Forest Research

Review of economic models for quantifying risk and uncertainty in forestry

Final Report

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April 2019

The Research Agency of the Forestry Commission

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Executive Summary

The report presents a broad review on natural hazards in forestry, economic approaches to risk and uncertainty, tipping points and resilience. The review is motivated in part by increases in risk and uncertainty in forestry related to ongoing climate change and the need to account for these in making forest planning and management decisions. This, together with a need to consider how the resilience of ecosystem services (ES) delivery by forests can best be valued provides the rationale for undertaking a review of economic approaches to risk and uncertainty. Ignoring risk and uncertainty in forestry could be expected to result in suboptimal choices, inefficient use of resources and poor investment decisions.

The review has three parts: 'Terms and Definitions', 'Natural Hazards' and 'Economic modelling approaches'.

The review of 'Terms and Definitions' shows that there are currently no universally accepted definitions for risk, uncertainty, or resilience, although there is more agreement on defining tipping points or critical thresholds. For the purposes of this study, risk is defined as a measure encompassing the probability and expected impact of uncertain events in the future. 'Soft' uncertainty is conceptualised as a set of (often subjective) probabilities of different potential future states of a system under study. 'Hard' uncertainty is characterised by situations where probabilities are unknown (also called Knightian uncertainty) due to lack of knowledge. This latter case is unhelpful for operational decision-making or modelling and is not considered in the report.

Resilience is a comparatively new concept and especially difficult to define as it is multidimensional, with potentially rich social content and context. From an ecological perspective, resilience can be defined as the amount of disturbance that an ecosystem can withstand without changing self-organized processes or structures and flipping into a different equilibrium. Tipping points are defined as critical points or a zone where a relatively rapid change occurs from one stable state to another with a small change in conditions. However, there generally appears insufficient evidence to identify tipping points in forest ecosystems, and how they will be affected by climate change. Some very preliminary work is reported but much more basic field work and data collection are required for delineation and definition of forestry systems.

The review of economic approaches to risk and uncertainty found that a variety of tools are available for modelling, including some novel developments, e.g. robust optimisation. However, there have been relatively few applications in forestry economics.

While no simple prescriptive answer can be given to the question of which economic modelling approach to risk and uncertainty should be chosen in each particular case, general recommendations are:

1. Consider modelling risk and uncertainty for problems where their influence is not negligible. [!]
2. The best approach to use will depend on factors such as the scale and type of problem and the nature of the uncertainty, as well as the availability of resources and skills, etc.
3. At a minimum, apply a scenarios and sensitivity analysis approach.
4. Where the problem warrants more in-depth analysis of issues of risk and uncertainty, select a relatively well tested approach initially, e.g. mean-variance portfolio. Others include (i) stochastic dynamic programming and related real option approaches – suitable for optimising a harvesting schedule; (ii) Markov Decision Process and related simulation approaches (Monte-Carlo and Markov Chain Monte-Carlo) – suitable for complex forest growth and dynamic simulation; and iii) Bayesian statistics – suitable for situations where a process of learning about the problem occurs over time and so reduces uncertainty of probability estimates of different potential future states and parameters.

Obstacles to a wider use of these tools are discussed. The overview presented will aid decision makers in choosing an appropriate tool and encourage them to take greater account of risk and uncertainty in making decisions.

A conceptual framework for the relationship between resilience and risk is proposed. While acknowledging the complexity of defining and measuring resilience, and that a number of issues are not yet fully resolved, a possible approach to valuing resilience is proposed. Specifically, a potential proxy for the value currently placed on the resilience of the forest ecosystem is the net cost (without accounting for the change in risk) of any actions that forest owners or managers take which are aimed specifically at increasing resilience. Such actions would include, for example, changing or diversifying tree species to reduce the expected impact of climate change. Future work would benefit from increasing linkages with other strands of the ongoing research on resilience.

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Introduction

A wide range of ecosystem services (ES) are provided by trees, woodlands and forests (TWF) in urban and rural locations. Quantifying and valuing (largely) non-market ES such as carbon sequestration, air filtration, health and recreation to ensure that they are accounted for in forestry management and planning decisions is challenging, especially in the context of climate change. The latter represents a major source of uncertainty and potential random shocks to TWF due to impacts on tree growth rates, frequency of storms and pest and disease outbreaks. In addition there is uncertainty in the supply of ES originating from the natural variability of forest ecosystems and also in demand for ES, which pose potential risks in forest management and planning decisions. The history of dealing with risk in forestry economics is a long one with a modern approach to the risk and investments in forestry appearing in the early 1980-s (Mills and Hoover, 1982).

Valuation of ecosystem services and benefits in the presence of risk is important to help underpin forest policy on woodland creation and land use change, as well as forest management for improving resilience. It is also important for pursuit of wider policy agendas including climate change mitigation and adaptation, natural capital accounting and urban planning.

The current study provides an initial step in exploring potential approaches to valuing forest resilience given risk and uncertainty.

Aims and Objectives

The key overarching objective was to review economic approaches for valuing and integrating a range of forest ecosystem services and benefits, including forest resilience, in the presence of risk.

Specific objectives for this study are:

1. To review major sources of risks and associated magnitudes of impacts to tree species which in turn may impact forest ES (e.g. timber provision, carbon sequestration,

biodiversity and recreation) and their resilience. Risks may include economic sources, e.g.: timber and carbon prices, interest rates, and environmental sources: forest growth (varying and impacted by climate changes, pests and diseases), wind and fire risks, etc. The current study focuses on environmental sources of risk.

2. To identify from literature review the most probable scenarios (including minimum and maximum potential impact scenarios) for various risks over the future where possible.

3. To identify and review the most useful existing economic methods, tools and models that could be used for valuing forest ES and resilience in the presence of risk.

Methodology

The study was primarily based on a review of international literature conducted in line with the Government Social Research Service (GSR) Rapid Evidence Assessment (REA) guidance (GSR, 2013). Details of the literature search protocol are presented in the annex.

A two-part literature review was conducted, including:

1. A brief literature review of the sources and magnitudes of risks for forest ES and resilience, noting any reported potential tipping points and / or critical thresholds.

2. A literature review of the most useful existing economic methods, tools and models for valuing forest ES and resilience in the presence of risk.

Literature review results

Results of the literature review are presented in three parts. First, we present terms and definitions for risk, resilience and tipping points as they feature in the economic and ecological literature. Second, we present findings on natural hazards for forests in the UK and potential tipping points. Third, we present economic modelling approaches developed to deal with risk and uncertainty and examples of their application in forestry.

Terms and Definitions

This section provides a brief review of terminology and definitions of risk, resilience and tipping points.

Uncertainty and Risk

Uncertainty results from the variability of natural systems and / or from a lack of information or knowledge about the system dynamics. It can arise as information is not available, or is not accessible, is of unknown accuracy, or is subject to differing interpretations, ambiguity. It can be simply due to the existence of a range of possibilities, including changes over time and space.

Exploring a conceptual basis for the systematic treatment of uncertainty in model-based decision support activities Walker et al., (2003) proposed a general definition of uncertainty as being "any deviation from the unachievable ideal of completely deterministic knowledge of the relevant system", i.e. a continuum between absolute determinism (but excluding this) and total ignorance.

'Pure uncertainty' (or 'hard uncertainty'), which is sometimes termed Knightian uncertainty (Knight, 1921), is conceptualised as completely distinct from risk. It arises where probabilities of future possible states of the world are completely unknown.

By contrast, 'soft' uncertainty is a key component of risk. It arises simply from the existence of more than one possibility (often called states of nature in economics) for an outcome and is measured by a set of probabilities assigned to the set of possibilities.

Defining uncertainty as a characteristic of situations where probabilities are unknown (Knightian uncertainty) is unhelpful for decision-making or for developing most economic models because in this case nothing can be deduced about the likelihood of particular outcomes. To proceed with quantitative analysis either some probability structure describing potential future states of the world is needed, or, alternatively, scenario analysis can be applied. In some cases, probabilities can be derived from experimental or modelling studies. Alternatively, they may be based on prior beliefs (as in the Bayesian approach).

However, distinctions between pure uncertainty, ('soft') uncertainty, and risk are not universally accepted and are far from being used consistently in the literature.

Risk is a term that has been defined in a numerous ways. For example, in the International Organization for Standardization (ISO) standard 31000 adopted in 2009 and related to risk management, risk is defined very broadly as the "effect of uncertainty on objectives" (https://www.iso.org/iso-31000-risk-management.html accessed 1.08.2017) (Purdy, 2010).

For the purposes of this study risk is considered a measure of the probability and impact of uncertain events in the future. There are two constituent parts of this conception of risk: (i) the probability of an uncertain future event happening; and (ii) the magnitude of impact it may cause, level of exposure (Yoe, 2011).

The probability of an occurrence (e.g. of a fire, windthrow event, drought, or pest or disease outbreak) is by definition determined by characteristics of an underlying stochastic process. Hence, the modelling of stochastic processes and representation of their essential characteristics (e.g. probability distribution) is an essential task for risk estimation.

The magnitude of the impact can either be a loss or gain in the outcome – although losses ('downside risks') tend to be the focus. It is generally determined by the system susceptibility/resilience and the level of exposure (Jactel *et al.*, 2009). Here susceptibility is understood in terms of how easily the forest is damaged by the disturbance, and exposure is the value at risk (Jactel *et al.*, 2009). It is this influence of resilience on the potential magnitude of impact (and sometimes on the probability of an event itself) that forms the major link between resilience and risk. (Figure 1 in the *Discussion* section characterises the relationship by using thick arrows originating from 'Forest Ecosystem' and 'Risk' circles that push against each other).

In mathematical terms, risk can be defined as the probability of event times the magnitude of impact:

Risk = Probability × Impact

From this it implies not only that both components need to be present, but also that each needs to have a non-zero value for a risk to be real. If there is non-zero probability of some hazardous event occurring but its impact on the system under investigation is zero then there is no real risk. Secondly, if an adverse event impact is non-zero but the probability of its happening for the system under investigation is zero or very close to zero, then there is no real risk.

Seven actions to manage risk at an organisational level are covered in ISO 31000:2009 (Purdy, 2010):

1. Avoiding the risk by deciding not to start or continue with the activity that gives rise to the risk
2. Accepting or increasing the risk in order to pursue an opportunity
3. Removing the risk source
4. Changing the likelihood / probability
5. Changing the consequences / impact
6. Sharing the risk with another party or parties (including contracts and risk financing)
7. Retaining the risk by informed decision

However, given that this review focuses on economic tools and approaches to modelling risk and valuing resilience, we do not pursue operational risk management issues further here.

Resilience

The concept of resilience, as well as those of tipping points and critical thresholds, is intrinsically linked to the idea that an ecosystem may operate in a number of different stable states. The change from one stable state to another is a critical phenomenon associated with the system reaching and/or crossing some tipping point or critical threshold.

The concept of resilience is related to the ability of a system to remain and function in (including a reasonably fast return to) a current equilibrium steady state in the face of some disturbances. The larger the disturbances a system can withstand without changing to another stable state, the stronger its resilience. The maximum level of disturbance that the system could withstand is directly linked to issues of tipping points and critical thresholds and the system's maximum capacity to absorb shocks.

Ecological resilience, also called ecosystem resilience, can be defined as the amount of disturbance that an ecosystem could withstand without changing self-organized processes and structures (defined as alternative stable states), i.e. before flipping into a different stable state (Gunderson, 2000).

An ecosystem itself is of course indifferent between its various possible steady states. It is a society's preferences that make some steady state (often the current one) more desirable than the others (Fuller and Quine, 2015), who together with (Brand and Jax, 2007) provide a review and examples of the resilience definitions.

The ecological resilience of a forest ecosystem is determined by its biodiversity (including species and genetic diversity), the regional pool of species, the size and connectivity of a system and the surrounding landscape (Thompson *et al.*, 2009). Being a complex system a forest is not described adequately by a simple static equilibrium but rather by a dynamic cycle (Drever *et al.*, 2006; Walker *et al.*, 2006). A typical cycle of resilience in response to a disturbance is (Fuller and Quine, 2015) resistance, recovery, and adaptation back to a pre-disturbance state or to a new alternative stable state - transformation.

Engineering resilience is a narrower concept in that it concerns only one steady state and is the capacity of a system to return to the pre-disturbance state and/or resist disturbance.

The concept of resilience in economics is often defined as "the ability of the system to withstand either market or environmental shocks without losing the capacity to allocate resources efficiently (the functionality of the market and supporting institutions), or to deliver essential services (the functionality of the production system)." (Perrings, 2006). It is interesting to note that a drive to efficiency through cost-cutting eliminates redundancies and diversity in a system and hence reduces its resilience (Walker *et al.*, 2006).

Tipping point

A tipping point or threshold is defined as a point or zone where relatively rapid change occurs from one stable state to another with a small change in conditions. It is "the critical point at which strong nonlinearities appear in the relationship between ecosystem attributes and drivers; once a tipping point threshold is crossed, the change to a new state is typically rapid and might be irreversible or exhibit hysteresis." (Brook *et al.*, 2013), potentially based on an earlier work (Scheffer *et al.*, 2001). At this critical point a tiny perturbation can qualitatively alter the state or development of a system (Lenton *et al.*, 2008; Lenton and Williams, 2013).

Natural hazards for forests

In this section we present major references on natural hazards occurring in forestry.

A number of recent reviews describe a range of natural hazards in forestry (Hanewinkel, Hummel and Albrecht, 2011; Jactel and Vodde, 2011; Jactel *et al.*, 2011; Lindner *et al.*, 2014). Uncertainties about climate change, one of the major drivers behind many risks in forestry, impacts and its implications for forest management are highlighted in (Lindner *et al.*, 2014) who caution that many studies underestimate the potential impacts. There is also a review (Jactel *et al.*, 2009) that consider the influences of biotic and abiotic risks on stand management.

A recent review article on forest disturbances under climate change (Seidl *et al.*, 2017) confirmed that our understanding of disturbance dynamics in response to climatic changes remains incomplete, particularly regarding large-scale patterns, interaction effects and dampening feedbacks. The study explored global climate change effects on important abiotic (fire, drought, wind, snow and ice) and biotic (insects and pathogens) disturbance agents. Warmer and drier conditions are expected to increase fire, drought and insect disturbances, while warmer and wetter conditions increase disturbances from wind and pathogens. Interaction effects between agents are likely to amplify disturbances, while indirect climate effects such as vegetation changes can dampen long-term disturbance sensitivities to climate. Future changes in disturbance are likely to be most pronounced in coniferous forests and the boreal biome.

Results from a recent synthesis study (Kautz *et al.*, 2017) on biotic disturbances (including insects, pathogens and wildlife herbivory) in Northern hemisphere forests suggest that overall 2.6% of forests were affected annually, although impacts vary a lot over space and time. Nevertheless, temporal trends show an increase over recent decades and that the forest area affected by biotic disturbances is larger than that affected by fire and other abiotic disturbances.

Recent analysis suggests that wind risk is one of the most significant threats in terms of timber volume damage to UK forests (EEA, 2010; Gardiner *et al.*, 2010). These European reports indicate higher future wind risk for Europe and especially for north-western Europe including the British Isles, given that most storms originate over the Atlantic before hitting Europe (Della-Marta and Pinto, 2009). Also, there is a link between past increases in forest storm damage and increased growing stock and average forest age across Europe (Gardiner *et al.*, 2010). This suggests a probable increase in vulnerability of the UK forests from an increasing growing stock and increasing average stand age and height. In addition, increasing mean winter rainfall for large parts of the UK envisaged under UKCP09 (http://ukclimateprojections.defra.gov.uk/) projections would lead to increased soil wetness and shallower water-tables, which could be expected to reduce tree anchorage.

The importance of wind risk management is emphasised by the assessment that storms are responsible for more than 50% of all primary abiotic and biotic damage by volume to European forests from catastrophic events (Gardiner *et al.*, 2010). However, (Deegen and Matolepszy, 2015) caution that the management of forests under storm risk is highly complex with economic calculations requiring more field data than is currently available. Nevertheless, in the UK there is a long tradition of managing forests in the face of wind risk and some useful tools and models were developed, for example, ForestGALES model (Gardiner and Quine, 2000; Gardiner *et al.*, 2006).

The summary of abiotic natural hazards faced by forestry in the UK is presented in (Nicoll, 2016). Below we reproduce a table from this study:

Table 1 Summary of abiotic risks, impacts and expected sector responses

Risk	Climate factors involved	Observed / expected impact	Sector Response
Drought damage	Reduced summer rainfall Higher summer temperature	Drought cracks / 'shake' Poor stem form/ timber quality Reduced timber growth and quality leading to Reduced economic return	Species change e.g. less use of drought intolerant species in drier south and east of UK Conversion to continuous cover forestry (CCF) and mixed species
Flooding and waterlogging	Increased rainfall	Woodlands increasingly used as part of natural flood management	Develop and implement natural flood management guidelines. Replacement with flood tolerant species.
Soil erosion / landslides	Increased storm frequency Increased rainfall Increased wind	Damage to infrastructure and urban areas	Develop protection forestry guidelines Risk assessment, removal of vulnerable trees, protect infrastructure with rock fall netting, tree topping, Encourage native woodland restoration on slopes to protect soil and protect infrastructure from landslides
Frost damage	Warmer winter temperature Warmer early spring temperature Maintained frost frequency	Frost damage to buds / shoots Poor stem form/ timber quality	Species change / avoid vulnerable species
Windthrow	Increased winter rainfall Increased storm frequency Increased windiness Increased air temperature	Faster growing trees reach vulnerable height sooner. Increased wind losses Damaged timber – reduced returns Increased operator risk	Species diversification, restricted thinning in exposed areas. Conversion to CCF Encourage understory development Implement wind risk DSS Implement contingency plans
Wildfire damage	Increased spring/ summer air temperature Reduced rainfall Increased fuel from insect / disease.	Reduced forest area Reduced production Risk to infrastructure and urban areas	Develop guidelines and implement contingency plans

Although not considered further in this report, biotic threats to forests and the severity of many plant disease epidemics are often driven by temperature, rainfall and soil moisture as these influence the production and release of spores. Expected warmer temperatures, particularly milder winters and warmer summers are expected to cause the spread of some pests and diseases from further south (Morison and Matthews, 2016; Wainhouse *et al.*, 2016).

Tipping points

In this section we present major references on ecological tipping points in forestry.

The majority of references uncovered on tipping points and critical thresholds deal with global climate change impacts, the carbon cycle or aquatic systems, and the concept of planetary boundaries (Lenton *et al.*, 2008; Biggs, Carpenter and Brock, 2009; Nobre and Borma, 2009; Anderies *et al.*, 2013; Brook *et al.*, 2013). A lack of supporting empirical evidence is generally accepted (Evans *et al.*, 2017) due to relative novelty and the complexity of identifying tipping points and critical thresholds in advance.

A British study is a rare piece of research focusing on a temperate forest ecosystem (Evans *et al.*, 2017) identified a number of potential thresholds. In particular, this research tested the hypothesis that threshold responses exist in measures of (1) biodiversity, (2) ecosystem function and (3) ecosystem condition within a temperate forest. The study examined a beech-dominated forest that is currently undergoing large-scale dieback in response to environmental change. The results confirmed the existence of several thresholds in biodiversity, namely: species richness of ectomycorrhizal fungi, epiphytic lichen and ground flora; for ecological condition: sward height, palatable seedling abundance; and a single threshold for ecosystem function: soil respiration rate.

A number of threshold responses in forest ecosystems as a result of deforestation and habitat fragmentation are reported in earlier studies: thresholds in forest structure (de Filho and Metzger, 2006; Rocha-Santos *et al.*, 2016), biodiversity loss (Fahrig, 2002; Ochoa-Quintero *et al.*, 2015) and ecosystem service provision (Bodin *et al.*, 2006). However, unlike (Evans *et al.*, 2017) these studies focused on the impacts of direct human-driven loss of forest cover or modified disturbance regimes at the landscape scale, and deal predominantly with tropical forests. For example, a quantitative assessments for the maintenance of the tropical forest suggest that 'tipping points' may exist for total deforested area (>40%) and for global warming greater than 3°C (Nobre and Borma, 2009).

Another synthesis paper (Reyer *et al.*, 2015) reflects on the current understanding of forest resilience and potential tipping points under environmental change over a wide range of spatio-temporal scales (local, regional and global). It argues that it is often unclear whether these changes reduce resilience or represent a tipping point. Tipping points may arise from interactions across scales, as processes such as climate change, land-use change, invasive species or deforestation gradually erode resilience and increase vulnerability to extreme events. Therefore, more studies addressing interactions across different spatio-temporal scales are needed to further our understanding.

An example of an economic type of a tipping point or a critical threshold is a price level below which business activity would be stop either because the level of profitability is low or zero, or due to bankruptcy resulting from a low revenue.

Economic modelling approaches to risk in forestry

This section reviews major economic approaches to risk measurement, risk and uncertainty modelling.

A number of recent reviews (Knoke, 2008; Hanewinkel, 2009; Hildebrandt and Knoke, 2011; Eeckhoudt and Louberge, 2012; Yousefpour *et al.*, 2012; Machina and Viscusi, 2013; Pasalodos-Tato *et al.*, 2013; Chudy, Sjølie and Solberg, 2016) help to summarise the major approaches to economic modelling under risk and uncertainty. The question as to which financial approaches could be used in evaluation of returns of mixed forests is partially addressed in Knoke (2008) who considers three approaches: standard mean-variance portfolio selection approach, stochastic dominance approach and information-gap decision theory. Approaches with risk premium, Monte-Carlo simulation, portfolio selection, stochastic dynamic programming and the real option theory are reviewed in Hanewinkel (2009). Focusing on investment decisions under uncertainty in forestry (Hildebrandt and Knoke, 2011) in a comprehensive review considered a large number of approaches: the expected utility framework, stochastic dominance, downside risk and lower partial moments, portfolio selection mean-variance approach, option pricing models and robust optimisation with information-gap decision theory.

Before diving into some of the more technical methods used for risk modelling, it is worth mentioning insurance – a common way of dealing with risk in economics, which is investigated in Holecy and Hanewinkel (2006) and Brunette *et al.*, (2015) in relation to forestry. The study by Holecy and Hanewinkel (2006) is an example of one of the first general forest insurance models that can serve as a basis to calculate risk premiums to insure the risk of forest destruction due to either a single cause or cumulative damaging factors.

Relatively low profitability in traditional forestry in comparison to the forest insurance premium is a major barrier to adopting risk insurance against fire and/or storm in some European countries (Brunette *et al.*, 2015). Applying an actuarial insurance model to a case study area of silver fir (*Abies Alba Mill*.) stands in the Paradise region of Slovakia, the study (Brunette *et al.*, 2015) estimated that gross insurance premiums range from €5.62/ha at a scale of 150,000 ha at age 150, to €6,312.81/ha at a scale of 15 ha at age 50. The gross insurance premium is composed of the risk premium, representing the risk to the insurer, and the net insurance premium, corresponding to the risk of the forest owner.

A review of decision-making approaches to handle uncertainty and risk in adaptive forest management under climate change (Yousefpour *et al.*, 2012) suggests that while many standard approaches assume that the parameters of the probability distributions or stochastic processes are known this may not hold for the case of risks related to climate change due to lack of comparable historical records. Therefore, one of the promising avenues for future research is developing optimisation methods which could include Bayesian theory which allows for learning and updating beliefs about parameter values and their distribution.

To aid forest managers in designing forest management plans, (Pasalodos-Tato *et al.*, 2013) reviewed different methods to handle risk at various spatial (stand, forest/landscape and regional) and temporal (strategic, tactical and operational planning) scales, also considering stakeholder participation processes, objectives and the goods and services covered. Many of these methods are reviewed below. The study also discussed obstacles to wider use of the methods for risk and uncertainty modelling which we present in the Discussion section.

Focusing on introducing risk in large-scale numerical forest sector models, a review by Chudy, Sjølie and Solberg (2016) identified fuzzy set theory and robust optimization techniques as promising new approaches, alongside methods already in use, like Monte Carlo simulation and, in particular, scenario and sensitivity analysis.

Other references (Eeckhoudt and Louberge, 2012; Machina and Viscusi, 2013) provide a more general introduction to risk and uncertainty in economics without focusing on forestry.

Uncertainty is a multi-facetted phenomenon. Three different aspects of uncertainty are (i) the perceived time horizon before there will be certainty on outcome, (ii) the spread of impacts across the set of alternative outcomes, and (iii) the subjective probability (belief) assigned to each outcome. The impact of each on forest management decisions in terms of harvest timing and choice of species for regeneration under climate change is explored in (Schou, Thorsen and Jacobsen, 2015). Their conclusion is that the longer is the period of time over which the climate change uncertainty could be resolved, the more the decisions will be based on *ex ante* expectations and beliefs.

Below we present tested and well established approaches mentioned most often. We summarise each approach together with its strengths and weaknesses.

There is a general agreement among economists and investors that the profitability of an investment portfolio should be measured by the expected return. However, there is no such agreement on how to measure risk, or a risk index (Levy, 2015). In his book Haim Levy provides a non-exhaustive list of twelve measures of risk, including:

1 Focusing on losses: a risk index (RI) that takes into account all negative or relatively low outcomes (e.g. below a riskless asset yield). However, this index cannot differentiate between an investment which has a small probability of a large loss (potentially spelling bankruptcy) and another with a large probability of a small loss (which although more probable on average would not spell a disaster for a business) if both have the same risk index value. E.g. $RI=-Prob * Loss = -[0.1*(-50\%)]= -[0.5*(-10\%)]=5\%$.
2 *Roy's safety first rule* (Roy, 1952): a risk index based on the probability that the future income will be lower than a specified critical threshold perceived by the investor. However, like the risk index based on losses, it does not take into account the size of the loss and preferences with respect to the threshold choice.
3 A risk index measured by one of the common *dispersion measures* (e.g. variance). However, this gives equal weight to positive and negative deviations.
4 A risk index measured by the *semi-variance* (*downside risk*) focusing on losses. This takes only the negative deviations from the mean into account, addressing some of the drawbacks of a variance measure which is indifferent between the negative and positive deviations. The notion of semi-variance is generalised in *lower partial moments* (LPM), which are used to explore the risk of falling below some selected critical threshold (x*). An example of an application in forestry is provided in (Hildebrandt and Knoke, 2011). However, downside risk is not universally accepted as an objective measure of risk and subjectivity remains in the choice of the critical threshold, x* (see more details in Annex 2).
5 A risk measure for an individual asset in a portfolio measured by the *Beta (β)* from the capital asset pricing model (CAPM) by Sharpe and Lintner (Sharpe, 1964; Lintner, 1965): this measures the sensitivity of the expected excess asset returns to the expected excess market returns: $\beta_i= Covariance(R_i, R_M) / Var(R_M)$, where R_i is return on asset *i*, R_M is return on optimal market portfolio. An asset with a high beta contributes more to the portfolio risk (variance). However, the CAPM assumes that the variance of returns is an adequate measurement of risk. This is implied by the assumption that returns are normally distributed, which is often not the case.
6 *Baumol's risk index* by William Baumol: this is based on the notion that risk is due to the possibility of getting less return than some critical threshold, "floor" (Baumol, 1963). However, the approach discounts the probability of earnings falling below the floor and subjectivity exists with respect to the choice of threshold.
7 *Value-at-Risk (VaR):* This is a risk index very widely used in financial institutions (e.g. banks). *VaR(α)* specifies the maximum possible loss in investment, given normal market conditions, in a set time period, e.g. a day, when *α* percent of the left tail of a distribution is ignored. It is related to the Baumol and Roy risk indices and has similar problems. For example, *VaR(α=1%)* means that the left tail corresponding to *α=1%* (i.e. 1% of total area under the curve), is ignored. The VaR calculation is based on the historic time series for returns for a particular industry or sector. Other methods used to estimate the distribution of returns include the Variance-Covariance method, which

assumes that returns are normally distributed, and Monte Carlo simulation. A recent example of an application in forestry is (Hahn *et al.*, 2014).

8 *Shortfall VaR* - also called Expected Shortfall and Conditional Value at Risk (CVaR): The expected shortfall at *α%* level is the expected return on the portfolio in the worst *α%* of cases. Shortfall VaR is an alternative to Value at Risk that is more sensitive to the shape of the tail of the loss distribution.

9 The *Minimax Regret criterion* by Leonard Savage for selecting among risky actions or investments (Savage, 1951): This approach is based upon choosing the investment that offers the minimum risk of possible losses due to a wrong choice; the regret measures the risk of making a wrong investment choice. According to this rule, losses are due to the alternative costs, or wrong investment choices. The investor calculates the maximum possible regret for each stock and the stock with the minimum of these maximum regrets should be chosen. However, (i) adding another stock may change the relative risk of the stocks itself even if the additional stock is irrelevant because it is not chosen; (ii) the criterion measures risk due to a wrong choice but it does not take into account the probability of the various outcomes.

10 *Risk premium* from the expected utility approach: this is the maximum amount that the investor is willing to pay for the insurance to eliminate the risk. If $EU(x)$ is the expected utility of x, which is a random return/income, then the risk premium ρ is determined by the equation: $U(Ex-\rho) = EU(x)$, where $U(x)$ is a utility function of x and Ex is the expected value of x. Risk premium is probably the most accurate measure of risk, as it measures the amount of money one is willing to pay to get rid of the risk (Levy, 2015). However, not all investors would agree on the shape of the utility function.

11 *Risk perception* from the behavioural economic approach: as each investor translates the ex-post data on historic returns into new probabilities for future returns, including ex-ante variance, in a different way, the perceived risk differs between investors. Risk is inherently subjective depending on our culture and beliefs, and may even depend on the period or point in time. Hence, the same drawback mentioned a number of times above applies in relation to subjectivity and lack of universal agreement on risk.

12 The "*Fear Index*"; this is related to the perceived risk but is more formally defined and could be calculated for different options. In the Black and Scholes option model (Black and Scholes, 1973) there is only one unknown parameter, the future volatility of the stock index price. Given values of all other parameters one can solve for the future volatility as perceived by investors. This value is called the "Fear Index" and can be considered as the average perceived risk.

Many of the approaches used in economics for dealing with risk are summarised in the table below are quite complex and mathematically involved therefore we put most of the details in Annex 2.

Summary of approaches

The review of economic approaches to dealing with risk covers a wide range of methods. However, given continuing research and development of novel approaches in this area, the number of methods applied can be expected to keep growing.

It has also not been possible to cover *all* methods and theories used in the decision making under uncertainty and risk in detail in a short review. A notable omission from the methods reviewed above is fuzzy set theory, which could be applied anywhere there is uncertainty, including MCDA (Kangas *et al.*, 2015).

Table 2 below summarises the advantages and disadvantages of the different approaches reviewed together with relevant references with a focus on forestry.

Table 2 Approaches to risk modelling – Summary [pp.: 19-33 to annex!]

Approach	Advantages	Disadvantages	References
Expected utility (EU)	• Widely accepted and applied in economic analysis • Foundation for the majority of other approaches	• None significant if one accepts the foundations of utility function theory, including underpinning assumptions (e.g. complete and convex preferences) • No universal agreement on the choice of the functional form for utility • Could be challenged by alternative approaches arising from behavioural economics and bounded rationality	• (von Neumann and Morgenstern, 1947; Savage, 1954) • These are not forestry specific references. Rather EU approach is the basis of New Classical Economics which became a mainstream economic approach since 1970s and is adopted by the majority of studies below.
The mean-variance approach	• Relative simplicity • Widely applied • Yields same results as Expected Utility when utility is quadratic or when returns are normally distributed and utility is exponential	• Using variance as a measure of risk puts equal weight on positive and negative deviations • Assumes the distribution of returns can be described with only two parameters • Cannot account for economic agents who exhibit preference for, or aversion to, skewness, or those who focus primarily on the worst outcomes, or those who always prefer larger payoffs	• (Knoke, 2008; Roessiger, Griess and Knoke, 2011; Roessiger *et al.*, 2013; Dragicevic, Lobianco and Leblois, 2016; Messerer, Pretzsch and Knoke, 2017)

The stochastic dominance criterion	• Does not require outcomes to be normally distributed • Only risk aversion is required for utility • Could be used for initial screening of alternatives with very few restrictions	• Often only partial ranking of alternatives is possible that distinguishes efficient (undominated) from inefficient (dominated) choice sets • Limited track record in forestry	• (Knoke, 2008)
The information-gap approach, minimax and robust optimisation	• Novel (at least for forestry) • Robust results • Can deal with severe uncertainty	• Complexity • Limited track record in forestry • Very conservative in terms of outcomes	• (Knoke, 2008) • (Palma and Nelson, 2009, 2010; Kašpar *et al.*, 2017; Messerer, Pretzsch and Knoke, 2017; Sanei Bajgiran, Kazemi Zanjani and Nourelfath, 2017; Uhde *et al.*, 2017)
The stochastic dynamic programming (SDP)	• Strong track record • Very flexible as can deal with a wide range of stochastic processes • Relatively simple initial problem setup	• Hard to solve • Numeric solutions are required for most but simplest cases • The 'curse of dimensionality': the cost of computing agents' expectations over all possible future states increases exponentially in the number of state variables (calls for a new solution approach 'Approximate Dynamic	• (Duku-Kaakyire and Nanang, 2004; Stainback and Alavalapati, 2004; Chladna, 2007; Insley and Lei, 2007; Daigneault, Miranda and Sohngen, 2010; Couture and Reynaud, 2011)

		Programming').	
The 'real' option approach	• Same as SDP	• Same as SDP	• As above for SDP
Markov Decision Process (MDP)	• Same as SDP	• Limited track record in forestry • Demanding data requirements, including historic time-series, which are required to produce believable transition probabilities between various states. • The 'curse of dimensionality' (similar to SDP) as the number of transition probabilities grows exponentially with a number of system states.	• (Forsell *et al.*, 2011; Zhou and Buongiorno, 2011; Buongiorno and Zhou, 2015; Zhou, 2015; Couture, Cros and Sabbadin, 2016; Buongiorno, Zhou and Johnston, 2017; Johnston and Withey, 2017)
The simulation approach (Monte-Carlo and Markov Chain Monte-Carlo)	• Widely applied	• Demanding data requirements (similar to MDP)	(Knoke and Wurm, 2006; Hyytiäinen and Haight, 2010; Kallio, 2010; Conedera *et al.*, 2011; Roessiger, Griess and Knoke, 2011; Moore *et al.*, 2012; Liénard and Strigul, 2016; Daniel *et al.*, 2017)
The scenarios approach and sensitivity analysis	• Relatively simple • Widely applied	• Imposed externally on the fully deterministic model • Simplistic • Only yields boundary solutions (which do not depend on uncertainty	• (Olsson, 2007; Sacchelli, Fagarazzi and Bernetti, 2013; Seidl and Lexer, 2013; de-Miguel *et al.*, 2014; Hynynen *et al.*, 2015; Zell and Hanewinkel, 2015; Holmström *et al.*,

		as a parameter) • Time consuming when a large number of scenarios for many variables need to be calculated.	2016; Heinonen *et al.*, 2017; Temperli *et al.*, 2017)
Bayesian statistics	• Allows for learning and updating beliefs, new evidence/data assimilation mechanisms to be modelled	• Limited track record in forestry • Complex • Difficult to solve	• (Yousefpour *et al.*, 2012, 2013, 2014, 2015; Grêt-Regamey, Brunner, Juerg Altwegg, *et al.*, 2013; Grêt-Regamey, Brunner, Jürg Altwegg, *et al.*, 2013; Liénard and Strigul, 2016; Reyer *et al.*, 2016)

Discussion and Recommendations

The review of natural hazards that affect forests showed that risk and uncertainty are intrinsic features of forest management decisions, especially in the context of climate change. They are therefore important aspects to take into account in modelling and decision making in forestry. Disregarding risk and uncertainty would tend to lead to sub-optimal (often significantly wrong) solutions and inefficient decisions on investment and resource use. For example, in forestry disregarding risk of windthrow would suggest a longer rotation length in anticipation of larger timber volumes but in fact could lead to much lower than expected returns if windthrow occured. Nevertheless, each of the review papers covered in this study reported a relatively low (although increasing) number of applications of the various approaches to modelling risk and uncertainty in forestry economics and management. Although scenario/sensitivity analysis is a relatively common approach in cost-benefit analysis used to aid public and private sector decision-making, this review found very few studies that apply risk and uncertainty modelling in forest planning and management. The main reasons discussed in the literature for the relatively few applications and obstacles to their use in practice in forestry are (Pasalodos-Tato *et al.*, 2013):

1 Complexity. The methods for formulating decision and optimisation problems involving uncertainty are typically quite complicated, both conceptually and mathematically, and hard to explain for non-specialists (Kangas and Kangas, 2004).

2 Technical implementation. Many approaches to modelling risk and uncertainty lead to very large-scale optimisation problems (significantly larger than deterministic ones) which are not trivial to solve especially using ordinary hardware. They tend to require specialist software and / or extensive programming. Often there is a lack of resources and skills required to implement the methods.

3 Knowledge gaps about the uncertainties and risks. For example, unlike inventory errors, the errors in forest growth models are not always well-known. The probability distributions of the various risks are commonly not known, and need to be approximated. The uncertainties associated with future timber prices are based on historical price information but the future price developments could be affected by some unknown factors making the assumed probability distributions uncertain. Other uncertainties relate to difficulty in defining benefits such as scenic beauty, sustainability, biodiversity and resilience.

4 Human factors relating to the decision makers. The preferences of a decision maker in terms of objective attitudes to risk are often difficult to describe and generally not explicitly elicited. The whole concept of uncertainty might be unfamiliar and vague to a decision maker. On a more fundamental level, given the attitudes towards uncertainty among decision makers, the mere existence of uncertainty might be an unfavorable fact. Mowrer (2000) stated that "certainly, no resource manager wants to

stand up in a public meeting and admit that they are not quite sure of the exact outcome of a proposed activity".

5 Cost-benefit trade-off. For some problems and decision-makers, rightly or wrongly, the expected economic losses due to given risk or uncertainty might be considered insignificant, or less than the costs of taking the uncertainty into account.

All these factors confirm that a trade-off exists between simplicity and complexity in deciding whether uncertainties are ignored or considered, and where considered, also in selecting which approach to uncertainty to adopt. At present decisions on considering or ignoring uncertainty have generally favoured simplicity (Pasalodos-Tato *et al.*, 2013).

The brief but quite wide ranging review of methods with references to their application in forestry provides a good starting point in choosing which approach is best to adopt in a particular decision-making context. While no simple prescriptive answer can be given to the question of which economic modelling approach to risk and uncertainty is recommended, general guidance would be:

1. take risk and uncertainty into account in modelling and decision-making for problems where their influence is significant.[CQ: worth saying?]

2. The choice of the best approach will depend on factors such as the spatial/temporal scale and the type of the problem, the nature of the uncertainty, the availability of resources and skills, etc.

3. At the minimum apply a scenarios and sensitivity analysis approach.

Of the relatively novel approaches, we would recommend (based on their potential applicability and the existing track record of applications in forestry) the use of: i) the stochastic dynamic programming and related real option approaches; ii) Markov Decision Process and related simulation approaches (Monte-Carlo and Markov Chain Monte-Carlo); and iii) Bayesian statistics.

Where resources and skills available are the limiting factor, more traditional approaches with a good track record should not be neglected, especially: i) the mean-variance approach (portfolio optimisation); and ii) scenario analysis.

In each case, one should make sure that the approach selected is applicable to the problem in hand and their major assumptions are not violated (e.g. for the MV approach, that returns are normally distributed).

If optimality is important, approaches based on the expected utility (e.g. SDP or MDP) should generally be preferred (where resources allow). However, if ranging of minimum and maximum boundaries is a key issue and probability distributions over potential outcomes are unknown, then scenarios and simulation approaches are a good choice.

Based on the review, we make the following qualified (by the above considerations) recommendation:

◇ **Recommendation:** *The best way to proceed with risk and uncertainty assessment and modelling, beyond what is suggested in the discussion above, is to start with a relatively well tested approach. These include (i) stochastic dynamic programming and related real option approaches – most suitable for optimising harvesting schedule; (ii) Markov Decision Process and related simulation approaches (Monte-Carlo and Markov Chain Monte-Carlo – most suitable for complex forest growth and dynamic simulation); and iii) Bayesian statistics – most suitable for situations where learning about problem's parameters, i.e. uncertainty reduction, occurs in the process.*

Implications for valuing forest resilience

The reviewed methods of risk should be place within the broader view of forest resilience. Hence, we propose the first insight how risk and resilience can influence each other. The following conceptual view (see Figure 1) show linkages between the economics of risk and forest resilience together with a tentative suggestion of how resilience could potentially be valued.

Near the centre of the figure we have a forest ecosystem, which could be characterised by its natural capital and / or the associated ecosystem services flows (for example, timber, woodfuel and other wood products, carbon sequestration, air filtration, recreation opportunities, etc.). In the lower left corner of the figure we have sources of random shocks and disturbances that could potentially impact the forest ecosystem. These shocks could be environmental and/or socio-economic in nature. The environmental shocks may be due to climate change, for instance. Social factors that may give rise to shocks include changes in forest management practices, perceptions and attitudes of the general population with respect to forests, for example a boom in biomass and bioenergy market. The shocks impact on the forest ecosystem which resists the change, hence, the 'Resilience' arrow opposing the 'Shocks'. . Forest resilience determines how well the system resists a shock either by recovering to the pre-shock state, adapting (potentially to a new forest type ecosystem, e.g. from pure conifers to mix forest) or transforming into a new equilibrium state (which may even be not forest). The interaction between resilience and shocks is shown as two thick green arrows pushing against each other in a centre of the diagram. Risk is the probability of a random event times the magnitude of impact and management actions could influence one or both of these components. Note that risk in turn may affect management actions, hence, double-sided arrow between two. At the top of the figure is the 'management objective and actions', which may be the expected returns or profitability where standard economic agents or investors are considered. The expected returns can be estimated using the economics approaches to modelling risk and uncertainty reviewed in this study.

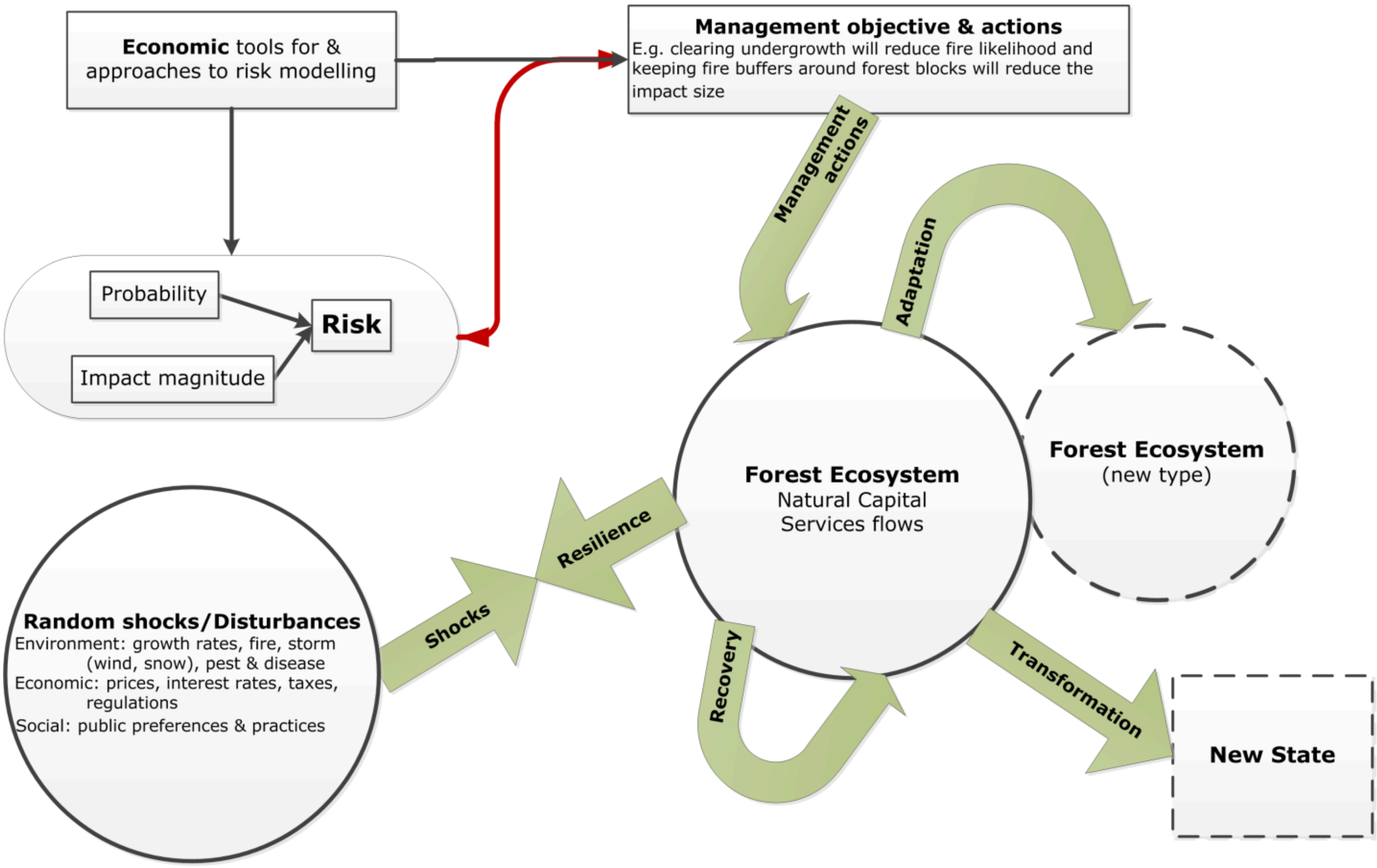

Figure 1 Risk and Resilience relationships in forestry. The interaction between resilience and shocks is shown as two thick green arrows pushing against each other in a centre of the diagram. One arrow 'Resilience' originates from 'Forest Ecosystem' circle. Another arrow 'Shocks' originates 'Random shocks/Disturbances' box. The concept of risk and associated economics tools help to operationalise management actions which in turn impact on forest resilience.

A potential proxy for the value currently placed on the resilience of the forest ecosystem could be computed by considering the net cost (without accounting for the change in risk) of any actions that forest owners or managers take aimed specifically at increasing resilience. Such actions would include, for example, changing or diversifying tree species to reduce the expected impact of climate change.

We believe that this approach may be a good starting point for future studies considering existing values placed on forest resilience.

An overall conclusion is that a range of economic tools and approaches to dealing with risk, uncertainty and tipping points exist, but considerable scope remains for further applied research to explore how these may be best adapted to the needs of the forestry sector. In general, risks should not be simply ignored in a decision making process and managers should be encourage to consider risks in their models and decisions, if not in a formal way, then at least using a deliberative approach as a first step to formal modelling. Research on resilience is not currently sufficiently advanced to make it fully operative and the work here should be continued and encouraged to bridge this gap. We hope that our conceptual framework for resilience and risk interactions will aid further consideration of these important topics.

Appendices

Annex 1 The literature search protocol

The table below summarises the search terms and their combinations used in the literature searches for the 'Economics of risk and tipping points' project.

Table 3 Search terms combinations

What	How	Structure
Risk* Uncertain* Stochastic* Random* "tipping point*"	Model* Optim* Simulat* Method* Markov Portfolio	Econom* AND forest*

Search terms across columns (horizontally) are combined with logical Boolean "AND" operator while terms within columns are combined with an "OR" operator. For example, a partial (picking only first few terms from columns) search query may look like: *risk* AND (model* OR optim* OR simulat*) AND (econom* AND forest*)*.

Full query: *(risk* OR uncertain* OR stochastic* OR random* OR "tipping point") AND (model* OR optim* OR simulat* OR method* OR Markov OR portfolio) AND (econom* AND forest*)*.

Databases searched: Scopus (www.scopus.com) and Forest Science Database CABI (www.cabi.org/forestscience). Focus was on newer papers.

Scopus results:

(risk* OR uncertain* OR stochastic* OR random* OR "tipping point") AND (model* OR optim* OR simulat* OR method* OR markov OR portfolio) AND (econom* AND forest*) AND PUBYEAR > 1996 AND (LIMIT-TO (LANGUAGE , "English") OR LIMIT-TO (LANGUAGE , "Russian"))

96,605 hits from 1997 to present (i.e. last 20 years), searched on the 5th of June 2017, limited to English and Russian languages.

Restricted full query search with subject area limitations: Agricultural and Biological Sciences, Environmental Science, Earth and Planetary Sciences, Social Sciences, Mathematics, Decision Sciences, Economics, Econometrics and Finance, Business, Management and Accounting, Multidisciplinary still yielded over 11 thousand hits. Checked top 800 hits to 2004.

As could be seen from the large number of hits an intrinsic commonality of the search terms did not allow for a reasonably small subset of the literature to be obtained. This is a problem that affected some other reviews as well, for example, (Machina and Viscusi, 2013, p. xx) went for a subset of leading journals only to restrict their number of hits to a manageable size. We decided to check a few hundreds of references with the highest citation scores.

CABI results:

(risk) AND (model*OR optim* OR simulat*) AND (forest* AND econom*) AND yr:[2000 TO 2017]*

1,245 hits searched on 23/05/2017. Closer inspection of top results showed a good agreement with Scopus results, on which this research is based.

Additional smaller searches were performed for topics of scenario analysis, Markov decision process, Monte-Carlo simulation, Bayesian approach and robust optimization.

Annex 2 Technical details for some risk measurements and approaches

Many of the approaches described below are quite complex and mathematically involved with even a simple demonstration for some potentially requiring a few pages. Hence, it was decided to omit such complexities. Instead if one is interested in any of the examples of application of a particular method in forestry it would be best to go to the original references provided for more detailed explanations.

Lower partial moments (LPM) as a measure of risk

The notion of semi-variance is generalised in *lower partial moments* (LPM), which are used to explore the risk of falling below some selected critical threshold (x*). $LPM_n(x^*;f) = \int_{-\infty}^{x^*}(x^* - x)^n f(x)dx.$, where f(x) is a probability density functions for x (usually financial returns). LPM_0 gives the probability of obtaining an outcome less or equal to x* (shortfall probability) without considering the amount of shortfall. LPM_1 gives the mean of the shortfalls below the critical reward (shortfall expected value), with equally weighted negative deviations. LPM_2 gives the mean squared deviation from x* (shortfall variance), with more emphasis on lower values (Hildebrandt and Knoke, 2011).

Expected utility approach – basic setup

In simple terms the expected utility approach calculates a mean value of utility over a set of potential future outcomes weighted by their probability (see *Annex 2* for a formal definition).

As illustrated for some of those discussed above, while many risk measures exist, all have their drawbacks. Choice of a particular risk index seems to be subjective and, hence require an introduction of individual utility function and associated risk preferences. Initial development of subjective expected utility theory was completed in the 1950s and was based on works by (Bernoulli, 1738; Ramsey, 1931; von Neumann and Morgenstern, 1947; Savage, 1954). It became a common approach in economic analysis in the 1960s with the mean-variance approach being used more widely initially.

The expected utility approach forms a foundation of many other approaches considered here. This is despite the fact that later research since 1950s showed that sometimes the real behaviour of investors is not fully rational and violate axioms required for the Neumann-Morgenstern utility function.

Formally, assume there are mutually exclusive outcomes $\{x_i\}$, which occur with probabilities $\{p_i\}$, where i = 1, 2, . . ., S, where S indexes the state of nature, $u(x_i)$ is a value of utility in the state i. u is a cardinal increasing function of x, where x stands for consumption, wealth or income. Then the expected utility (EU) value is given by (in a discreet case):

Equation 1 Expected utility - discreet case

$$E[u(x)] = \sum_{i=1}^{S} p_i u(x_i)$$

An integral form is used for continuous variables case:

Equation 2 Expected utility - continuous case

$$E[u(x)] = \int_a^b u(x) dF(x)$$

Where F(x) denotes the cumulative distribution function (CDF) associated with a particular random variable, x.

The expected utility theorem (von Neumann and Morgenstern, 1947) states that an individual makes decisions under risk as if he or she maximized the expected value of a cardinal utility function of outcomes (Eeckhoudt and Louberge, 2012).

An individual is risk averse if that person starting from a position of certainty rejects the addition of any fair gamble to that certain starting position. For example, a risk averse investor will choose between two projects with the same expected returns the one with lower uncertainty/risk. In the EU approach risk aversion is equivalent to the concavity of the Neumann-Morgenstern utility function u(x) used to compute expected utility.

The absolute risk aversion measure by Arrow and Pratt is defined for a utility function *u(x)* as (Pratt, 1964; Arrow, 1965):

Equation 3 Arrow and Pratt absolute risk aversion index

$$A(x) = -\frac{u''(x)}{u'(x)}$$

A risk-averse agent would accept to pay ρ, the risk premium, to replace a random outcomes of x by its expectation E(x) received with certainty:

U(Ex-ρ) = EU(x).

Interestingly, the risk premium and the Arrow and Pratt absolute risk aversion index are directly related in case of small risks. Consider a random variable z with zero mean and risk measured by variance σ_z^2 added to a non-random wealth x. A risk averse individual would be prepared to pay risk premium (ρ) to get rid of random variable z defined by the following equation: *u(x-ρ) = Eu(x+z)*. One could show using Taylor series approximation that this risk premium is proportional to the product of this absolute risk aversion measure (A(x)) and the size of the risk (σ_z^2):

$$\rho(x) \approx \frac{1}{2} A(x) \sigma_z^2$$

The mean-variance approach

This approach is a subset of the expected utility approach. It is adapted to financial decision making and restricts the functional form of the utility function used in the analysis.

An alternative to the expected utility approach in applied economic analysis is the mean-variance (MV) approach (Meyer, 2014). It was developed only slightly later than the expected utility approach in the late 1950s and mostly based on works by Markowitz and Tobin (Markowitz, 1952, 1959; Tobin, 1958).

The major assumption of the MV approach is that agent's preferences over random variables can be represented by a utility or another ranking function that depends only on the mean (μ) and standard deviation (σ) of the possible outcomes.

A common form for an MV utility function is $V(\sigma, \mu) = \mu - \lambda \cdot \sigma^2$. For this particular form, $\lambda > 0$ characterizes the decision maker as risk averse, i.e. lower variance is preferred to higher variance.

The MV approach is particularly well suited to portfolio selection problems where portfolio returns could be sufficiently well described by two quantities: mean return and its variance.

The probability distribution of a real-valued random variable represented by a density function and its shape can be described a set of numbers, in statistics called moments. For example, the zeroth moment is the total probability (i.e. one, an area under the curve), the first moment is the mean, the second central moment is the variance, the third central moment is the skewness, and the fourth central moment (with normalization and shift) is the kurtosis. Higher moments exist but are rarely considered in applied analysis.

While variance measures how widespread or concentrated the distribution is around its mean, the skewness measures the lop-sidedness of the distribution; any symmetric distribution, including a normal distribution, will have a third central moment, if defined, equal to zero.

In a similar way to the concept of skewness, kurtosis is a descriptor of the shape of a probability distribution with respect to the tails of the distribution. Higher kurtosis is the result of infrequent extreme deviations (or outliers), as opposed to frequent modestly sized deviations. The kurtosis of any univariate normal distribution is 3. Distributions with kurtosis greater than 3 produce more and more extreme outliers, i.e. have higher probabilities of extreme events, than does the normal distribution.

There are a number of weaknesses in the MV approach (Meyer, 2014). First, by design all alternatives with the same mean and variance are automatically ranked the same. Therefore, it is not possible to account for alternatives with various measures of skewness and kurtosis. For example, positively skewed outcomes are ranked the same as ones which are negatively skewed as long as the mean and variance of the two are the same. For example, consider two payoff alternatives. Alternative A: earn −1 with probability .9999 and 9999 with probability .0001. Alternative B: obtaining −9999 with probability .0001 and 1 with probability .9999. Both have the same mean value, 0, and the same variance, 9999, and hence are ranked the same by the MV approach making decision makers indifferent between A and B. However, in reality a probability of a huge loss in case B makes majority of decision makers to prefer A to B. Another example shows that indifference curves in the MV approach are not consistent with preferring higher outcomes (Meyer, 2014, p. 103).

The expected utility approach is significantly more flexible and by the choice of the utility function can account for individuals who exhibit preference for, or aversion to, skewness, or those who focus intensely on the worst outcomes, or those who always prefer larger payoffs.

The expected utility and MV approaches yield equivalent results when the utility function is quadratic. Additionally, it could be shown (Freund, 1956) that when a random variable is normally distributed an exponential utility, which is a utility function with constant absolute risk aversion (CARA), would yield same results as the MV approach.

Knoke (2008) provides examples of the mean-variance, stochastic dominance and information-gap approaches applied in forestry context.

A recent study of the forest planning by means of the Markowitz mean-value (M-V) portfolio model (Dragicevic, Lobianco and Leblois, 2016) considered three different productivity measures (wood production, carbon sequestration and the market value of the wood) and their respective variances in a mixed forest, in ninety French administrative departments. By weighting the forest productivity with factors of future climate change effects the study found that unlike maximizing wood productivity or carbon sequestration, which leads to similar portfolios, maximizing the economic value of wood production decreases both the levels of wood production and carbon sequestration.

The stochastic dominance criterion

Sometimes one may have good information on probability of different returns for various forest configurations and less information on specific form of the agents' utility function. Stochastic dominance approach lets one to rank various forest configurations with a minimal assumptions on the utility function, for example, we only need to agree that higher returns are preferred to lower returns and for the same returns a less risky one is preferred.

Risk aversion, which could be calculated for a particular utility function, characterises an individual's attitude to risk and was considered above to be the second constituent part of the expected utility calculation. The expectation itself is measured by the probability distribution of a random variable over potential outcomes. Probability distributions are often characterised using a cumulative distribution function (CDF).

The CDF of a real-valued random variable X (F_X), or just distribution function of X, evaluated at x, is the probability that X will take a value less than or equal to x.

Equation 4 Cumulative distribution function (CDF)

$$F_X(x) = P(X \leq x) = \int_{-\infty}^{x} f_X(t)dt$$

Here f_X is the probability density function of a continuous random variable *X*.

One interesting and important question is what are the necessary and sufficient conditions for one random variable or CDF to be preferred or considered equivalent to another by all decision makers with specific risk preferences? This issue could be addressed within a general concept of stochastic dominance of first, second and other degrees (Levy, 2015). Early contributions on first and second degree stochastic dominance conditions and their use to define one random variable stochastically dominating another were made during the 1960s (Hadar and Russell, 1969; Hanoch and Levy, 1969).

Consider two stochastic variables defined by their respective CDFs, F(x) and G(x).

Definition of first degree stochastic dominance (FSD) (Meyer, 2014):

F(x) dominates G(x) in the first degree if G(x) ≥ F(x) for all x.

The implications in an expected utility decision model comes from a theorem that indicates that all decision makers who prefer larger outcomes, that is, those for whom u′(x) ≥ 0, also consider that F(x) dominates G(x), with FSD capturing their preference both for larger outcomes and for higher likelihood of the larger outcomes.

FSD yields a partial order over CDFs. It only provides a ranking for random variables whose CDFs do not cross.

Second degree stochastic dominance (SSD) is related to the concept of an increase in risk developed by Rothschild and Stiglitz (Rothschild and Stiglitz, 1970, 1971). They gave a few definitions of what it means for one random variable to be riskier than another. Consider two stochastic variables, x and y, defined by their respective CDFs, F(x) and G(x). For example, y can be equal to x with some added noise. One can define y as riskier than x if switching from x to y reduces expected utility for all risk averse persons: $\int_a^b u(x)dF(x) \geq \int_a^b u(x)dG(x)$ for all concave u(x).

Definition of second degree stochastic dominance (SSD):

F(x) dominates G(x) in the second degree if $\int_a^s [G(x) - F(x)]dx \geq 0$ for all s in [a, b] and $\int_a^b [G(x) - F(x)]dx = 0$.

A relevant theorem links this with the expected utility decision model, it assumes u′(x) ≥ 0 and u′′(x) ≤ 0 for all x. One random variable can dominate another in the second degree because it is larger, less risky, or as a result of a combination of both. When F(x) dominates G(x) in SSD, the mean of F (μ_F) is at least as large as the mean of G (μ_G), that is $\mu_F \geq \mu_G$.

Knoke (2008) provides examples of the stochastic dominance approach applied in a forestry context. The study analysed portfolio returns of tree species utilising existing financial data on Norway spruce (*Picea abies*) and European beech (*Fagus sylvatica*) using Monte-Carlo simulations for risks of wind damage, snow breakage, insect attacks and timber price fluctuations. Although in all cases pure forests did not outperform mixed forests, SSD was not able to rank consistently all mixtures. Nevertheless, SSD showed that mixed forests with 20% to 30% of Norway spruce were dominating pure stands and some other mixtures. That is, for any level of financial return, a pure forest was dominated by a mixed forest of 80% European beech and 20% Norway spruce.

The information-gap approach, minimax and robust optimisation

Three approaches, the information-gap approach, minimax principle and robust optimisation, are closely linked and motivated by the need to deal with severe uncertainty.

Info-gap decision theory (IGDT) is a "non-probabilistic decision theory that seeks to optimize robustness to failure under severe uncertainty" (https://en.wikipedia.org/wiki/Info-gap_decision_theory, accessed 26 Sep. 17). It has been developed since the 1980s by Yakov Ben-Haim (Ben-Haim, 2005, 2006, 2010).

IGDT works by using 3 linked models (described further below). It starts with a model for the situation, where some parameter or parameters are unknown (Uncertainty model). It then takes an estimate for the parameter, which is assumed to be substantially wrong, and analyses how sensitive the outcomes in the model are to the error in this estimate (Robustness model). A decision is taken that optimises robustness (Decision-making model).

i) Uncertainty model: Starting from the estimate, the uncertainty model measures how distant other values of the parameter are from the estimate. As uncertainty increases, the set of possible values increase.

ii) Robustness model: Given the uncertainty model and a minimum level of desired outcome, the robustness model quantifies the maximum level of uncertainty consistent with achieving this minimum level of outcome. (This is called the robustness of the decision.)

iii) Decision-making model: To decide, one optimizes robustness on the basis of the robustness model. Thus, for a desired minimum outcome, the choice which is most robust (can stand the most uncertainty) and still give the desired outcome is selected (the robust-satisficing action). (There is an alternative approach based on the *opportuneness* of the decision, which is not considered here.)

IGDT came under some serious criticism very early on (Sniedovich, 2007, 2010) with the major critical point being that it is in the main not different from Wald's maximin model (also called minimax criterion, see below). As for robustness in IGDT it is well served by a well-established robust optimisation technique (Ben-Tal and Nemirovski, 2002; Ben-

Tal, Ghaoui and Nemirovski, 2009) and is not different from a classic stability radius model, which appeared in the 1960s (Wilf, 1960) and became popular in in control theory and optimization in 1980s (Hinrichsen and Pritchard, 1986; Zlobec, 1988) and is an instance of Wald's maximin model (Sniedovich, 2010).

Minimax principle is a decision rule for minimizing the possible loss for a worst case (maximum loss) scenario. When dealing with gains, it is referred to as "maximin" - to maximize the minimum gain. In decision theory it is also known as Wald's maximin model (Wald, 1945). It is a non-probabilistic decision-making model according to which decisions are ranked on the basis of their worst-case outcomes – the optimal decision is one with the least worst outcome. It is one of the most important models in robust decision making in general and robust optimization in particular.

Often the Maximin model is described as a game between two players: the Decision Maker (DM) and Nature. DM controls the decision variable, Nature controls the state variable. Let X be the decision space, S(x) be the state space associated with decision x and *f(x, s)* be the reward/payoff function associated with a particular decision and state. The game proceeds in the following steps.

Step 1: DM selects a decision $x \in X$ aiming to maximize his reward.

Step 2: Nature selects the worst state in S(x), call it *s(x)*, aiming to minimize the payoff awarded to DM. Note that Nature knows the decision taken by DM.

Step 3: A payoff *f(x, s(x))* is awarded to DM.

In its classic form the mathematical formulation of this game is given by:

Equation 5 Maximin model - Classic format

$$\max_{x \in X} \min_{s \in S(x)} f(x,s)$$

There is an equivalent mathematical programming (MP) format:

Equation 6 Maximin model - MP format

$$\max_{x \in X, v \in R} \{v : v \leq f(x,s), \forall s \in S(x)\}$$

where *R* denotes the real line.

Given an optimal solution (x^*, v^*) the worst payoff v^* associated with decision x^* is then: $v^* = f(x^*, v^*) = min_{s \in S(x^*)} f(x^*, s)$.

The conversion to MP format is based on the fact that for any function *g(y)*:

$$\min_{y \in Y} g(y) \equiv \max_{v \in R} \{v : v \leq g(y), \forall y \in Y\}$$

Where the ≡ sign indicates that these models are equivalent.

The MP format allows for easy inclusion of constraints. For example, if one needs to impose the following constraint: $g(x, s) \in G(x), \forall s \in S(x)$, the Maximin model in the MP becomes:

Equation 7 Maximin model with constraint - MP format

$$\max_{x \in X, v \in R} \{v: v \leq f(x,s), g(x,s) \in G(x), \forall s \in S(x)\}$$

Robust optimisation (RO) is an approach with its origins in the 1970s when it was originally developed in the field of robust control. In more recent years (late 1990s and 2000s) it has been developed as an approach to optimization under uncertainty, in which the uncertainty is not stochastic, but rather deterministic and set-based (Bertsimas, Brown and Caramanis, 2011) (and the references therein). Its modern presentation is by (Ben-Tal and Nemirovski, 2002; Ben-Tal, Ghaoui and Nemirovski, 2009). RO considers optimization problems for which inputs are not precise but have a given level of uncertainty, and the optimization problem's constraints must not be violated for all possible values of the data contained within the uncertainty intervals (Hildebrandt and Knoke, 2011).

(Palma and Nelson, 2009) is one example of the application of robust optimisation in forestry to schedule harvest decisions when the timber yield and demands for wood products are uncertain.

Other recent examples are (Palma and Nelson, 2010; Kašpar *et al.*, 2017; Messerer, Pretzsch and Knoke, 2017; Sanei Bajgiran, Kazemi Zanjani and Nourelfath, 2017; Uhde *et al.*, 2017).

Major disadvantages of all 3 methods (IGDT, minimax and robust optimisation) are the relatively limited track record of their application in forestry and their inherent mathematical and computational complexities.

The stochastic dynamic programming

The stochastic dynamic programming approach is based on deterministic dynamic programming but with model state variables (e.g. prices, interest rates or growth rates) described by stochastic processes.

Dynamic programming (DP) is particularly well suited to deal with sequential problems (of which forest management is one) and allows for incorporation of uncertainty associated with the long-term nature of forestry, for example concerning future prices, interest rates and changing yields as the climate changes. DP is well developed for applications in both discrete and continuous time settings. It was developed in the 1950s by Bellman and others (Bellman, 1954, 1957) and applied initially in engineering (http://en.wikipedia.org/wiki/Bellman_equation, accessed 26 Sep. 17). The DP method breaks the whole sequence of decisions into smaller sub-problems with just two components. Richard Bellman's "Principle of Optimality" suggests how to do this:

Principle of Optimality: *An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.* (Bellman, 1957)

The problem is split into the immediate decision and a value function that embeds the consequences of all subsequent decisions, starting with the position that results from the immediate decision (Dixit and Pindyck, 1994, Chapter 4). The value function is the best possible value of the objective being optimised. For a stochastic environment when the processes one models are non-deterministic and random the DP approach becomes stochastic dynamic programming (SDP). In SDP a conditional expectation of a value function is used in the Bellman equation. In continuous time, the resulting partial differential equation is often called a Hamilton–Jacobi–Bellman (HJB) equation (http://en.wikipedia.org/wiki/Hamilton-Jacobi-Bellman_equation, accessed 26 Sep. 17). Its solution yields the value function which is the optimal value of the objective for a given problem and from which an optimal control can be derived. The HJB equation is usually solved backwards in time and except for special cases requires a numerical treatment (Judd, 1998; Miranda and Fackler, 2002). The DP approach has become much more widely accepted and its application in economics has increased since the late 1980s when a number of examples demonstrated how to employ DP to economic issues (Stokey, Lucas and Prescott, 1989).

A great variety of stochastic processes are possible for SDP modelling. Their general functional form in stochastic differential equations is given by:

$dX_t = \mu(X_t, t)dt + \sigma(X_t, t)dW_t$, where X_t is the continuous time stochastic process, dW_t is a Wiener process (Standard Brownian motion).

Over a small time interval δ X_t changes its value by an amount that is normally distributed with expectation $\mu(X_t, t)\,\delta$ and variance $\sigma(X_t, t)^2\,\delta$ and is independent of the past behaviour of the process. The stochastic process X_t is called a diffusion process, and satisfies the Markov property (i.e. its future and past states are independent, only present state matters for future evolution). Any functional form is allowed for the mean (μ) and the variance (σ).

Some notable examples of stochastic processes used in applied SDP modelling are: Geometric Brownian motion and the Ornstein–Uhlenbeck process.

Geometric Brownian motion ($dX_t = \mu\, X_t\, dt + \sigma\, X_t\, dW_t$) is a continuous-time stochastic process in which the logarithm of the randomly varying quantity follows a Brownian motion with drift, which itself is a continuous-time version of a random walk with drift. It is frequently used to model securities prices, interest rates, output prices and wages.

This is the equation for the dynamics of the price of a stock in the Black–Scholes options pricing model of financial mathematics.

The Ornstein–Uhlenbeck process (named after Leonard Ornstein and George Eugene Uhlenbeck) describes the velocity of a massive Brownian particle under the influence of friction ($dX_t = \theta (\mu - X_t) dt + \sigma X_t dW_t$). As a result of friction over time, the process tends to drift towards its long-term mean and is called mean-reverting. The process can be considered to be a modification of the random walk in continuous time in which there is a tendency of the walk to move back towards a central location, with a greater attraction when the process is further away from the centre. The Ornstein–Uhlenbeck process can also be considered as the continuous-time analogue of the discrete-time AR(1) process (an auto-regressive process with one lag). The Ornstein–Uhlenbeck process is also used to model interest rates, currency exchange rates, and commodity prices stochastically. The parameter μ represents the equilibrium or mean value supported by fundamentals; σ the degree of volatility around it caused by shocks, and θ the rate by which these shocks dissipate and the variable reverts towards the mean.

A major problem with the DP approach (and others based on DP like the option approach described below) is the 'curse of dimensionality': the cost of computing agents' expectations over all possible future states increases exponentially in the number of state variables. This makes large-scale realistic DP problems nearly impossible to solve. This called for a new solution approach 'Approximate Dynamic Programming' (Powell, 2007). The ADP approach uniquely integrates four distinct disciplines-Markov design processes, mathematical programming, simulation, and statistics to allow for solutions to DP with a large number of states.

The option approach

The option approach originates in finance. Its key feature is the recognition of the fact that under risk and uncertainty the flexibility (i.e. option) in timing of decisions has value.

Another method used in forest management and investment appraisal is the real option approach (or for short, 'option' approach). The most recent review of its application in forestry is (Chaudhari, Kane and Wetzstein, 2016). The approach is based on the theory of financial options valuation and is relatively new for forestry with the majority of early applications dating to 1990s (Hildebrandt and Knoke, 2011). Its relevance stems from the nature of investment decisions in forestry. Most investment decisions in forestry have three important characteristics:

1. The investment is partially or completely irreversible, i.e. the initial cost is at least partially irrecoverable.
2. There is uncertainty over the future return.

3 There is some flexibility in timing of significant investment decisions (planting, thinning, harvesting). One can postpone action to get more information about the future.

The ability to delay some irreversible investment actions is akin to a financial call option that gives the right to buy an underlying asset at a certain price in a certain period and offers managerial flexibility (e.g. options to wait/delay, to abandon, to change the amount invested, etc.,), which has a value that can be evaluated. In general, the option value increases with the size of the sunk cost and with the level of uncertainty over the future (Dixit and Pindyck, 1994).

Option pricing yields a new and useful view of uncertainty. In particular, it demonstrates the economic value of flexibility in the decision making process in an uncertain environment (Hildebrandt and Knoke, 2011).

Two common techniques to value real options are DP and contingent claim (CC) approaches (Insley and Wirjanto, 2010). The major drawback of the DP approach is its use of an exogenous constant discount rate which reflects the opportunity cost of capital for investments of similar risk. It implies that the risky investment project under consideration has a constant volatility over its lifetime (Insley and Wirjanto, 2010). This is too restrictive and is likely to introduce bias into the valuation.

The contingent claims approach originated in 1970s in finance. It assumes that a sufficiently rich set of markets in risky assets exist that allow for exact replication of the risky component of the project under consideration. Therefore, a riskless portfolio can be assembled that consists of a risky project and assets following the project's uncertainty. In equilibrium with no arbitrage opportunities, this portfolio must earn the risk free rate of interest, which allows the value of the risky project to be determined. The no-arbitrage assumption makes it unnecessary to invoke some risk adjusted discount rate as used in the DP approach. However, if a portion of the return from holding the risky asset is due to an unobservable convenience yield, it is still necessary to estimate either that convenience yield or a market price of risk, which is often problematic (Insley and Wirjanto, 2010). The disadvantages of the CC approach are the need for estimating the convenience yield and the assumption that a sufficiently rich set of markets in risky assets exists to replicate exactly the risky component of the project. Both of these requirements are not necessary for DP. Notice that both DP and CC approaches to option valuation lead ultimately to a partial differential equation that needs to be solved numerically, except in special cases.

Empirical results (Insley and Wirjanto, 2010, pp.: 170-174) show that in general the DP and CC approaches to real option valuation can produce significantly different results. This was the case for the value of bare land (or LEV) and implied risk adjusted discount rates. However, critical harvesting prices (i.e. prices when delaying harvesting by one more period does not bring more value and hence it is optimal to harvest) were quite close (within 2%) for both approaches.

A number of studies (Duku-Kaakyire and Nanang, 2004; Stainback and Alavalapati, 2004; Chladna, 2007; Insley and Lei, 2007; Daigneault, Miranda and Sohngen, 2010; Couture and Reynaud, 2011) investigate risk and forest management using DP and an Option approach.

Markov Decision Process (MDP)

MDP is a mathematical framework for modeling decisions under uncertainty.

In a Markov decision process (MDP), an agent chooses action a_t at time t based on observing state s_t. The agent then receives a reward r_t. The state evolves probabilistically based only on the current state and action taken by the agent. The assumption that the next state does not depend on any previous state or action, i.e. history, is the Markov assumption. The core focus of MDPs is to identify an optimal "policy" for the decision maker: a function *π(s)* that the decision maker will choose when in state *s*. Note that once a Markov decision process is combined with a policy in this way, this fixes the action for each state and the resulting combination behaves like a Markov chain. The goal is to choose a policy *π* that will maximize some cumulative function of the random rewards, typically the expected discounted sum over a potentially infinite horizon (https://en.wikipedia.org/wiki/Markov_decision_process, accessed 26 Sep. 17).

An important link exists between MDP and dynamic programming: solutions to MDP problems could be obtained by solving a corresponding Bellman equation from DP.

Examples of recent applications in forestry include (Forsell *et al.*, 2011; Zhou and Buongiorno, 2011; Buongiorno and Zhou, 2015; Zhou, 2015; Couture, Cros and Sabbadin, 2016; Buongiorno, Zhou and Johnston, 2017; Johnston and Withey, 2017).

In an interesting application, uncertainty in climate policy was translated into a limited number of scenarios regarding the timing and magnitude of policy regime switches. This model was then incorporated into an MDP model of forest management, which accounted for multiple forms of risk and uncertainty affecting forest functioning and management (Zhou, 2015).

(Buongiorno and Zhou, 2015) is a further example of how MDP can offer a rigorous and practical way of developing optimum management strategies based upon a combination of ecological and economic objectives, including diversity of tree species and size, landscape diversity, old growth preservation, and carbon sequestration.

A MDP approach can also be used to analyse multi-objective forest management, such as for timber and carbon, in the presence of risk (Johnston and Withey, 2017) and the influence of the attitude to risk of decision makers in managing mixed forests (Buongiorno, Zhou and Johnston, 2017).

A less favourable aspect of MDP models is the high data requirements (e.g. for prices and their variation, for tree growth and its variation, etc.), including historic time-series, which are required to produce believable transition probabilities between various states. Another issue is the high computational burden, as the number of transition probabilities grows exponentially with a number of system states. Although of itself it is less of an issue with increasing computer power, computational complexity makes working out why particular results were obtained nearly impossible. MDP models are in essence black-box models where it is impossible to understand why an output is as it is. The number of system states determines how finely one can partition the system with respect to the variables of interest (e.g. tree age classes, number of tree species, timber price levels, etc.).

The simulation approach (Monte-Carlo and Markov Chain Monte-Carlo)

The estimation of probability distributions is a crucial first step for many risk approaches and Monte-Carlo simulation (MCS) is one of the most widely applied techniques used to derive these. MCS is a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results (Kroese *et al.*, 2014). Monte Carlo methods are mainly used in three distinct problem classes: optimization, numerical integration, and generating draws from a probability distribution (https://en.wikipedia.org/wiki/Monte_Carlo_method, accessed 26 Sep. 17). Random fluctuations in uncertain input parameters (e.g. timber prices and their volatility, including interaction/correlation for prices for different species in mixed forests) are simulated and the results of a large number of repeated simulations used to derive the relevant probability distribution functions (e.g. for financial returns) (Hildebrandt and Knoke, 2011).

When the probability distribution of a variable can be parametrized, a Markov chain Monte Carlo (MCMC) sampler is often used. MCMC is a technique used to solve the problem of sampling from a complicated distribution. The central idea is to design a judicious Markov chain model with a prescribed stationary probability distribution. That is, in the limit, the samples being generated by the MCMC method will be samples from the desired (target) distribution. MCMC is applied in estimations of multi-dimensional integrals in Bayesian statistics, including Bayesian networks and hierarchical models.

A Markov chain is a Markov process. This is a stochastic process that satisfies the Markov property that one can make predictions of the future of the process based solely on its present state as precisely as one could if the process's full history were known. Predictions are derived independently from such history (i.e. conditional only on the present state of the system), as its future and past states are independent. Modern references on Monte-Carlo are (Brooks *et al.*, 2011; Kroese, Taimre and Botev, 2011).

(Knoke and Wurm, 2006) is an example of an application in forestry of MCS, where it is used for optimal portfolio selection. Market (timber price fluctuation) and natural hazard (insects, snow and wind) risks as well as their correlation were considered in an evaluation of forestry management strategies for mixed forests.

(Roessiger, Griess and Knoke, 2011) explore with a help of MCS whether clear-felling and mono-species forests are optimal in the presence of risk focusing upon VaR (value at risk). The study showed that 'near-natural' selective harvesting in a mixed forest (42% Norway spruce and 58% European beech) is the optimal choice, particularly for cautious, and thus risk-avoiding small forest owners who do not have the opportunity to diversify risks in ways that are available to owners of large-scale forest properties. MCS was used to simulate the annualised NPV of various management alternatives, timber prices fluctuations and natural hazards.

Researching old-growth boreal stands harvesting in Quebec (Canada) (Moore *et al.*, 2012) used MCS to compare the long-term profitability (or financial returns measured as NPV) of selection felling with that of a clear-felling approach.

Another study (Liénard and Strigul, 2016) investigated the consequences of global warming scenarios in Quebec forests using an inhomogeneous Markov chain model. The model predicts changes in the fire rate in Quebec hardwood forests as well as possible growth enhancements due to increasing CO2 and temperature.

Other recent examples of MCS applications in forestry include (Hyytiäinen and Haight, 2010; Kallio, 2010; Conedera *et al.*, 2011; Daniel *et al.*, 2017).

A less favourable aspect of the MCS approach is that, like MDP models, there are high data input and computational requirements.

The scenarios approach, sensitivity analysis, and multicriteria decision analysis (MCDA)

Scenario analysis is a relatively simple way to account for uncertainty and is widely used. In this approach a user generates a set of different scenarios for the process of interest (e.g. tree growth rates, prices, costs, catastrophic events, risk preferences, etc). Often these realisations of the processes over time can be directly used in the objective function for the problem under investigation. The most common approach is to produce 'Central', 'Low' and 'High' scenarios to cover the range of uncertainty, where deemed sufficient for the problem at hand.

Examples of recent applications in forestry are (Olsson, 2007; Sacchelli, Fagarazzi and Bernetti, 2013; Seidl and Lexer, 2013; de-Miguel *et al.*, 2014; Hynynen *et al.*, 2015; Holmström *et al.*, 2016; Heinonen *et al.*, 2017; Temperli *et al.*, 2017).

The impact of climate change on forest productivity and as a result on harvested volume and net present value (NPV) is investigated in (Zell and Hanewinkel, 2015) using scenario analysis and a simulation over the period 2010-2500, and a climate change scenario based on the A1B emissions with a HeadCM3 model chain. The results show that storm frequency has a major impact on all output variables, followed by forest management treatment. Compared with storm frequency and treatment, change in precipitation and temperature is less influential. There is a clear negative climate change effect on harvest levels for the spruce and mixed stand, while Douglas fir shows a distinct positive reaction.

Sensitivity analysis is a common way to consider uncertainty by changing input variables and evaluating the effects on target variables. It explores the stability of the solution to the uncertainty in input parameters.

Multicriteria decision analysis (MCDA) developed to help decision makers choose between actions that require reaching a compromise among objectives with different weights (Malczewski, 2006; Zavadskas and Turskis, 2011; Kangas *et al.*, 2015; Kaliszewski, Miroforidis and Podkopaev, 2016) has been adapted to produce a multicriteria risk analysis (MCRA) approach. MCRA has been used in relation to forest health in Europe to compare the risk of damage to silvicultural systems of different management intensity (Jactel *et al.*, 2012). The study found two cases of low overall risk and one case of high risk. The first low risk case was in short-rotation forests for biomass production, and was due to reduced susceptibility of stands to the majority of hazards. The second low risk case was at the opposite end of the management intensity gradient, in close-to-nature systems, and was due to a lower stand value being exposed to damage. The high risk case was associated with intensive even-aged forestry, irrespective of tree species and bioclimatic zone.

Bayesian statistics

Bayesian inference or learning (BL) is an area of statistics where the probability of a hypothesis is updated according to Bayes' theorem when new evidence becomes available. Bayesian learning is a very attractive mechanism to model how some agents (e.g. forest and /or land owners) update their beliefs about climate change as more and more knowledge becomes available about the impacts on forest ecology. A Bayesian approach was also mentioned in a number of papers referenced here as potentially a promising tool for modelling the impact of uncertainty on decision making (Yousefpour *et al.*, 2012, 2013, 2014). Other recent references on the Bayesian approach are (Grêt-Regamey, Brunner, Juerg Altwegg, *et al.*, 2013; Grêt-Regamey, Brunner, Jürg Altwegg, *et al.*, 2013; Yousefpour *et al.*, 2015; Liénard and Strigul, 2016; Reyer *et al.*, 2016). A number of these references (Grêt-Regamey, Brunner, Juerg Altwegg, *et al.*, 2013; Grêt-Regamey, Brunner, Jürg Altwegg, *et al.*, 2013) show that a Bayesian approach works well with GIS and spatial mapping which illustrates its potential versatility in analysing forestry decisions.

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