropriate growth models and of detailed costs models for forestry operations apart from clear fell harvesting scenarios, essential for further model development and application. Further research by growth modellers and silviculturists on these issues is likely to be needed to address these evidence gaps.

Developing multi-stand / CCF version of optimal rotation length prototype model

Introduction

Optimal rotation models aim to help ensure the efficient use of scarce resources in terms forestry management. Forest Research has developed a prototype forest optimal rotation length model that includes climate change adaption and climate change mitigation elements. The model currently integrates timber production with carbon sequestration and substitution, together with windthrow risk considerations.

The prototype model supports forestry policies in the UK that have increasingly broadened in recent decades from a traditional focus on timber production to also consider biodiversity, carbon and other benefits of forestry. It also supports adoption of an ecosystem services approach to account for the multiple benefits of woodlands.

However, the initial prototype model focuses purely on single-species even-aged forests. The focus of Forest Policy in the UK has increasingly shifted towards interest in diversifying homogeneous forests to increase biodiversity and other ecosystem services provided by forests and to strengthen resilience to climate change and pest and diseases. The shift is reflected in the UK Forestry Standard (Forestry Commission, 2011), a range of forestry policy documents (e.g. Forestry Commission Scotland, 2006; Forestry Commission Wales, 2009), as well as goals set by the Ministerial Conference on the Protection of Forests in Europe (MCPFE). Structural diversification, including through Continuous Cover Forestry (CCF), is one element of this broader policy agenda. Interest in bringing unmanaged forests, many of which are uneven-aged, into management, is a further reason for greater interest in forest management issues of complex forests with more than one age class and species present.

Diversification of forest age structure is often impeded by strongly held opinions that managing uneven-aged forests is less profitable than managing mono-age plantations on a clear-cut basis. Contrary to this, recent reviews (Knoke *et al.*, 2008; Kuuluvainen, Tahvonen and Aakala, 2012) found that uneven-aged forestry was fully competitive with even-aged management in a number of cases, sometimes outperforming it. Similarly, recent research on CCF (Pukkala and von Gadow, 2012; Davies and Kerr, 2015; Jacobsen, Jensen and Thorsen, 2016) shows that often the transformation need not be costly and land values can be higher after transformation than for even-aged forests.

This is a two-part report consisting of a literature review, followed by development and analysis of an initial forest age-class model. In the first part of the report based on the literature review we outline how the initial prototype optimum rotation length model can be extended to apply to CCF. This draws upon results of a literature review on terminology, definitions and approaches to economic modelling of uneven aged forests and CCF. Major research gaps are also identified.

We argue that a multi-stand optimal rotation length approach based on forest age-class models can be consistent with a minimalist definition of CCF. This is the case so long as three conditions (constituting a minimal CCF) are met: stand size is limited to below 0.25 ha; adjacent stands are constrained to be of different ages; and the felling of adjacent stands in the same period is not permitted. This is the approach recommended for the next stages of development of the prototype model.

Broader forms of CCF extend to mixed species uneven aged stands and to greater degrees of mixing of different tree species and canopy layers (see the *Continuous cover forestry (CCF) definitions* section). Mixed forests often provide a wider range of ecosystem services than single-species even-aged forests, with some evidence that mixed, uneven-aged forests are more biodiverse and more resilient with respect to climate change and many other abiotic and biotic risks (see *Annex 3 References on benefits of mixed complex forests*). Moreover, there is evidence on public preference to a more natural looking forests (Edwards *et al.*, 2012).

The initial literature review undertaken for this extends to approaches to modelling these more complex forest types.

This report proceeds as follows: Aims and objectives of the study are presented. The next section briefly explains the methodology and how the literature review was conducted. The following section presents the results with three subsections: terms and definitions; benefits of mixed complex forests; and economic modelling approaches to complex forests. We conclude with discussions and recommendations. Next we proceed with an initial development and analysis of the forest age-class model and present first results.

Aims and Objectives

The key objective is to develop and demonstrate a prototype optimal forestry management model which could work in a multi-stand / CCF setting integrating a range of forest ecosystem services and benefits, e.g. timber and carbon initially, with potential inclusion of biodiversity and recreation in future developments of the model.

Specific objectives of the first stage of this study were:

1 To gain a better understanding of the meaning of multi-stand / CCF approaches: multi-age and/or multi-species stands. Consider existing approaches to economic modelling of complex forest stands and recommend the most suitable one.

2 Develop a prototype optimal rotation length model that accounts for timber production, carbon sequestration and potentially other ecosystem services (ES) provided by forests in a multi-stand/CCF setting.

3 Investigate how multi-age/multi-species forests and CCF as modelled by the prototype model compares in economic terms with a traditional 'plant-grow-clearfell' intensive productive forestry.

4 Demonstrate the prototype model application on real data for a typical CCF example (subject to data availability from CCF field trials).

This report addresses the first and second (partially) of these specific objectives. Due to unforeseen complexity of initial model development, no time was left from that initially allocated to proceed with its expansion to include carbon sequestration (which had to be left for the next development stage). For the same reason collection of data sources on CCF economic performance in UK field trials and comparison with the model output were not feasible.

Methodology

The study was primarily based on a review of international literature conducted in line with the Government Social Research Service (GSR) Rapid Evidence Assessment (REA) guidance (GSR, 2013). Details of the literature search protocol are presented in Annex 1.

Literature review results

Results of the literature review are presented in three parts. First, we present terms and definitions discussing various types of complex forests as they feature in the economic modelling and ecological literature. Second, we present findings on benefits of mixed complex forests. Third, we present economic modelling approaches showing how a classical optimal rotation length model developed for a mono-aged, mono-species forest stand could be modified to address the needs of economic modelling of complex forests. Other modelling approaches are presented as well.

Terms and Definitions

Defining various forest types is not a trivial task. As shown in a recent study (Bravo-Oviedo *et al.*, 2014) there exists a large variety of forest types and of typologies even across the EU, a historically and culturally similar group of countries when compared to the rest of the world.

Numerous terms are used to describe non-homogeneous forests that differ in terms of age and species structure, stand or forest and techniques to manage them. Terms include: multi-stand forests, mixed forests, even-aged and uneven-aged forests, complex forests, natural forests, continuous cover forestry, close-to-nature forestry etc.

We define some of these terms used in the literature below.

Mixed forests definition

Mixed forests, where 'mixed' refers only to species composition, are considered to occur where no single species occupies more than 80 per cent of the stand (Anonymous, 2013). The definition focuses on species composition of a forest and not on age structure.

Forest age structure definitions

When describing forest age structure, in terms of the age composition of the trees, the following terms are used: multi-age, even-aged and uneven-aged forests. The first two terms are used for homogeneous forest patches/stands which are not intimately mixed spatially. For example, multi-stand even-aged forest refers to a forest comprised of many even-aged stand units of different ages. By contrast, uneven-aged forests is used to describe more complex forests, including 'natural forests' in which trees of all ages are intimately mixed.

Continuous cover forestry (CCF) definitions

CCF is not a forest type but a management approach to forestry.

"Continuous cover forestry involves the maintenance of a forest canopy during the regeneration phase with a consequent presumption against clearfelling…" (Mason, Kerr and Simpson, 1999). Clearfelling is defined as the cutting-down of all trees on an area of more than 0.25 ha.

“CCF is an approach to forest management in which the forest canopy is maintained at one or more levels without clearfelling.” (Forestry Commission, 2008). We will term these two definitions as minimalist definitions of CCF.

In principle, CCF can be applied to a single species forest. A single species CCF forest would be quite unusual, given that CCF approach promotes diversity, but may occur during transformation phase from mono-species, mono-aged forest. This situation is quite common in the UK with Sitka spruce stands.

The definitions show that a great variation in the level of complexity is possible: from the simplest single-species, single-age stand to a natural forest, which could be characterised as mixed species and uneven-aged.

For the purpose of this study we summarise the definitions of different forest types in Table 4 in *Annex 2 Definitions*. No universal agreement still exists on CCF.

Benefits of mixed uneven-aged forests

A literature review identified examples of the following benefits of mixed forests compared to a mono-age monoculture managed on a clearcut basis:

- more biodiversity and resilience to threats from climate change and overall higher levels of various ecosystem services, including aesthetic, recreational and cultural values;
- more resilient with respect to storms and wind risk;
- more resilient to pest damage with overall less pest damage;
- mixed stands are more productive than single species stands;
- reduced economic risk and uncertainty.

However, one must be cautious and note that some of these benefits are potentially very context specific and my not be applicable to all combinations and comparisons. Nevertheless, desirable features such as higher biodiversity and resilience to a wide range of risks make mixed forests a highly significant research topic. Details of the review are presented in the *Annex 3* References on benefits of mixed complex forests.

Economic models of multi-stand and mixed species forests

A number of recent reviews (Amacher, Ollikainen and Koskela, 2009; Pukkala and von Gadow, 2012; Liang and Picard, 2013) help to summarise major economic approaches to modelling optimal forest management of multi-stand and mixed species forests.

Solutions to problems of optimal forest management of homogeneous stands, i.e. comprised of single species and trees of the same age, exist, including that provided by the classic Faustmann formula. The latter solution also applies in case of a multi-stand even-aged forest comprised of many even-aged stands. For homogeneous stands where timber production is the sole concern, for example, each stand could simply be managed according to an optimal Faustmann solution. So long as there are no significant interactions between management of adjacent stands (e.g. relating to windthrow risk), optimal forest management could be considered simply to reflect an aggregation of optimal management decisions for the individual stands. Alternatively, if one is also interested in an optimal steady state age-class structure, forest age-class models (Tahvonen, 2004a, 2004b; Uusivuori and Kuuluvainen, 2005) could be applied. Forest age-class models are not suitable for forests comprising of intimately mixed species, but may be useful during a transition phase from a mono-aged forest to one comprising of stands of different ages (Yousefpour and Hanewinkel, 2009) and for cases fitting a minimalist definition of CCF. Age-class models can also incorporate uncertainty (Couture and Reynaud, 2008). Forests featuring an intimate mix of species and ages (Pommerening and Murphy, 2004) cannot be adequately described by age-class models but could be managed on the basis of more complex matrix models developed for uneven-aged forestry.

For mixed uneven-aged forests new economic optimisation tools are necessary. Uneven-aged forest management refers to a situation when trees of all (or at least of more than one) ages are present on any given area of land in a forest, and tree sizes are intimately mixed. Below we describe three possible approaches:

1. forest age-class models (Lyon and Sedjo, 1983, 1991, Mitra and Wan, 1985, 1986, Tahvonen, 2004a, 2004b; Uusivuori and Kuuluvainen, 2005);
2. modified or generalised Faustmann solution (Chang, 1998, 2013; Buongiorno, 2001; Chang and Gadow, 2010; Buongiorno and Zhou, 2011; Parajuli and Chang, 2012);
3. matrix models (Buongiorno and Michie, 1980; Haight and Getz, 1987; Buongiorno *et al.*, 1995; Sánchez Orois and Rodríguez Soalleiro, 2002; Rojo and Orois, 2005; Rollin *et al.*, 2005; Zhao, Borders and Wilson, 2005; Hao *et al.*, 2005; Liang, Buongiorno and Monserud, 2005; Yang and Kant, 2008; Martin Bollandsås, Buongiorno and Gobakken, 2008; Tahvonen, 2009; Tahvonen *et al.*, 2009, 2010;

Liang, 2010; Liang and Picard, 2013; Rämö and Tahvonen, 2014, 2016; Roessiger *et al.*, 2016).

One of the specific problems for management of an uneven-aged forest is to agree on the optimal long-run or steady state structure of the forest in terms of the age class composition or the diameter class distribution, since any structure could be achieved theoretically through a series of harvests. Some advocate an ideal inverse J-shaped diameter distribution, others point to the fact that field measurement of yield in some virgin forests are best fitted by a bimodal Weibull-function (Pukkala and von Gadow, 2012). Alternatively it is possible to leave the long-run structure of the forest to be determined within the optimisation process. A selective harvesting approach is one of the defining characteristics of the management of both uneven-aged and mixed species forests, differentiating their management from a clearcut system (see *Annex 4* Economic modelling of complex forests for more details).

Forest age-class models

Forest age-class models assume that a forest can be divided into stands of unique age class. It uses a classic framework of utility maximisation with an intertemporal budget constraint. The forest owner is assumed to maximise their utility of consumption or amenities (including carbon sequestration) subject to a budget constraint by choosing in each period of time the shares to be harvested in each age class in each period. The budget constraint depends on harvest shares, timber volume (determined by a growth function), timber (and carbon sequestration) prices and planting costs.

Forest age-class models can be used to analyse and design carbon policies (Uusivuori and Laturi, 2007; Couture and Reynaud, 2008).

Modified and generalised Faustmann

The first examples of modification of the classic Faustmann model appeared in the late 1990s. Key assumptions of the classic Faustmann model require that the timber prices, interest rate, stand volume growth, and regeneration/replanting costs are the same in every rotation. It was shown (Chang, 1998) that this assumption could be relaxed so that these variables could differ from one rotation to another. Furthermore, as future prices and interest rates are uncertain, these can be treated as stochastic variables, with Faustmann's formula generalised using a Markov decision process (MDP) (Buongiorno, 2001; Buongiorno and Zhou, 2011).

A generalized Faustmann model was developed for management of uneven-aged forests, allowing the number of years and the level of residual growing stock to vary from one cutting cycle to the next (Chang and Gadow, 2010). This generalisation could be further extended to explore carbon sequestration in uneven-aged forest management (Parajuli and Chang, 2012).

Matrix models

Matrix models are among the most widely used approaches for the economic analysis of uneven-aged management and are able to account for the effects of in-growth (i.e. natural and artificial regeneration) and selective harvesting.

Uneven-aged forests can be modelled using a matrix model (Amacher, Ollikainen and Koskela, 2009) - also known as a transition matrix model, matrix population or growth model (Liang and Picard, 2013):

- Matrix models show how trees in one diameter class transition to the next with a growth function potentially accounting for interactions between classes.
- In economic models based on this matrix transitional dynamic an owner must choose the number of trees to harvest in each diameter class and the frequency of harvesting over time.
- There is no requirement to remove all trees on the site and the "selective harvesting" of trees in the stand is a key concept of optimal uneven-aged management.

As complex (i.e. uneven-aged and mixed in terms of species) forests cannot be described with a single variable, like age, matrix models require more data than simpler models (see *Annex 4* Economic modelling of complex forests for details).

Whichever modelling approach one chooses the crucial input is a growth model, which estimates and predicts regeneration, growth and mortality of trees. A simple typology of growth models is reproduced from (Pukkala and von Gadow, 2012, Ch. 6) in Table 5 of *Annex 4* Economic modelling of complex forests.

Discussion

Just as there is a continuum of forest types stretching from the simplest single-species even aged forest to natural forests comprising mixed species with uneven-aged stands, there are economic modelling approaches of varying complexity developed to explore management of different forest types. Leaving aside approaches related to simulation and modelling of forests under stochastic conditions, three approaches seem most relevant to developing the CCF optimal management model: forest age-class models, generalised Faustmann models and matrix models.

Having the advantages of their relative simplicity and less demanding data requirements, age-class models may be a good starting point for extending optimal stand management to a forest level that could aid strategic planning and help forest managers on the ground in achieving the best possible use of their resource.

Based on the preliminary literature review for this study, the following recommendation was made to initially address objectives two to four of this research project:

◇ **Recommendation 1:** *Given resources and scientific knowledge constraints the best way to proceed with the current project is to start with developing a forest age-class model which would ultimately include carbon sequestration and windthrow risk as well.*

Forest age-class model

Following the above recommendation, we present in this section an age-class model of a forest with multiple stands of a single tree species of different ages. To fit with the minimal definition of CCF, the size of each stand is assumed to be under 0.25 ha, with adjacent stands of different age classes, and clearfelling on adjacent areas that would create clearfell sites greater than 0.25 ha not being permitted. The latter restriction on adjacent clearfelling is of an explicit spatial nature, with a special treatment required for potential clearfell patches at the forest boundary. For simplicity a non-spatial model was initially developed that does not allow such spatial constraints to be implemented in the current version of the model. However, the larger and more age diverse the forest the less binding this restriction would be, because it would be easier to find non-adjacent patches for harvesting. Therefore, although it is not possible to model the spatial adjacency rule in the current set up, one may expect the model results to be applicable for larger CCF forests.

Model setup

The model used in this study is taken from Uusivuori and Kuuluvainen (2005). In this model the landowner is assumed to maximize the utility of managing a forest consisting of *n* stands of *n* distinct age classes. At time zero the initial age class distribution for all stands is given by the vector $x_0 = (x_{01}, x_{02}, \dots x_{0n})$, where an element x_{0j} gives the initial land area in hectares occupied by the age class $j=1,\dots,n$ ordered from the youngest ($j=1$) to the oldest ($j=n$). At any time *t* the forest is described by the vector $x_t = (x_{t1}, x_{t2}, \dots x_{tn})$. The aggregate growth of the forest is defined by the vector $q = (q_1, q_2, \dots q_n)$ which gives the timber volume per hectare in each age-class. At the beginning of any given period of time *t* the landowner chooses the harvesting shares, a_{ti}, for each of the n age classes. Each share must lie between 0 and 1, 0 means that age class *i* is not harvested at all and 1 means that age class *i* is fully clear cut at time t. The total volume of timber in the forest at time *t* is given by $Q_t = q \cdot x_t$. After harvesting shares are chosen the harvest (followed by processing and consumption of wood products) occurs. The state of the forest is then updated to the next time period $t+1$ (x_{t+1}) according to the equation below and the decision cycle repeats. The age-class dynamics describes an evolution of the vector x_t at periods greater than *t* and is given by a matrix equation:

Equation 1 Stand dynamics

$$x_{t+1} = A_t \cdot x_t$$

The matrix is given by:

Equation 2 Transition matrix

$$A_t = \begin{bmatrix} a_{t1} & a_{t2} & a_{t3} & & a_{t\,n-1} & a_{tn} \\ 1-a_{t1} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1-a_{t2} & 0 & & 0 & 0 \\ & \vdots & & \ddots & \vdots & \\ & & & & 0 & 0 \\ & 0 \quad 0 \quad 0 & & \cdots & 1-a_{t\,n-1} & 1-a_{tn} \end{bmatrix}$$

The A_t matrix determines the periodic relations between the different age classes. The first row gives the periodic harvest shares (a_{ti}) of each age class and the following rows give the unharvested shares ($1-a_{ti}$) of subsequent age classes.

Optimisation problem

Let the discount factor be: $\beta = 1/(1+\rho)$, where $0 < \rho < 1$ describes the representative landowner's rate of time preference. Let *p* denote the timber price, *r* denote the real interest rate, and *k* denote the (re-)planting costs per hectare for each stand, all of which are assumed to be constant over time.

The landowner's dynamic optimization problem is to choose harvesting shares in each age class at each point in time to maximise his utility of consumption (u(c_t)) and of amenity value (M(Q_t)) from the standing forest volume (Q_t) according to:

Equation 3 Optimisation problem

$$\max_{\{a_{ti}\}} \sum_{t=0}^{\infty} \beta^t \cdot [u(c_t) + M(Q_t)]$$

Subject to the following constraints:

Equation 4 Optimisation constraints

$$c_0 = \sum_{i=1}^{n} a_{0i} x_{0i} (pq_i - k) + w_0 - w_1$$

$$c_t = \sum_{i=1}^{n} a_{ti} x_{ti} (pq_i - k) + w_t (1 + r) - w_{t+1}, \qquad t \geq 1$$

Here consumption is equal to the net value of harvest revenues ($p \cdot a_{ti} x_{ti} q_i$) less costs ($a_{ti} x_{ti} \cdot k$), plus an exogenous income, the difference between this period wealth (w_t) and next period wealth(w_{t+1}), with initial non-forest wealth (w_0) given. Also we assume that harvesting shares are between zero and one and a land area for each age class is non-negative: $a_{ti} \geq 0$, 1- $a_{ti} \geq 0$ and $x_{0i} \geq 0$. Equation 1 and 2 also need to be added together with the forest total volume definition (Q_t).

First order optimality conditions require that:

Equation 5 First order condition for consumption

$$\frac{u'(c_t)}{u'(c_{t+1})} = \frac{1 + r}{1 + \rho}$$

This is a standard intertemporal consumption choice condition, which implies constant consumption rule, when r = ρ. Assuming this and for simplification no amenity utility, $M(Q_t)$, the second first order condition is:

Equation 6 First order condition for harvest shares

$$(pq_i - k)(1 + r)\frac{(1 + r)^i}{(1 + r)^i - 1} - (pq_{i+1} - k)\frac{(1 + r)^{i+1}}{(1 + r)^{i+1} - 1} \begin{Bmatrix} > 0 \\ < 0 \end{Bmatrix} \; as \; \begin{Bmatrix} a_{ti} = 1 \\ a_{ti} = 0 \end{Bmatrix}$$

Since the model is based on assuming discrete time, it is not feasible to set the left-hand side (LHS) expression to zero. In the full model with amenities a solution, $0 < a_{ti} < 1$, is possible. However, one can find a pair of time periods when the LHS changes sign from negative to positive. That is it is profitable to fell the stand in age class i when the marginal return from harvesting is greater than the marginal return from not harvesting, $a_{ti} = 1$. In the opposite case it is optimal to abstain from felling and set $a_{ti} = 0$.

Model results: numerical analysis

We started with a numerical analysis of a simplified version of the full model in a long-run steady-state (Equation 6).

The model estimates are presented for the case of unthinned Sitka spruce yield class (YC) 14, planted at 2m spacing (i.e. 2,500 trees per hectare). Other parameter values used in developing the prototype model are listed in Table 1 below. The values chosen are always within the source range and are considered to represent typical (rounded) values.

Table 1 Values for the prototype model inputs

Parameter	Value	Source
Planting costs	£2,000 per ha	FCS (2011) Standard costs
Timber price, Coniferous Standing Sales Price	£15 per m^3 overbark	Coniferous Standing Sales Price Index for Great Britain (Forestry Commission, 2015)
Discount rate	3.50%	Initial rate from HM Treasury's Green book (HM Treasury, 2003)

Three options were considered with respect to age-class categories: 1) five age-classes; 2) ten age-classes; and 3) fifteen age-classes. The oldest age class in all cases included all trees of age 100 and older. Other age classes contained trees of ages between 0 and 100 with the interval divided into equal non-overlapping age-classes according to the total number of age-classes. For example, for option 1 the five age-classes were those shown in Table 2 below:

Table 2 Age-classes

Age-class	Trees of age
1	0 – 24
2	25 - 49

3	50 - 74
4	75 - 99
5	100 and older

The long-run steady-state harvesting solution involves the following harvesting shares: $a = [0, 0, 1, 1, 1]$. This implies that between age-classes two and three one moves from no harvesting to full harvesting of entire stands. This transition is not far off the Faustmann optimal rotation length solution of 43 years. The difference could be due to the discrete nature of the problem for age-classes and should get smaller as one increases a number of age-classes.

Uusivuori and Kuuluvainen (2005) showed that the equilibrium age-class distribution is independent of the initial age-class distribution and depends only on harvesting shares and the total forest land area. They showed that the long-run steady-state can be either cyclical or noncyclical. In the case of a noncyclical equilibrium the long-run age-class distribution and harvesting shares are constant. The distribution by age-classes in terms of land shares will not typically be even and one may observe old-growth preservation coexisting with timber harvesting. This is so because in a noncyclical steady-state a constant harvesting policy would always have a stand that is cut in part: $0 < a_i < 1$. This may be important for implementation of carbon sequestration and biodiversity conservation policies. In the case of a cyclical equilibrium the long-run age-class distribution and the periodic harvesting shares oscillate in stable cycles.

Interestingly, in their *Proposition 1* Uusivuori and Kuuluvainen (2005) showed that although a noncyclical equilibrium would typically be characterised by an uneven age-class distribution it is possible to identify a harvesting policy such that the equilibrium forest will be normal, i.e. forest land is evenly allocated among age-classes. An example of such policy would be the following harvesting shares values: $a_n + a_{n-1} = 1$ and the other shares are zero.

A normal forest structure features prominently in forestry because it provides a constant flow of harvest over time. This feature is often considered ideal or very desirable by policy makers and forest planners.

Below we illustrate an example of convergence to a normal forest structure for our five age-classes example with *Sitka* spruce YC 14. We start with an 'old forest' where land areas are distributed by age-classes as follows: $x_0 = [0, 0, 0, 0.5, 0.5]$, i.e. only the two oldest age-classes are present and occupy equal areas. We fix harvesting policy through harvesting shares as: $a = [0, 0, 0, 0.5, 0.5]$. We trace the dynamics of land shares for a twenty time periods. A convergence to a normal forest where all age-classes occupy equal areas is observed:

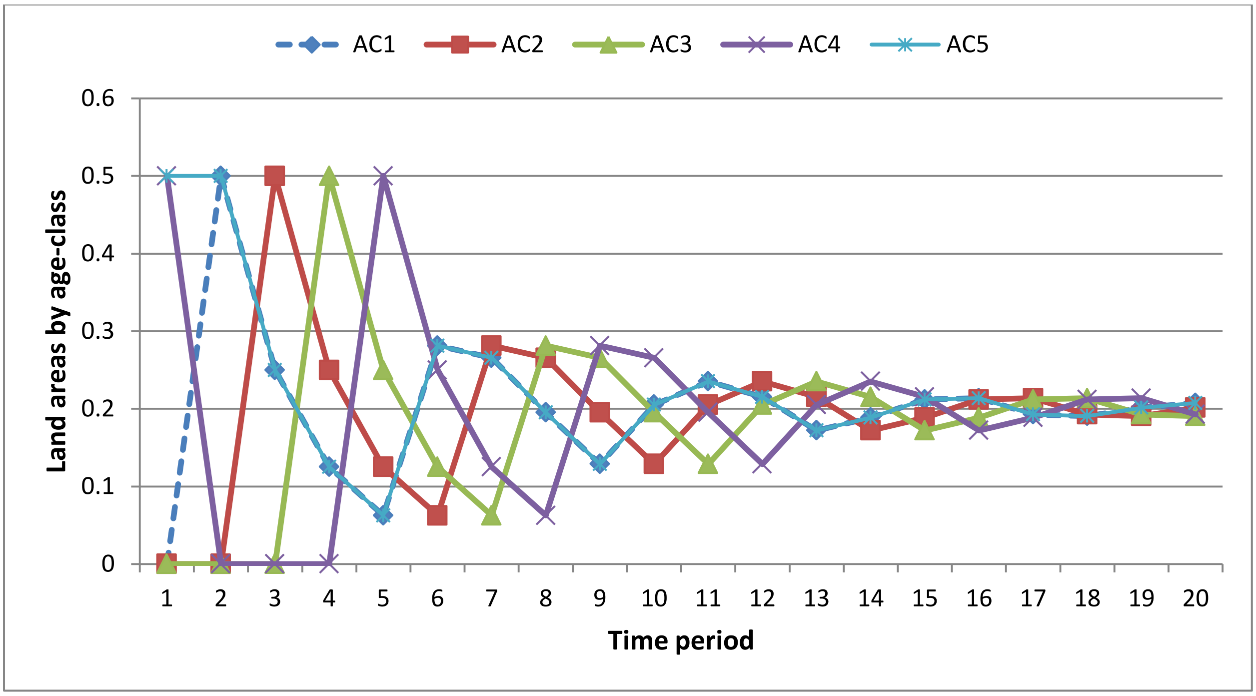

Figure 1 Convergence to a normal forest structure in a case with five age-classes

As can be seen from Figure 1, significant convergence already occurs from around period ten, with all age-classes ultimately converging to equal shares of 20% of the total area (which could be any number of hectares in practice).

Discussion of future modelling options

A prototype age-class model of a forest (i.e. one comprising multiple stands of different ages) has been developed and explored. Our initial numerical analysis has focused on a long-run steady-state of a simplified version of the full model (Uusivuori and Kuuluvainen, 2005). Results derived for the case of unthinned Sitka spruce YC 14 are in agreement with earlier studies confirming the correctness of our analysis.

Working with this age-class model prompted the following recommendation, which partly mirrors and reinforces the initial recommendations for the project.

◇ **Recommendation 2:** *It would be useful to further develop our understanding of the age-class model similar to Uusivuori and Kuuluvainen (2005), especially the numerical* *solution procedure for the full model with the amenity value of the volume of standing trees in the forest also included. This would allow us to explore potential for incorporating an amenity function that depends on the size of clear-fell areas and on the proportion of old trees in the forest, as well as how carbon sequestration (and potentially biodiversity) influences the optimal harvesting schedule. The dynamics of forest transformation to a desired structure and associated harvesting strategies could also be investigated in this case.*

The initial literature review showed that the development of age-class models is only a first step towards uneven-aged forest modelling. Development of *a forest age-class* model will help to build up knowledge and skills necessary for subsequent development and application of models able to deal with a wider range of factors and forest types and CCF management appoaches. It is also the only step forward available when there are no tree growth models which exhibit dependence on stand density variables.

The Generalised Faustmann formula is capable of providing solutions in the case of changing prices and costs and interest rates. Therefore, it could be used to implement the discount rate schedule as recommended by the Treasury (HM Treasury, 2003). There are two examples of using Faustmann's generalisation for an uneven-aged forest management (Chang and Gadow, 2010; Parajuli and Chang, 2012). However, it is not used widely, with a majority of papers by the same lead author. Therefore, its potential for use in further development of the prototype model seems limited, but this may usefully be explored further.

The current state of knowledge and data availability suggest that matrix models are the best middle ground for modelling uneven-aged forestry management including CCF approaches. They are detailed enough for a lot of interesting economic interventions and scenario analysis and still manageable in terms of complexity and data requirements. Matrix models are capable of addressing a much wider range of mixed and uneven aged forest types than forest age-class models and would help with economic management beyond 'minimal CCF' case studies. The number of references with matrix models for uneven-aged forestry management is significantly larger than that for age-class models indicating their much wider spread and application.

◇ **Recommendation 3:** *After developing a forest age-class model version, the development of a matrix model for uneven-aged forestry management should be considered. The feasibility of this will depend crucially on availability of growth model(s) capable of accounting for species interactions and/or spatial structure (i.e. distance-dependence) of a stand like MOSES GB currently being calibrated for some of the major GB species by Forest Research.*

◇ **Recommendation 4:** *Once the re-calibration for Britain of the MOSES GB growth model permits, it would be most beneficial to our understanding and application of optimal forest management to uneven-aged and CCF forestry to proceed as soon as feasible from a forest age-class modelling to an exploration and development of matrix models based upon the MOSES GB growth model.*

Research Gaps

Major research and data gaps identified relate to a lack of appropriate growth models (with stand density and tree distance dependencies both for single and mixed species forests), although ongoing work on the MOSES GB growth model may address some of the issues in the near future, detailed timber product prices and cost models of forestry operations (for other than clearcut harvesting scenarios) essential for model development and application. Further research with ecologists, growth modellers and field operators on these issues is likely to be needed to address these knowledge gaps.

Further investigation is also needed to see how models may be adapted to deal with risks and uncertainty due to climate change (possibly through impacts on growth and wind risk) and due to changes in the economic environment (prices, costs and interest rate). This could be done by using stochastic programming approaches, including an options approach, or through Monte-Carlo simulations.

Conclusion

The study represents an initial exploratory step by Forest Research into the multi-stand and CCF optimal forest management research area. The preliminary literature review helped clarify definitions and terminology used in the literature. It also identified evidence of the multiple benefits associated with natural forests that are often lacking in current forestry based on single species even-aged stands. Finally, three potential avenues for economic modelling of complex forests were identified: forest age-class models, generalised Faustmann formula models and matrix models.

The project concluded with a brief numerical investigation of a steady-state solution of a simplified age-class model for a typical British forest Sitka spruce yield class 14.

To assist forest managers in ensuring efficient use of resources, further work on multi-stand forest modelling, uneven-aged forestry and CCF is both necessary and timely.

Appendices

Annex 1 The literature search protocol

The table below summarises the search terms and their combinations used in the literature searches.

Table 3 Search terms combinations

What	How	Structure
Forest*	Model* Optim* Simulat* Econom* Process based Matrix Markov	Uneven AND (age* OR size*) Continuous cover Selecti* AND (fell* OR thin* OR harvest* OR single tree) Multi AND (stand* OR age*) Mix* Gap AND (cut* OR fell* OR harvest*)

Search terms across columns (horizontally) are combined with logical Boolean "AND" operator while terms within columns are combined with an "OR" operator. For example, a partial (picking only the first few terms from columns) search query may look like: forest* AND (model* OR optim* OR simulat*) AND ((uneven AND (age* OR size*)) OR continuous cover).

Full query: *forest* AND (model* OR optim* OR simulat* OR economy* OR "process based" OR matrix OR Markov) AND ((uneven AND (age* OR size*)) OR continuous cover OR mix* OR (multi AND (stand* OR age*)) OR (selecti* AND (fell* OR thin* OR harvest* OR single tree)) OR (gap AND (cut* OR fell* OR harvest*)))*.

Databases searched: Scopus and Forest Science Database CABI (www.cabi.org/forestscience). Focus was on newer papers.

Scopus results: 1,006 hits from 1985 to present, searched on 12/08/2016. Full query search string was used with subject area limitations: Agricultural and Biological Sciences, Environmental Science, Earth and Planetary Sciences, Social Sciences, Mathematics, Decision Sciences, Economics, Econometrics and Finance, Business, Management and Accounting, Multidisciplinary. Checked 800 hits from 2016 to 2004.

CABI results: searched on 16/08/2016, 62,085 hits for a search string:

(forest) AND (model* OR optim* OR simulat* OR economy* OR "process based" OR matrix OR Markov) AND (uneven* OR continuous cover OR mix* OR multi* OR selecti* OR single tree) AND yr:[1999 TO 2016]*

Clearly, it was not possible to sift through so many entries. Restricting search to only last 10 years (*2005 TO 2016*) and topic to *Economics* only yielded 3,955 hits, of which over 500 most recent and relevant were selected and inspected for selection.

Annex 2 Definitions

A comprehensive review of the history and definitions of CCF is provided in a number of studies (Pommerening and Murphy, 2004; Helliwell and Wilson, 2012; Pukkala and von Gadow, 2012). A review on the implementation of CCF in the British Isles is provided in (Mason, 2015). It is important to note that there is still no universal agreement on defining CCF.

Generally CCF systems "involve continuous and uninterrupted maintenance of forest cover and which avoid clearcutting" (Pommerening and Murphy, 2004). Other features of some CCF are:

- whole ecosystem management with close-to-nature silviculture/forestry/management;
- structural and species diversity with uneven-aged/multi-aged forestry;
- retention of mature trees and deadwood and green tree retention;
- promotion of native tree species/provenances and broadleaves; and
- harvesting and thinning by selective cutting based on individual trees and not on area.

CCF is seen as a suitable holistic approach to forestry when managing for multiple objectives (Pommerening and Murphy, 2004). The majority of definitions emphasise the concept of continuity of woodland conditions over time. For example, according to Helliwell and Wilson (2012):

"Forest management that works with the characteristics of the site and with tree species that are well adapted to the location, and which maintains forest cover permanently. … will normally involve a mixture of tree species and ages. Management is based on the selection and favouring of individual trees (of all sizes) rather than the creation of areas of uniform tree size and spacing… Stand structure will be permanently irregular…".

Uneven-aged silviculture is distinct from CCF. CCF is considered a more holistic approach to the whole ecosystem at a site whereas uneven-aged silviculture is used as a tool (Helliwell and Wilson, 2012).

A broad definition of a mixed forest is proposed in a large EU-wide study (Bravo-Oviedo *et al.*, 2014) as: "a forest unit, excluding linear formations, where at least two tree species coexist at any developmental stage, sharing common resources (light, water, and/or soil nutrients). The presence of each of the component species is normally quantified as a proportion of the number of stems or of basal area, although volume, biomass or canopy cover as well as proportions by occupied stand area may be used for specific objectives. A variety of structures and patterns of mixtures can occur, and the interactions between the component species and their relative proportions may change over time."

Table 4 below summarises the definitions of the various forest types. We focus on three main characteristics: number of species and age classes and forest structural diversity both vertical (usually in terms of number of canopy layers and overall canopy cover) and horizontal (in terms of how intimately are species and age classes or tree sizes, diameters, mixed spatially):

Table 4 Forest types definitions

Number of Species	Age classes	Structure	Forest Type
One	One	Single canopy layer	Even-aged, single-species
One	More than one	More than one canopy layer but not intimately[a] mixed spatially, i.e. there are identifiable patches of same age	Multi-age, multi-stand forest
One	More than one	More than one canopy layer and intimately mixed spatially	Uneven-aged forest
More than one	One	Potentially[b] more than one canopy layer but not intimately mixed spatially[a]	Mixed even-aged multi-stand forest
More than one	One	Potentially more than one canopy layer and intimately mixed spatially	Mixed even-aged forest
More than one	More than one	More than one canopy layer but not intimately mixed spatially	Mixed uneven-aged multi-stand forest
More than one	More than one	More than one canopy layer and intimately mixed spatially	Mixed uneven-aged forest (also Natural forest)[c]

Notes: a) there is no strict definition but from (Mason, Kerr and Simpson, 1999) it follows that identifiable homogenous patches should be less than 0.25 ha in size;

b) since different species of the same age may have different heights;

c) close-to-nature forestry and CCF management approaches would normally be associated with this forest type, although as we noted in the main text CCF could be applied even to mono-species stands.

Annex 3 References on benefits of mixed complex forests

A literature review identified the following benefits of mixed forests compared to a single species even aged forest managed on a clearcut basis:

- more biodiversity (Gamfeldt *et al.*, 2013; Lafond *et al.*, 2014; Redon *et al.*, 2014; Calladine *et al.*, 2015; Dănescu, Albrecht and Bauhus, 2016)
- more resilience to threats from climate change (e.g. droughts) (Gauthier *et al.*, 2015; Thurm, Uhl and Pretzsch, 2016)
- better balance between carbon sequestration and timber production (Seidl *et al.*, 2007; Buongiorno *et al.*, 2014)
- more resilient with respect to storms and wind risk (Mason, 2002, 2015; Schütz *et al.*, 2006; Schelhaas, 2008; Jactel *et al.*, 2009, 2017; Griess and Knoke, 2011; Griess *et al.*, 2012; Jönsson, Lagergren and Smith, 2013; Hanewinkel *et al.*, 2014; Lafond *et al.*, 2014; Pukkala, Laiho and Lähde, 2016);
- more resilient to pest damage and less pest damage overall (Jactel, Brockerhoff and Duelli, 2005; Jactel *et al.*, 2009, 2017);
- more productive than single species stands (Forrester *et al.*, 2006; Lähde, Laiho and Lin, 2010; Pretzsch *et al.*, 2013, 2015; Pukkala, Lähde and Laiho, 2013);
- lower economic risk and uncertainty (Knoke and Plusczyk, 2001; Knoke *et al.*, 2008; Griess and Knoke, 2013; Wagner *et al.*, 2014).

Annex 4 Economic modelling of complex forests

While modelling complex forests where there is no clear end-state and one needs to decide on the characteristics of the steady-state for a managed forest. The classical Plenter (from German *Plenterung* – a selection system for picking trees anywhere without proper planning or control for single stem harvesting) forest is based on a long-term vision of an ideal forest structure, implemented by the inverse J-shaped diameter distribution (Figure 2), often represented by a negative exponential DBH-class (Diameter at Breast Height) distribution. Once attained, that structure is to be maintained in perpetuity by selective harvesting (Pukkala and von Gadow 2012). However, there are arguments against adopting steady state of an inverse J-shaped diameter distribution on biological and economic grounds. For example some virgin forests, especially multi species ones, are found to have a bi-modal distribution of trees by diameter class (Pukkala and von Gadow 2012, Ch. 2). Therefore, dynamic optimisation models for uneven-aged stand management were developed (Haight and Monserud, 1990; Tahvonen *et al.*, 2010) that do not require a predefined steady state.

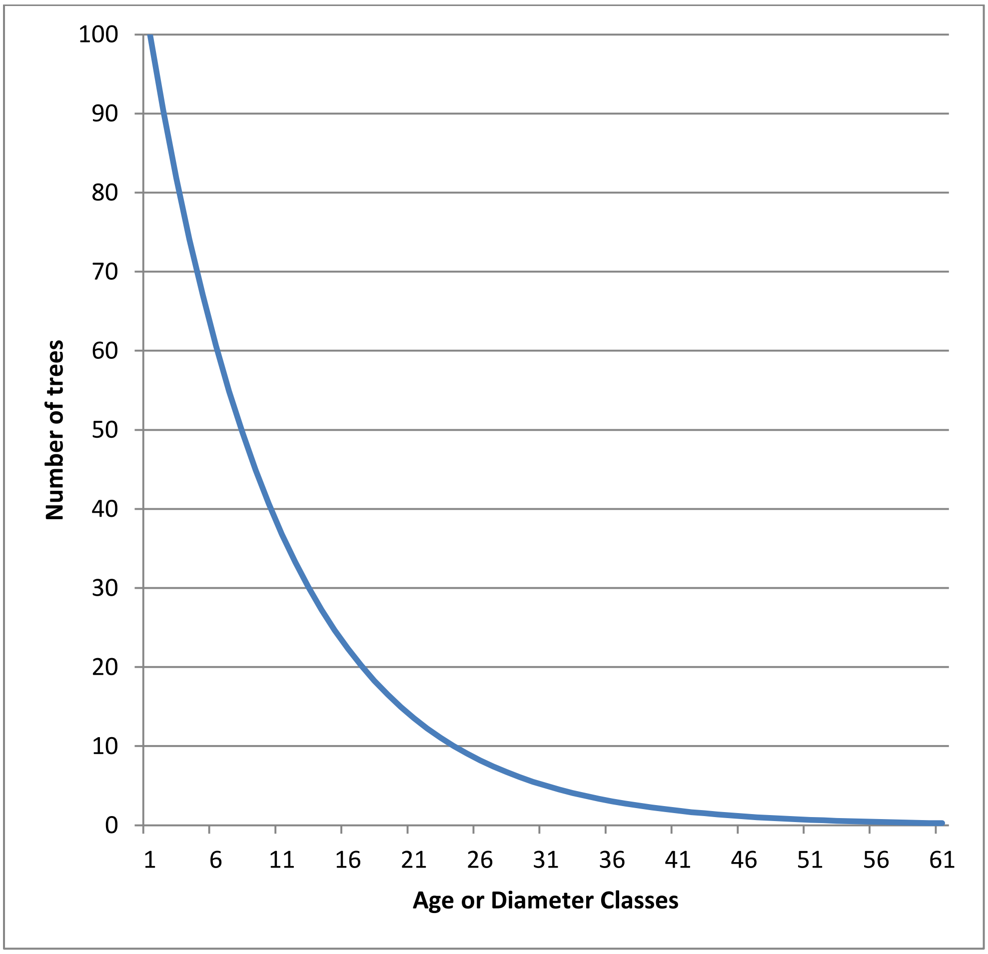

Figure 2 An ideal balanced target structure of a 'natural' *Plenter* forest with selective harvesting represented by an inverse J curve.

The forest age-class model can be used to analyse whether a normal forest structure is indeed optimal from an economic point of view. A normal forest solution provides an even flow of timber volume harvested in each period. If a stand is economically optimal for harvesting at an age of T years then the even-flow solution suggests having T stands in a forest (each stand occupying 1/T of total forest area) ranging in age from 1 to T years.

It is important to stress that for forest age-class models the assumption of a utility-maximising agent with an intertemporal budget constraint is very important; without it, e.g. maximising only the net present value of harvest revenues, an age-class structure of a forest would not matter and one should just apply the classic Faustmann rotation age solution to the whole forest (Amacher, Ollikainen and Koskela, 2009, Ch. 8).

Transitional matrix models for uneven-aged forests have the following data requirements, for single species models in discrete time settings (Tahvonen and Rämö, 2016):

1. Harvesting cost model: for thinnings and clearcuts
2. Stand growth as a matrix transition model: natural regeneration, transition to the next size class and mortality rate, DBH distribution
3. Tree height model to compute tree volumes per size class
4. Stand density: basal area and basal area per tree in a size class
5. Site productivity
6. Replanting cost
7. Economic data: interest rates, prices for logs, small-diameter logs (round wood) and saw timber

Examples of matrix models (Buongiorno and Michie, 1980; Haight and Getz, 1987; Buongiorno et al., 1995; Sánchez Orois and Rodríguez Soalleiro, 2002; Rojo and Orois, 2005; Rollin et al., 2005; Zhao, Borders and Wilson, 2005; Hao et al., 2005; Liang, Buongiorno and Monserud, 2005; Yang and Kant, 2008; Martin Bollandsås, Buongiorno and Gobakken, 2008; Tahvonen, 2009; Tahvonen et al., 2009, 2010; Liang, 2010; Liang and Picard, 2013; Rämö and Tahvonen, 2014, 2016; Roessiger et al., 2016).

Matrix model are most suitable for modelling optimal forest management with CCF approach.

A simple typology of growth models reproduced from (Pukkala and von Gadow, 2012, Ch. 6) is shown below:

Table 5 Typology of stand growth models

Model Type	**Description**
Density-free whole stand models or univariate models	Describe the development of stand volume as a function of time; have been used to study optimal rotation lengths under even-aged management.
Variable-density whole stand models	More detailed models are required to study uneven-aged management. They include stand density as an independent

	variable, often measured as stand basal area.
Matrix models	Also called transition matrix or stage-structure models describe the stand state with a tree size distribution. Trees are classified in discrete size classes, typically characterized by tree diameter measured at breast height. Each class is represented by average tree volume, tree height, and number of trees. Growth is described as the transition from one class to another at discrete time intervals. Recruitment and survival functions define in-growth and mortality. These models are widely used for the economic analysis of uneven-aged management being able to account for the effects of in-growth and selection harvests.
Individual-tree models (distance-independent)	Describe a forest stand using a list of tree records. Each tree is characterized by a number of state variables reflecting its current dimensions (diameter, height, crown ratio etc.) and the total number of its kind in the stand. The tree vectors evolve over time due to in-growth, growth, mortality and harvesting. In-growth, growth and mortality are specified as functions of stand density variables.
Individual-tree models (distance-dependent)	Also called spatial models. Here growth depends explicitly on a tree's location, height, and crown relative to its neighbours.
Process-based models	A most detailed and advanced stand growth models build up from basic ecophysiological and biophysical processes that govern biomass development in different compartments of the tree: roots, stem, branches and foliage.

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